# CMAT Newsletter: December 2005

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## Contents

# 1 General Remarks

Some bugs are fixed - again. Among others, some related to the glim function applied to models with a string type response variable.
The glim function now also computes the logistic regression model for multinomial scaled response variables.
The univar function now also works when the data contain missing values. Since univar computes univariate statistics for each column, the length of the columns (number of observations) is adjusted for missing values.
The functions rspfil and wspfil have been renamed to rspfile resp. wspfile. There are a few more options now for the wspfile function which is now able to write data sets in the following formats:

1. in triplet coordinate form $(ir, ic, val)$ for a nonzero value in row $ir$ and column $ic$.
2. in row oriented Harwell-Boeing form:
   (a) the cumulative numbers of nozeros in rows
   (b) the $N_nonzero$ column numbers of nonzeros
   (c) the $N_nonzero$ values of nonzeros
3. in column oriented Harwell-Boeing form:
   (a) the cumulative numbers of nozeros in columns
   (b) the $N_nonzero$ row numbers of nonzeros
   (c) the $N_nonzero$ values of nonzeros

See the example in $cmat\backslash tana\backslash alan\backslash alan1.inp$:

```
/* iknd: sparse coordinate triplets */
optn = [ -1 ,   /* no response */
          0 ,   /* ytyp, not valid */
          1 ,   /* coordinate output */
          1 ];  /* index base for Fortran */
wspfile(jobs12,"spj12_1.txt"," ",optn);


/* Harwell-Boeing: row oriented */
optn = [ -1 ,   /* no response */
          0 ,   /* ytyp, not valid */
          2 ,   /* rowwise Harwell-Boeing output */
          1 ];  /* index base for Fortran */
wspfile(jobs12,"spj12_2.txt"," ",optn);
```

```
/* Harwell-Boeing: column oriented */
optn = [ -1 ,   /* no response */
          0 ,   /* ytyp, not valid */
          3 ,   /* columnwise Harwell-Boeing output */
          1 ];  /* index base for Fortran */
wspfile(jobs12,"spj12_3.txt"," ",optn);
```

### 1.1 New Functions

The following new functions are implemented:

**anacor** for simple correspondence analysis. (See GIFI project.)

**anaprof** for (multiple) correspondence analysis of profile data. (See GIFI project.)

**cndcov** various algorithms for computing the conditional covariance matrix and the conditional mean, especially efficient for ARMA time series processes. See Monahan's paper in JSS 2005.

**princals** for principal component analysis of categorical data. (See GIFI project.)

### 1.2 Fixed Bugs

1. A number of bugs in the `glim` function were fixed.

## 2 Modifications of Features

### 2.1 Batch Processing

Executing an input script `foo.inp` with CMAT generates two files, `foo.log` and `foo.txt`. In the past, eventually existing files with those names were deleted. Now, existing files are renamed into `foo.blg` and `foo.bxt` overriding eventually existing files with such names. This modification makes it much easier for me to compare two sets of output benchmarks for changes after some development cycle.

### 2.2 Extensions to `glim()` Function

The logistic model was extended for multinomial response. The following methods were added:

| Option | Second Column | Meaning |
|---|---|---|
| "yord" | | ordinal scaling of response |
| "ynom" | | nominal scaling of response |

The following example is that of PROC LOGISTIC. The results here are different from those of PROC LOGISTIC, since the `glim` function applies another design for the categorical predictors.

```
school = [ 1 "regular"   "self"  10 ,  1 "regular"   "team"  17 ,
           1 "regular"   "class" 26 ,  1 "afternoon" "self"   5 ,
           1 "afternoon" "team"  12 ,  1 "afternoon" "class" 50 ,
           2 "regular"   "self"  21 ,  2 "regular"   "team"  17 ,
           2 "regular"   "class" 26 ,  2 "afternoon" "self"  16 ,
           2 "afternoon" "team"  12 ,  2 "afternoon" "class" 36 ,
           3 "regular"   "self"  15 ,  3 "regular"   "team"  15 ,
           3 "regular"   "class" 16 ,  3 "afternoon" "self"  12 ,
           3 "afternoon" "team"  12 ,  3 "afternoon" "class" 20 ];


cnam = [ "School" "Program" "Style" "Count" ];
school = cname(school,cnam);
print "Data=", school;
nr = nrow(school); nc = ncol(school); print "nr,nc=", nr,nc;


clas = [ 1 2 3 ];
modl = "3 =  1 2 ";
optn = [ "print"           5 ,
         "link"       "logit" ,
         "dist"       "binom" ,
         "ynom"               ,
         "freq"            4 ,
         "seed"          123 ,
         "tech"      "trureg" ];
gof = glim(school,modl,optn,clas);
```

```
                       *****************
                       Model Information
                       *****************

                 Number Valid Observations   18
                 Response Variable         Y[3]
                 N Independend Variables      2
                 Error Distribution   MULTINOML
                 Link Function            LOGIT

                 Frequ. Variable Column       4
                 Significance Level:  0.0500000
                 Design Coding:       Full-Rank
                 Hessian Matx. for Optimization
```

```
Hessian Matx. for Covariance M
No Variable Selection Process

*************
Model Effects
*************

Intercept + C1 + C2

**********************
Class Level Information
**********************

Class  Level    Value

 Y[3]      3  class self  team
 C[1]      3        1       2       3
 C[2]      2  afternoon regular

*********************
Parameter Information
*********************

Parameter | Meaning
----------------------------------------
        1 | Intercept
        2 | School
        3 | School
No Param. | School
        5 | Program
No Param. | Program

************************************************
Sums of Weights and Frequencies for Class Levels
************************************************
           Total Frequencies = 338

Variable       Value    Nobs  Proportion      Freq
Y[3]           class       6   33.333333       174
               self        6   33.333333        79
               team        6   33.333333        85

C[1]               1       6   33.333333       120
```

| | | | | |
|---|---|---|---|---|
| | 2 | 6 | 33.333333 | 128 |
| | 3 | 6 | 33.333333 | 90 |
| C[2] | afternoon | 9 | 50.000000 | 175 |
| | regular | 9 | 50.000000 | 163 |

********************
Goodness of Model Fit
********************

| | | | |
|---|---|---|---|
| Log Likelihood | -333.46694 | Degrees of Freedom | 14 |
| Deviance | . | Pearson ChiSquare | . |
| SSE | 375.00000 | MSE = SSE/nobs | 20.833333 |
| AIC (Intercept) | 699.40431 | AIC (All Param.) | 682.93388 |
| SBC (Intercept) | 707.05040 | SBC (All Param.) | 713.51824 |
| -2logL (Intercept) | 695.40431 | -2logL (All Param.) | 666.93388 |
| -2logL (ChiSqu.) | 28.470430 | Pvalue (df= 6) | 0.0000766 |
| Score ChiSqu. Test | 27.119014 | Pvalue (df= 6) | 0.0001376 |
| Wald Test | 25.588515 | Pvalue (df= 6) | 0.0002656 |

*****************************************
Analysis of Effects and Parameter Estimates
*****************************************

| Parameter | DF | Estimate | Std_Error | WaldChiSq | Pr>ChiSq |
|---|---|---|---|---|---|
| Response Level 1 | | | | | |
| Intercept | 1 | 1.308827 | 0.259607 | 25.417505 | 0.000000 |
| School | 1 | -0.180107 | 0.317197 | 0.3224071 | 0.570165 |
| School | 1 | -0.655596 | 0.339472 | 3.7296166 | 0.053456 |
| School | 0 | 0.000000 | . | . | . |
| Program | 1 | -0.742632 | 0.270573 | 7.5331801 | 0.006057 |
| Program | 0 | 0.000000 | . | . | . |
| Response Level 2 | | | | | |
| Intercept | 1 | -0.661928 | 0.364426 | 3.2991588 | 0.069315 |
| School | 1 | 0.902717 | 0.403433 | 5.0067908 | 0.025248 |
| School | 1 | 0.659116 | 0.418685 | 2.4782735 | 0.115429 |
| School | 0 | 0.000000 | . | . | . |
| Program | 1 | 4.8e-003 | 0.318917 | 2.27e-004 | 0.987971 |
| Program | 0 | 0.000000 | . | . | . |

Covariance Matrix and ASE Based on Hessian Matrix

```
              ******************************
              Confidence Limits of Parameters
              ******************************
```

| Parameter | Estimate | LowWaldCL | UppWaldCL |
|---|---|---|---|
| Response Level 1 | | | |
| Intercept | 1.3088271 | 0.80000740 | 1.81764678 |
| School | -0.1801073 | -0.80180207 | 0.44158740 |
| School | -0.6555962 | -1.32094967 | 0.00975726 |
| School | 0.0000000 | . | . |
| Program | -0.7426318 | -1.27294500 | -0.21231857 |
| Program | 0.0000000 | . | . |
| Response Level 2 | | | |
| Intercept | -0.6619277 | -1.37618876 | 0.05233346 |
| School | 0.9027170 | 0.11200197 | 1.69343199 |
| School | 0.6591161 | -0.16149123 | 1.47972335 |
| School | 0.0000000 | . | . |
| Program | 0.0048082 | -0.62025859 | 0.62987499 |
| Program | 0.0000000 | . | . |

Wald Confidence Intervals Based on Hessian Matrix

```
           ************************************
           Odds Ratio and Standardized Estimates
           ************************************
```

| Parameter | OddsRatio | StndEst | LowerCL | UpperCL |
|---|---|---|---|---|
| Response Level 1 | | | | |
| School | 0.8351806 | . | 0.44851997 | 1.55517394 |
| School | 0.5191325 | . | 0.26688173 | 1.00980502 |
| School | . | . | . | . |
| Program | 0.4758599 | . | 0.28000579 | 0.80870703 |
| Program | . | . | . | . |
| Response Level 2 | | | | |
| School | 2.4662949 | . | 1.11851507 | 5.43811224 |
| School | 1.9330829 | . | 0.85087399 | 4.39173056 |
| School | . | . | . | . |

```
Program          1.0048198     .      0.53780535  1.87737587
Program              .         .          .            .
```

```
               ******************************
               Evaluation of Training Data Fit
               ******************************
```

| Index | Value | StdErr |
|---|---|---|
| Absolute Classification Error | 165 | . |
| Classification Accuracy | 51.18343195 | . |
| Goodman-Kruskal Lambda C\|R | 0.000000000 | 0.000000000 |
| Goodman-Kruskal Lambda SYM | 0.000000000 | 0.000000000 |

Classification Table

| Observed | Predicted class | self | team | TotSum | OffSum |
|---|---|---|---|---|---|
| class | 158 | 16 | 0 | 174 | 16 |
| self | 64 | 15 | 0 | 79 | 64 |
| team | 70 | 15 | 0 | 85 | 85 |
| TotSum | 292 | 46 | 0 | 338 | |
| OffSum | 134 | 31 | 0 | | 165 |

Scoring for the first level only:

| Nobs | Yobs | Ypred | Level1 | LowerCL | UpperCL |
|---|---|---|---|---|---|
| 1 | 1 | 0.00 | 0.537076 | 0.427241 | 0.646911 |
| 2 | 2 | 0.00 | 0.537076 | 0.427241 | 0.646911 |
| 3 | 0 | 0.00 | 0.537076 | 0.427241 | 0.646911 |
| 4 | 1 | 0.00 | 0.709477 | 0.619530 | 0.799424 |
| 5 | 2 | 0.00 | 0.709477 | 0.619530 | 0.799424 |
| 6 | 0 | 0.00 | 0.709477 | 0.619530 | 0.799424 |
| 7 | 1 | 0.00 | 0.392366 | 0.292573 | 0.492159 |
| 8 | 2 | 0.00 | 0.392366 | 0.292573 | 0.492159 |
| 9 | 0 | 0.00 | 0.392366 | 0.292573 | 0.492159 |
| 10 | 1 | 0.00 | 0.576384 | 0.474877 | 0.677892 |
| 11 | 2 | 0.00 | 0.576384 | 0.474877 | 0.677892 |
| 12 | 0 | 0.00 | 0.576384 | 0.474877 | 0.677892 |
| 13 | 1 | 1.00 | 0.313556 | 0.208552 | 0.418560 |

| | | | | | |
|---|---|---|---|---|---|
| 14 | 2 | 1.00 | 0.313556 | 0.208552 | 0.418560 |
| 15 | 0 | 1.00 | 0.313556 | 0.208552 | 0.418560 |
| 16 | 1 | 0.00 | 0.490373 | 0.371111 | 0.609636 |
| 17 | 2 | 0.00 | 0.490373 | 0.371111 | 0.609636 |
| 18 | 0 | 0.00 | 0.490373 | 0.371111 | 0.609636 |

********************************
LR Statistics for Type 1 Analysis
********************************

| Effect | Deviance | DF | ChiSquare | Pr>Chi |
|---|---|---|---|---|
| Intercept | . | 0 | . | . |
| School | . | 1 | 17.3764248 | 0.0000 |
| Program | . | 2 | 11.0940053 | 0.0039 |

********************************
LR Statistics for Type 3 Analysis
********************************

| Effect | DF | ChiSquare | Pr>Chi |
|---|---|---|---|
| School | 4 | 16.3853834 | 0.0025 |
| Program | 2 | 11.0940053 | 0.0039 |

We also applied the model for the three level response Iris data. However, it seems that model and data are quasi separable and the Hessian matrix is at the optimal solution higly rank deficient. That means there are many optimal parameter sets with the same value of the objective function (log likelihood). The optimal parameter sets are different for different starting values of the optimization.

## 2.3 Extensions to `svd()` Function

The function now has the following syntax:

<sval, v, u> = svd(a<,"eco"|p<optn>>)

The last newsletter contained the extension of the `svd()` function for alternative regression method estimation. The following methods were added:

| Option | Second Column | Meaning |
|---|---|---|
| `"meth"` | string | computational method |
| | ”l_2” | least squares ($L_2$) estimation |
| | ”l_1” | least absolute value (LAV, $L_1$) estimation |
| | ”l_i” | Chebyshev regression, $L_\infty$ estimation |
| | ”l_p” | general $L_p$ norm regression, where $p \geq 1$ |
| | ”odi” | linear orthogonal distance regression, total least squares (TLS), errors-in-variables regression (EIV) |
| | ”hub” | robust (Huber) regression where *large* residuals enter the objective function with smaller weights than small residuals $r_i$. |
| | ”lms” | resistant least median squares regression |
| | ”lts” | resistant least trimmed squares regression |

Now some additional least squares estimation methods where added for large sparse matrces which were already implemented in the `svdtrip()` function, which also required option for specifying related method control parameters:

| Option | Second Column | Meaning |
|---|---|---|
| `"meth"` | string | computational method |
| | ”arp” | Arnoldi estimation as in Arpack |
| | ”bls” | Block Lanczos Estimation |
| | ”las” | Single Lanczos Estimation |
| | ”sis” | Rutishauser-Ritz Subspace Iteration |
| | ”tms” | Trace Minimization Method |

This extension makes the `svdtrip()` function obsolete

< s,v,u > = svdtrip(a,”meth”<,optn>)

when called with a matrix argument. However, the use of `svdtri()` with a function argument

< s,v,u > = svdtrip(funa,”meth”<,optn<,funata>>)

is still not implemented in the `svd()` function.

# 3 New Developments

## 3.1 Function `anacor`

---

< scor,vars,rprm,cprm,sprm > = anacor(a,optn)

**Purpose:** The `anacor` function is part of the early Gifi suite of programs for data analysis of data of mixed type, i.e. categorical (ordinal and nominal) and numeric data. The `anacor` function performs *correspondence analysis* The analysis is done for an integer interval of $[1 \leq dim1 \leq dim2 \leq MIN(nr, nc)]$ the number of factors.

There are a number of methods implemented for computing the singular value decomposition of a $nr \times nc$ matrix. Among those are methods for sparse and very large data matrices using (block) Lanczos and Arnoldi iteration. The latter are useful only if a small number of the largest singular values and corresponding eigenvectors are computed. That would be possible by specifying a small value for the `dim2` option except for the following problem. When the covariance matrix of singular values is requested by specifying the `conf` option, it is necessary to compute the full nonzero singular value decomposition which for non-rank-deficient data matrices would be of dimension $\min(nr, nc)$. That could be much larger than the specification of `dim2` and too much for the sparse iterative methods.

**Input:** **a** This argument specifies a $nr \times nc$ data matrix which usually is a frequency table. But it may also be a matrix of similarity, dissimilarity or distance matrices.

**optn** This argument must be specified in form of a two column matrix where the first column defines the option as string value (in quotes) and the second column can be used for a numeric or string specification of the option. See table below for content.

| Option | Second Column | Meaning |
|---|---|---|
| "conf" | | compute variance matrices |
| "dim1" | int | lower bound for number of factors |
| "dim2" | int | upper bound for number of factors |
| "perm" | | perform permutation of rows and columns |
| "print" | int | amount of printed output |
| "pvars" | | printed covariances and correlations |
| "pperm" | | print permuted objects |
| "plottran" | int | plots of transformations<br>=0: no plot, =1: only rows, =2: only cols,<br>=3: separate for rows and cols<br>=4: combined for rows and cols |
| "plot2dim" | int | plots 2 dimensional graphs<br>=0: no plot, =1: only rows, =2: only cols,<br>=3: separate for rows and cols<br>=4: combined for rows and cols |
| "plotscor" | | plot of row versus column scores |
| "scale" | string | "rows" : normalization in centre of gravity of rows<br>"cols" : normalization in centre of gravity of columns<br>"symm" : symmetric normalization<br>"both" : symmetric normalization<br>"none" : no scaling is done |

**Output:** The following are the output objects:

**scores** This is a $(1 + nrow + ncol) \times ndim$ super matrix consisting of one row vector of eigenvalues, a $nrow \times ndim$ matrix of row scores and a $ncol \times ndim$ matrix of column scores stacked vertically. Here $ndim = nfac - ffac + 1$ where $ffac$ is the first factor and $nfac$ is the last factor in the analysis.

**vars** This is a $(1 + nrow + ncol) \times nd2$ super matrix consisting of one row vector of the $nd2$ lower diagonal entries of a covariance matrix of eigenvalues, a $nrow \times nd2$ matrix of covariances of row scores and a $ncol \times nd2$ matrix of covariances of column scores.

**rprm** $rprm[nrow, 2]$ the indices and values of permutet row marginals

**cprm** $cprm[ncol, 2]$ the indices and values of permutet column marginals

**sprm** $data[ihlp1[nrow], ihlp2[ncol]]$ the permutet input data

**Restrictions:** 1. String data are not permitted.

2. Missing values are not permitted.

**Relationships:** svd(), pca(), anaprof(), princals(), homals()

**Examples:** 1. Occupational Mobility Data: Father and Son, p.278:

```
a = [ 50  19   26    8   18    6    2 ,
      16  40   34   18   31    8    3 ,
      12  35   65   66  123   23   21 ,
      11  20   58  110  223   64   32 ,
      14  36  114  185  714  258  189 ,
       0   6   19   40  179  143   71 ,
       0   3   14   32  141   91  106 ];
 cnam = [ "S_PROF" "S_EXEC" "S_HSUP" "S_LSUP" "S_SKIL" "S_SEMI" "S_UNSK" ];
 rnam = [ "F_PROF" "F_EXEC" "F_HSUP" "F_LSUP" "F_SKIL" "F_SEMI" "F_UNSK" ];
 a = cname(a,cnam);
 a = rname(a,rnam);

 optn = [ "dim1"         1 ,
          "dim2"         3 ,
          "cross"   "cols" ,
          "print"        3 ,
          "pvars"        2 ,
          "conf"           ];
 < scor,vars > = anacor(a,optn);
 print "Scores=", scor;
 print "Variances=", vars;

          *********************************************
          Largest Singular Values and Principal Inertia
               Total Chi-Square=1361.74 with df=36
          *********************************************

                 Singular             Principal       Chi-
             N      Value Difference    Inertia     Square
             1 0.52562901            0.27628586 966.171648
             2 0.26743383 0.25819518 0.07152085 250.108421
             3 0.16522054 0.10221329 0.02729783 95.4604961
               Total Number of Observations: 3497

              *************************************
              Scree Plot of Largest Singular Values
              *************************************

    N      Value +-----------+-----------+-----------+-----------+
    1 0.52562901 ************************************************
    2 0.26743383 ************************
    3 0.16522054 ***************

Row Marginals
```

-------------

```
  1 :       129      150      345      518     1510
  6 :       458      387
```

Column Marginals
----------------

```
  1 :       103      159      330      459     1429
  6 :       593      424
```

Row Scores

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| F_PROF | 4.020749242 | 2.870021203 | -1.268857969 |
| F_EXEC | 2.112958308 | -1.638985702 | 3.262263390 |
| F_HSUP | 0.660239034 | -1.474110778 | 0.164074913 |
| F_LSUP | 0.063348219 | -0.865061220 | -1.145624907 |
| F_SKIL | -0.329793820 | -0.017029461 | -0.378195441 |
| F_SEMI | -0.667769471 | 0.865303324 | 0.673379792 |
| F_UNSK | -0.755530122 | 1.192999004 | 1.224388108 |

Column Scores

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| S_PROF | 2.318906605 | 0.872173531 | -0.263829081 |
| S_EXEC | 1.051208263 | -0.451360821 | 0.523967077 |
| S_HSUP | 0.491237150 | -0.290588858 | 0.027171643 |
| S_LSUP | 0.019267373 | -0.281812566 | -0.153530422 |
| S_SKIL | -0.159777482 | -0.043687427 | -0.093674078 |
| S_SEMI | -0.318824849 | 0.240720842 | 0.099623765 |
| S_UNSK | -0.376312078 | 0.299199675 | 0.189034945 |

Variance Matrix of Singular Values

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 0.000531781 | 0.000288890 | 9.2923e-005 |
| Fac_2 | 0.000288890 | 0.000559585 | 0.000231625 |
| Fac_3 | 9.2923e-005 | 0.000231625 | 0.000513074 |

Variances of Row Scores

| F_PROF | F_EXEC | F_HSUP | F_LSUP |
|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| Fac_1_1 | 0.081855425 | 0.080990406 | 0.022569559 | 0.007268491 |
| Fac_2_1 | -0.034780567 | 0.110919643 | 0.007254362 | -0.002832254 |
| Fac_2_2 | 0.144387092 | 0.545791802 | 0.037002910 | 0.053630097 |
| Fac_3_1 | 0.079018758 | 0.018983600 | 0.016630991 | 0.003630225 |
| Fac_3_2 | 0.016963214 | 0.093088772 | -0.026521934 | -0.029067647 |
| Fac_3_3 | 0.363222001 | 0.165449461 | 0.308073690 | 0.072894994 |

Variances of Row Scores

| | F_SKIL | F_SEMI | F_UNSK |
|---|---|---|---|
| Fac_1_1 | 0.000787315 | 0.004416815 | 0.006216498 |
| Fac_2_1 | -0.000391839 | -0.003233458 | -0.005645256 |
| Fac_2_2 | 0.009638588 | 0.030568316 | 0.053017549 |
| Fac_3_1 | -0.000207274 | 0.000901578 | 0.001431948 |
| Fac_3_2 | 0.003046257 | 0.007925926 | -0.001317746 |
| Fac_3_3 | 0.019293234 | 0.125423623 | 0.127266529 |

Variances of Column Scores

| | S_PROF | S_EXEC | S_HSUP | S_LSUP |
|---|---|---|---|---|
| Fac_1_1 | 0.048749413 | 0.017929778 | 0.005576864 | 0.002056258 |
| Fac_2_1 | -0.000129509 | 0.012817314 | 0.002803126 | -0.000488593 |
| Fac_2_2 | 0.016319385 | 0.044702679 | 0.006077402 | 0.002860795 |
| Fac_3_1 | 0.011159106 | 0.002563618 | 0.000776686 | 0.000651190 |
| Fac_3_2 | 0.004875261 | -0.001326060 | -0.001119747 | -0.001594215 |
| Fac_3_3 | 0.011140736 | 0.007967263 | 0.006598786 | 0.004769579 |

Variances of Column Scores

| | S_SKIL | S_SEMI | S_UNSK |
|---|---|---|---|
| Fac_1_1 | 0.000332529 | 0.000927055 | 0.001092366 |
| Fac_2_1 | -0.000168611 | -6.0235e-005 | -0.000371870 |
| Fac_2_2 | 0.000955102 | 0.001615517 | 0.002957538 |
| Fac_3_1 | -1.7951e-005 | -1.8472e-005 | -4.3788e-005 |
| Fac_3_2 | -5.7768e-005 | 0.000103467 | 7.5581e-005 |
| Fac_3_3 | 0.000538649 | 0.003422213 | 0.004070712 |

Correlation Matrix of Singular Values

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | 0.529581701 | 0.177896976 |
| Fac_2 | 0.529581701 | 1.000000000 | 0.432277121 |
| Fac_3 | 0.177896976 | 0.432277121 | 1.000000000 |

Correlations of Row Scores : Row=1

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | -0.319925416 | 0.458268908 |
| Fac_2 | -0.319925416 | 1.000000000 | 0.074072652 |
| Fac_3 | 0.458268908 | 0.074072652 | 1.000000000 |

Correlations of Row Scores : Row=2

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | 0.527567667 | 0.163994434 |
| Fac_2 | 0.527567667 | 1.000000000 | 0.309778390 |
| Fac_3 | 0.163994434 | 0.309778390 | 1.000000000 |

Correlations of Row Scores : Row=3

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | 0.251026606 | 0.199447816 |
| Fac_2 | 0.251026606 | 1.000000000 | -0.248404781 |
| Fac_3 | 0.199447816 | -0.248404781 | 1.000000000 |

Correlations of Row Scores : Row=4

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | -0.143451745 | 0.157711210 |
| Fac_2 | -0.143451745 | 1.000000000 | -0.464896995 |
| Fac_3 | 0.157711210 | -0.464896995 | 1.000000000 |

Correlations of Row Scores : Row=5

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | -0.142241498 | -0.053182312 |
| Fac_2 | -0.142241498 | 1.000000000 | 0.223386717 |
| Fac_3 | -0.053182312 | 0.223386717 | 1.000000000 |

Correlations of Row Scores : Row=6

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | -0.278276681 | 0.038305334 |
| Fac_2 | -0.278276681 | 1.000000000 | 0.128004309 |
| Fac_3 | 0.038305334 | 0.128004309 | 1.000000000 |

Correlations of Row Scores : Row=7

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | -0.310957446 | 0.050909306 |
| Fac_2 | -0.310957446 | 1.000000000 | -0.016042234 |
| Fac_3 | 0.050909306 | -0.016042234 | 1.000000000 |

Correlations of Column Scores : Column=1

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | -0.004591590 | 0.478837070 |
| Fac_2 | -0.004591590 | 1.000000000 | 0.361567204 |
| Fac_3 | 0.478837070 | 0.361567204 | 1.000000000 |

Correlations of Column Scores : Column=2

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | 0.452733656 | 0.214492104 |
| Fac_2 | 0.452733656 | 1.000000000 | -0.070265451 |
| Fac_3 | 0.214492104 | -0.070265451 | 1.000000000 |

Correlations of Column Scores : Column=3

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | 0.481491477 | 0.128032008 |
| Fac_2 | 0.481491477 | 1.000000000 | -0.176818959 |
| Fac_3 | 0.128032008 | -0.176818959 | 1.000000000 |

Correlations of Column Scores : Column=4

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | -0.201449189 | 0.207935472 |
| Fac_2 | -0.201449189 | 1.000000000 | -0.431582144 |
| Fac_3 | 0.207935472 | -0.431582144 | 1.000000000 |

Correlations of Column Scores : Column=5

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | -0.299190132 | -0.042416260 |
| Fac_2 | -0.299190132 | 1.000000000 | -0.080540222 |
| Fac_3 | -0.042416260 | -0.080540222 | 1.000000000 |

Correlations of Column Scores : Column=6

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | -0.049219837 | -0.010370531 |
| Fac_2 | -0.049219837 | 1.000000000 | 0.044003859 |
| Fac_3 | -0.010370531 | 0.044003859 | 1.000000000 |

Correlations of Column Scores : Column=7

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | -0.206891358 | -0.020765314 |
| Fac_2 | -0.206891358 | 1.000000000 | 0.021782838 |
| Fac_3 | -0.020765314 | 0.021782838 | 1.000000000 |

The following is the *scores* output object:

Scores=

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| EValue | 0.52563 | 0.26743 | 0.16522 |
| F_PROF | 4.02075 | 2.87002 | -1.26886 |
| F_EXEC | 2.11296 | -1.63899 | 3.26226 |
| F_HSUP | 0.66024 | -1.47411 | 0.16407 |
| F_LSUP | 0.06335 | -0.86506 | -1.14562 |
| F_SKIL | -0.32979 | -0.01703 | -0.37820 |
| F_SEMI | -0.66777 | 0.86530 | 0.67338 |
| F_UNSK | -0.75553 | 1.19300 | 1.22439 |
| S_PROF | 2.31891 | 0.87217 | -0.26383 |
| S_EXEC | 1.05121 | -0.45136 | 0.52397 |
| S_HSUP | 0.49124 | -0.29059 | 0.02717 |
| S_LSUP | 0.01927 | -0.28181 | -0.15353 |
| S_SKIL | -0.15978 | -0.04369 | -0.09367 |
| S_SEMI | -0.31882 | 0.24072 | 0.09962 |
| S_UNSK | -0.37631 | 0.29920 | 0.18903 |

The following is the *variances* output object:

Variances=

| | Fac_1_1 | Fac_2_1 | Fac_2_2 |
|---|---|---|---|
| EValue | 0.00053 | 0.00029 | 0.00056 |
| F_PROF | 0.08186 | -0.03478 | 0.14439 |
| F_EXEC | 0.08099 | 0.11092 | 0.54579 |
| F_HSUP | 0.02257 | 0.00725 | 0.03700 |
| F_LSUP | 0.00727 | -0.00283 | 0.05363 |

```
  F_SKIL |   0.00079  -0.00039   0.00964
  F_SEMI |   0.00442  -0.00323   0.03057
  F_UNSK |   0.00622  -0.00565   0.05302
  S_PROF |   0.04875  -0.00013   0.01632
  S_EXEC |   0.01793   0.01282   0.04470
  S_HSUP |   0.00558   0.00280   0.00608
  S_LSUP |   0.00206  -0.00049   0.00286
  S_SKIL |   0.00033  -0.00017   0.00096
  S_SEMI |   0.00093  -0.00006   0.00162
  S_UNSK |   0.00109  -0.00037   0.00296

         |   Fac_3_1   Fac_3_2   Fac_3_3
----------------------------------------
  EValue |   0.00009   0.00023   0.00051
  F_PROF |   0.07902   0.01696   0.36322
  F_EXEC |   0.01898   0.09309   0.16545
  F_HSUP |   0.01663  -0.02652   0.30807
  F_LSUP |   0.00363  -0.02907   0.07289
  F_SKIL |  -0.00021   0.00305   0.01929
  F_SEMI |   0.00090   0.00793   0.12542
  F_UNSK |   0.00143  -0.00132   0.12727
  S_PROF |   0.01116   0.00488   0.01114
  S_EXEC |   0.00256  -0.00133   0.00797
  S_HSUP |   0.00078  -0.00112   0.00660
  S_LSUP |   0.00065  -0.00159   0.00477
  S_SKIL |  -0.00002  -0.00006   0.00054
  S_SEMI |  -0.00002   0.00010   0.00342
  S_UNSK |  -0.00004   0.00008   0.00407
```

2. Number of PhD degrees awarded in US during 1973-1978

```
a = [  4489  4303  4402  4350  4266  4361 ,
       4101  3800  3749  3572  3410  3234 ,
       3354  3286  3344  3278  3137  3008 ,
       2444  2587  2749  2878  2960  3049 ,
       3338  3144  2959  2791  2641  2432 ,
       1222  1196  1149  1003   959   959 ];
cnam = [ "Life Sci" "Physical S" "Social Sci"
         "Behav Sci" "Engineer"  "Mathemat" ];
rnam = [ "1973" "1974" "1975" "1976" "1977" "1978" ];
```

```
         ********************************************
         Largest Singular Values and Principal Inertia
              Total Chi-Square=383.856 with df=25
```

```
          ********************************************

            Singular                       Principal        Chi-
        N      Value Difference Proportion    Inertia     Square
        1 0.05845076            0.73654776 0.00341649 368.653059
        2 0.00860761 0.04984314 0.10846599 7.409e-005 7.99471913
        3 0.00693996 0.00166766 0.08745157 4.816e-005 5.19698254
        4 0.00414269 0.00279726 0.05220279 1.716e-005 1.85183960
        5 0.00121670 0.00292599 0.01533188 1.480e-006 0.15973756
              Total Number of Observations: 107904

              ************************************
              Scree Plot of Largest Singular Values
              ************************************

     N      Value +-----------+-----------+-----------+-----------+
     1 0.05845076 ***********************************************
     2 0.00860761 *******
     3 0.00693996 *****
     4 0.00414269 ***
     5 0.00121670

Row Marginals
-------------

  1 :   2.617e+004  2.187e+004  1.941e+004  1.667e+004  1.731e+004
  6 :        6488

Column Marginals
----------------

  1 :   1.895e+004  1.832e+004  1.835e+004  1.787e+004  1.737e+004
  6 :   1.704e+004

                          Row Scores

                     Fac_1        Fac_2        Fac_3
          Row_1  -0.025812509  0.008097054 -0.007139298
          Row_2   0.041272687 -0.002420026 -0.005901793
          Row_3  -0.001351685 -0.011412977  0.005962506
          Row_4  -0.110005522 -0.001299364  0.004216297
          Row_5   0.070379158 -0.003670965  0.000752890
          Row_6   0.063941656  0.022762407  0.018014032
```

Column Scores

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Col_1 | 0.084027199 | 0.003251780 | -0.010816818 |
| Col_2 | 0.050893110 | 0.002938972 | 0.009951913 |
| Col_3 | 0.014822865 | 0.000792730 | 0.007083209 |
| Col_4 | -0.024241440 | -0.012925692 | -0.002323323 |
| Col_5 | -0.051248913 | -0.008190145 | -0.001102286 |
| Col_6 | -0.086413449 | 0.014275787 | -0.002736653 |

Variance Matrix of Singular Values

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 9.1788e-006 | -2.3009e-008 | -1.0325e-007 |
| Fac_2 | -2.3009e-008 | 9.0575e-006 | -1.9789e-009 |
| Fac_3 | -1.0325e-007 | -1.9789e-009 | 9.4513e-006 |

Variances of Row Scores

| | Row_1 | Row_2 | Row_3 | Row_4 |
|---|---|---|---|---|
| Fac_1_1 | 2.9523e-005 | 3.5671e-005 | 4.3725e-005 | 4.2086e-005 |
| Fac_2_1 | 5.8502e-007 | 1.7927e-006 | 1.0999e-007 | -2.8792e-006 |
| Fac_2_2 | 0.000152770 | 0.000115494 | 0.000140415 | 7.3833e-005 |
| Fac_3_1 | -1.8103e-006 | 1.1717e-006 | 3.3108e-007 | 5.1439e-006 |
| Fac_3_2 | 0.000101410 | -2.4103e-005 | 0.000117830 | 9.7454e-006 |
| Fac_3_3 | 9.3647e-005 | 4.1314e-005 | 0.000203886 | 4.1488e-005 |

Variances of Row Scores

| | Row_5 | Row_6 |
|---|---|---|
| Fac_1_1 | 4.5451e-005 | 0.000149109 |
| Fac_2_1 | 3.9161e-006 | -9.9159e-006 |
| Fac_2_2 | 6.3591e-005 | 0.000882679 |
| Fac_3_1 | -1.9677e-006 | -1.1287e-005 |
| Fac_3_2 | 5.2293e-006 | -0.000694830 |
| Fac_3_3 | 0.000111087 | 0.000619277 |

Variances of Column Scores

| | Col_1 | Col_2 | Col_3 | Col_4 |
|---|---|---|---|---|
| Fac_1_1 | 3.7411e-005 | 4.3691e-005 | 4.5520e-005 | 4.8257e-005 |
| Fac_2_1 | -2.3803e-006 | -5.2554e-007 | 6.1720e-008 | -2.8709e-006 |
| Fac_2_2 | 0.000307861 | 0.000294571 | 0.000194654 | 5.5801e-005 |

| | | | | |
|---|---|---|---|---|
| Fac_3_1 | 8.2949e-006 | -2.5794e-006 | -5.3900e-007 | -4.6212e-007 |
| Fac_3_2 | 6.3420e-005 | -4.7472e-005 | -8.1249e-006 | -5.2023e-005 |
| Fac_3_3 | 3.0906e-005 | 7.3600e-005 | 0.000106845 | 0.000215670 |

Variances of Column Scores

| | Col_5 | Col_6 |
|---|---|---|
| Fac_1_1 | 4.8542e-005 | 4.7897e-005 |
| Fac_2_1 | -3.1625e-006 | 9.9639e-006 |
| Fac_2_2 | 5.4315e-005 | 4.1902e-005 |
| Fac_3_1 | -1.2592e-006 | -5.5501e-007 |
| Fac_3_2 | -1.6414e-005 | 6.9501e-005 |
| Fac_3_3 | 0.000133775 | 0.000229174 |

Correlation Matrix of Singular Values

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | -0.002523435 | -0.011085204 |
| Fac_2 | -0.002523435 | 1.000000000 | -0.000213880 |
| Fac_3 | -0.011085204 | -0.000213880 | 1.000000000 |

Correlations of Row Scores : Row=1

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | 0.008711087 | -0.034428520 |
| Fac_2 | 0.008711087 | 1.000000000 | 0.847838471 |
| Fac_3 | -0.034428520 | 0.847838471 | 1.000000000 |

Correlations of Row Scores : Row=2

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | 0.027930432 | 0.030522364 |
| Fac_2 | 0.027930432 | 1.000000000 | -0.348930846 |
| Fac_3 | 0.030522364 | -0.348930846 | 1.000000000 |

Correlations of Row Scores : Row=3

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | 0.001403709 | 0.003506471 |
| Fac_2 | 0.001403709 | 1.000000000 | 0.696393560 |
| Fac_3 | 0.003506471 | 0.696393560 | 1.000000000 |

Correlations of Row Scores : Row=4

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | -0.051651287 | 0.123100887 |
| Fac_2 | -0.051651287 | 1.000000000 | 0.176081257 |
| Fac_3 | 0.123100887 | 0.176081257 | 1.000000000 |

Correlations of Row Scores : Row=5

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | 0.072843224 | -0.027691696 |
| Fac_2 | 0.072843224 | 1.000000000 | 0.062217004 |
| Fac_3 | -0.027691696 | 0.062217004 | 1.000000000 |

Correlations of Row Scores : Row=6

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | -0.027332605 | -0.037142186 |
| Fac_2 | -0.027332605 | 1.000000000 | -0.939798229 |
| Fac_3 | -0.037142186 | -0.939798229 | 1.000000000 |

Correlations of Column Scores : Column=1

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | -0.022180079 | 0.243941925 |
| Fac_2 | -0.022180079 | 1.000000000 | 0.650164254 |
| Fac_3 | 0.243941925 | 0.650164254 | 1.000000000 |

Correlations of Column Scores : Column=2

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | -0.004632540 | -0.045486991 |
| Fac_2 | -0.004632540 | 1.000000000 | -0.322403846 |
| Fac_3 | -0.045486991 | -0.322403846 | 1.000000000 |

Correlations of Column Scores : Column=3

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | 0.000655679 | -0.007728794 |
| Fac_2 | 0.000655679 | 1.000000000 | -0.056339059 |
| Fac_3 | -0.007728794 | -0.056339059 | 1.000000000 |

Correlations of Column Scores : Column=4

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | -0.055324976 | -0.004529843 |
| Fac_2 | -0.055324976 | 1.000000000 | -0.474218407 |
| Fac_3 | -0.004529843 | -0.474218407 | 1.000000000 |

Correlations of Column Scores : Column=5

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | -0.061589719 | -0.015625675 |
| Fac_2 | -0.061589719 | 1.000000000 | -0.192564561 |
| Fac_3 | -0.015625675 | -0.192564561 | 1.000000000 |

Correlations of Column Scores : Column=6

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | 0.222409791 | -0.005297446 |
| Fac_2 | 0.222409791 | 1.000000000 | 0.709233323 |
| Fac_3 | -0.005297446 | 0.709233323 | 1.000000000 |

Scores=

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| EValue | 0.05845 | 0.00861 | 0.00694 |
| Row_1 | -0.02581 | 0.00810 | -0.00714 |
| Row_2 | 0.04127 | -0.00242 | -0.00590 |
| Row_3 | -0.00135 | -0.01141 | 0.00596 |
| Row_4 | -0.11001 | -0.00130 | 0.00422 |
| Row_5 | 0.07038 | -0.00367 | 0.00075 |
| Row_6 | 0.06394 | 0.02276 | 0.01801 |
| Col_1 | 0.08403 | 0.00325 | -0.01082 |
| Col_2 | 0.05089 | 0.00294 | 0.00995 |
| Col_3 | 0.01482 | 0.00079 | 0.00708 |
| Col_4 | -0.02424 | -0.01293 | -0.00232 |
| Col_5 | -0.05125 | -0.00819 | -0.00110 |
| Col_6 | -0.08641 | 0.01428 | -0.00274 |

Variances=

| | Fac_1_1 | Fac_2_1 | Fac_2_2 |
|---|---|---|---|
| EValue | 0.000009 | -2.3e-008 | 0.000009 |
| Row_1 | 0.000030 | 0.000001 | 0.000153 |

```
Row_2 |   0.000036   0.000002   0.000115
Row_3 |   0.000044   0.000000   0.000140
Row_4 |   0.000042  -0.000003   0.000074
Row_5 |   0.000045   0.000004   0.000064
Row_6 |   0.000149  -0.000010   0.000883
Col_1 |   0.000037  -0.000002   0.000308
Col_2 |   0.000044  -0.000001   0.000295
Col_3 |   0.000046   6.2e-008   0.000195
Col_4 |   0.000048  -0.000003   0.000056
Col_5 |   0.000049  -0.000003   0.000054
Col_6 |   0.000048   0.000010   0.000042
```

## 3.2 Function `anaprof`

< scor,vars > = anaprof(a,optn)

**Purpose:** The `anaprof` function is part of the early Gifi suite of programs for data analysis of data of mixed type, i.e. categorical (ordinal and nominal) and numeric data. The `anaprof` function performs *correspondence analysis* of profile data. The analysis is done for an integer interval $[1 \leq dim1 \leq dim2 \leq MIN(nr, nc)]$ of the number of factors. A *profile* is a row vector of binary variables indicating either the occurence or non-occurence of a specific feature.

There are a number of methods implemented for computing the singular value decomposition of a $nr \times nc$ matrix. Among those are methods for sparse and very large data matrices using (block) Lanczos and Arnoldi iteration. The latter are useful only if a small number of the largest singular values and corresponding eigenvectors are computed. That would be possible by specifying a small value for the `dim2` option except for the following problem. When the covariance matrix of singular values is requested by specifying the `conf` option, it is necessary to compute the full nonzero singular value decomposition which for non-rank-deficient data matrices would be of dimension $\min(nr, nc)$. That could be much larger than the specification of `dim2` and too much for the sparse iterative methods.

**Input:** **a** This argument specifies a $nr \times nc$ data matrix which is either a profile matrix with an additional frequency column or a single frequency vector.

**optn** This argument must be specified in form of a two column matrix where the first column defines the option as string value (in quotes) and the second column can be used for a numeric or string specification of the option. See table below for content.

If `a` is only a $N$ frequency vector a complete profile matrix is assumed with $n$ variables defined as $N = 2^n$, i.e. the number of input frequencies must be a power of two.

| Option | Second Column | Meaning |
|---:|---:|---|
| `"conf"` | | compute variance matrices |
| `"dim1"` | int | lower bound for number of factors |
| `"dim2"` | int | upper bound for number of factors |
| `"freq"` | int | column number of frequency in input data |
| `"no0fr"` | int | skip profiles with zero frequency |
| `"orig"` | | implementation like in the original Fortran code |
| `"print"` | int | amount of printed output |
| `"pvars"` | | printed covariances and correlations |
| `"pperm"` | | print permuted objects |
| `"plottran"` | int | plots of transformations<br>=0: no plot, =1: only rows, =2: only cols,<br>=3: separate for rows and cols<br>=4: combined for rows and cols |
| `"plot2dim"` | int | plots 2 dimensional graphs<br>=0: no plot, =1: only rows, =2: only cols,<br>=3: separate for rows and cols<br>=4: combined for rows and cols |
| `"plotscor"` | | plot of row versus column scores |
| `"scale"` | string | shifting the singular value weight to left and right score matrix<br>(only if `orig` is not specified)<br>"rows" : normalization in centre of gravity of rows<br>"cols" : normalization in centre of gravity of columns<br>"symm" : symmetric normalization<br>"both" : symmetric normalization<br>"none" : no scaling is done |

Note, that only when specifying the `orig` option will yield the same results as the original `ANAPROF` Fortran code in the Gifi package. The differences have to do with the obvious mistake to substitute eigenvalues for the singular values in the original program (whereas the eigenvalues are the squared singular values). Also, for the `orig` option the `scale` option is not ignored and symmetric normalization

**Output:** The following are the output objects:

**scores** This is a $(1 + nrow + ncol) \times ndim$ super matrix consisting of one row vector of eigenvalues, a $nrow \times ndim$ matrix of row scores and a $ncol \times ndim$ matrix of column scores stacked vertically. Here $ndim = nfac - ffac + 1$ where $ffac$ is the first factor and $nfac$ is the last factor in the analysis.

**vars** This is a $(1 + nrow + ncol) \times nd2$ super matrix consisting of one row vector of the $nd2$ lower diagonal entries of a covariance matrix of eigenvalues, a $nrow \times nd2$ matrix of covariances of row scores and a $ncol \times nd2$ matrix of covariances of column scores.

**rprm** $rprm[nrow, 2]$ the indices and values of permutet row marginals

**cprm** $cprm[ncol, 2]$ the indices and values of permutet column marginals

**sprm** $data[ihlp1[nrow], ihlp2[ncol]]$ the permutet input data

**Restrictions:** 1. String data are not permitted.

2. Missing values are not permitted.

**Relationships:** svd(), pca(), anacor(), princals(), homals()

**Examples:** 1. Sugiyama Profile Data (GIFI) : nrow=64, ncol=7:

```
a = [ 1 1 1 1 1 1  042 ,  1 1 1 1 1 0  033 ,  1 1 1 1 0 1  006 ,  1 1 1 1 0 0  01
      1 1 1 0 1 1  012 ,  1 1 1 0 1 0  029 ,  1 1 1 0 0 1  008 ,  1 1 1 0 0 0  08
      1 1 0 1 1 1  051 ,  1 1 0 1 1 0  069 ,  1 1 0 1 0 1  020 ,  1 1 0 1 0 0  05
      1 1 0 0 1 1  034 ,  1 1 0 0 1 0  124 ,  1 1 0 0 0 1  027 ,  1 1 0 0 0 0  31
      1 0 1 1 1 1  001 ,  1 0 1 1 1 0  002 ,  1 0 1 1 0 1  000 ,  1 0 1 1 0 0  00
      1 0 1 0 1 1  001 ,  1 0 1 0 1 0  011 ,  1 0 1 0 0 1  007 ,  1 0 1 0 0 0  05
      1 0 0 1 1 1  008 ,  1 0 0 1 1 0  023 ,  1 0 0 1 0 1  007 ,  1 0 0 1 0 0  03
      1 0 0 0 1 1  010 ,  1 0 0 0 1 0  055 ,  1 0 0 0 0 1  013 ,  1 0 0 0 0 0  19
      0 1 1 1 1 1  011 ,  0 1 1 1 1 0  007 ,  0 1 1 1 0 1  002 ,  0 1 1 1 0 0  00
      0 1 1 0 1 1  004 ,  0 1 1 0 1 0  008 ,  0 1 1 0 0 1  004 ,  0 1 1 0 0 0  04
      0 1 0 1 1 1  072 ,  0 1 0 1 1 0  126 ,  0 1 0 1 0 1  045 ,  0 1 0 1 0 0  14
      0 1 0 0 1 1  080 ,  0 1 0 0 1 0  258 ,  0 1 0 0 0 1  137 ,  0 1 0 0 0 0  76
      0 0 1 1 1 1  000 ,  0 0 1 1 1 0  002 ,  0 0 1 1 0 1  000 ,  0 0 1 1 0 0  00
      0 0 1 0 1 1  004 ,  0 0 1 0 1 0  013 ,  0 0 1 0 0 1  006 ,  0 0 1 0 0 0  03
      0 0 0 1 1 1  033 ,  0 0 0 1 1 0  048 ,  0 0 0 1 0 1  038 ,  0 0 0 1 0 0  06
      0 0 0 0 1 1  042 ,  0 0 0 0 1 0  096 ,  0 0 0 0 0 1  090 ,  0 0 0 0 0 0  71

cnam = [ "A" "B" "C" "D" "E" "F" "Freq" ];
a = cname(a,cnam);
```

First, lets use the `orig` option:

```
optn = [ "freq"        7 ,
         "orig"          ,
         "dim1"        1 ,
         "dim2"        3 ,
         "scale"   "symm" ,
         "print"       3 ,
         "pvars"       2 ,
         "conf"          ];
< scor,vars > = anaprof(a,optn);
```

```
print "Scores=", scor;
print "Variances=", vars;
```

```
                ********************************
                Mapping of Categories to Columns
                ********************************

               Column Variable Category        Name
                    1        1        1           0
                    2                 2           1

                    3        2        1           0
                    4                 2           1

                    5        3        1           0
                    6                 2           1

                    7        4        1           0
                    8                 2           1

                    9        5        1           0
                   10                 2           1

                   11        6        1           0
                   12                 2           1


         *********************************************
         Largest Singular Values and Principal Inertia
         *********************************************

     Singular                          Principal       Chi-
N       Value Difference Proportion    Inertia     Square
1 0.26817940            0.26817940 0.07192019 1193563.02
2 0.20273470 0.06544469 0.20273470 0.04110136 969982.344
3 0.15761969 0.04511501 0.15761969 0.02484397 412717.253
4 0.13440816 0.02321153 0.13440816 0.01806555 263846.232
5 0.12007017 0.01433799 0.12007017 0.01441684 130546.044
6 0.11698789 0.00308228 0.11698789 0.01368617 1.104e-022
        Total Number of Observations: 4253


            *************************************
            Scree Plot of Largest Singular Values
            *************************************
```

```
N      Value +-----------+-----------+-----------+-----------+
1 0.26817940 ************************************************
2 0.20273470 ************************************
3 0.15761969 ****************************
4 0.13440816 ************************
5 0.12007017 *********************
6 0.11698789 ********************
```

Row Marginals
-------------

| | | | | | |
|---|---|---|---|---|---|
| 1 : | 252 | 198 | 36 | 102 | 72 |
| 6 : | 174 | 48 | 492 | 306 | 414 |
| 11 : | 120 | 324 | 204 | 744 | 162 |
| 16 : | 1902 | 6 | 12 | 0 | 54 |
| 21 : | 6 | 66 | 42 | 354 | 48 |
| 26 : | 138 | 42 | 210 | 60 | 330 |
| 31 : | 78 | 1164 | 66 | 42 | 12 |
| 36 : | 30 | 24 | 48 | 24 | 264 |
| 41 : | 432 | 756 | 270 | 852 | 480 |
| 46 : | 1548 | 822 | 4560 | 0 | 12 |
| 51 : | 0 | 24 | 24 | 78 | 36 |
| 56 : | 180 | 198 | 288 | 228 | 384 |
| 61 : | 252 | 576 | 540 | 4308 | |

Column Marginals
----------------

| | | | | | |
|---|---|---|---|---|---|
| 1 : | 2893 | 1360 | 1623 | 2630 | 3790 |
| 6 : | 463 | 3277 | 976 | 2944 | 1309 |
| 11 : | 3438 | 815 | | | |

Row Scores

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Row_01 | 3.249282419 | 1.023943278 | 1.163582937 |
| Row_02 | 2.460210062 | 1.884213784 | 0.111517463 |
| Row_03 | 2.348393657 | 1.433502863 | 1.268006540 |
| Row_04 | 1.559321300 | 2.293773369 | 0.215941066 |
| Row_05 | 2.292827108 | 1.533821554 | 0.798727998 |
| Row_06 | 1.503754751 | 2.394092060 | -0.253337477 |
| Row_07 | 1.391938346 | 1.943381139 | 0.903151601 |
| Row_08 | 0.602865989 | 2.803651645 | -0.148913873 |
| Row_09 | 2.493812685 | -0.744139367 | 0.272926777 |

| | | | |
|---|---|---|---|
| Row_10 | 1.704740328 | 0.116131139 | -0.779138697 |
| Row_11 | 1.592923922 | -0.334579782 | 0.377350381 |
| Row_12 | 0.803851565 | 0.525690724 | -0.674715094 |
| Row_13 | 1.537357374 | -0.234261091 | -0.091928162 |
| Row_14 | 0.748285017 | 0.626009415 | -1.143993637 |
| Row_15 | 0.636468611 | 0.175298494 | 0.012495441 |
| Row_16 | -0.152603746 | 1.035569001 | -1.039570033 |
| Row_17 | 2.699062089 | 1.044811599 | 2.985246747 |
| Row_18 | 1.909989732 | 1.905082105 | 1.933181273 |
| Row_19 | 1.798173326 | 1.454371185 | 3.089670351 |
| Row_20 | 1.009100969 | 2.314641691 | 2.037604876 |
| Row_21 | 1.742606778 | 1.554689875 | 2.620391808 |
| Row_22 | 0.953534421 | 2.414960382 | 1.568326333 |
| Row_23 | 0.841718015 | 1.964249461 | 2.724815411 |
| Row_24 | 0.052645658 | 2.824519967 | 1.672749937 |
| Row_25 | 1.943592354 | -0.723271046 | 2.094590588 |
| Row_26 | 1.154519997 | 0.136999460 | 1.042525113 |
| Row_27 | 1.042703591 | -0.313711460 | 2.199014191 |
| Row_28 | 0.253631234 | 0.546559046 | 1.146948717 |
| Row_29 | 0.987137043 | -0.213392770 | 1.729735648 |
| Row_30 | 0.198064686 | 0.646877737 | 0.677670174 |
| Row_31 | 0.086248280 | 0.196166816 | 1.834159252 |
| Row_32 | -0.702824077 | 1.056437322 | 0.782093777 |
| Row_33 | 2.708802328 | -0.193440529 | 1.155119715 |
| Row_34 | 1.919729971 | 0.666829977 | 0.103054241 |
| Row_35 | 1.807913566 | 0.216119057 | 1.259543319 |
| Row_36 | 1.018841209 | 1.076389563 | 0.207477844 |
| Row_37 | 1.752347017 | 0.316437747 | 0.790264776 |
| Row_38 | 0.963274660 | 1.176708254 | -0.261800699 |
| Row_39 | 0.851458255 | 0.725997333 | 0.894688379 |
| Row_40 | 0.062385898 | 1.586267839 | -0.157377095 |
| Row_41 | 1.953332594 | -1.961523174 | 0.264463555 |
| Row_42 | 1.164260237 | -1.101252667 | -0.787601919 |
| Row_43 | 1.052443831 | -1.551963588 | 0.368887159 |
| Row_44 | 0.263371474 | -0.691693082 | -0.683178315 |
| Row_45 | 0.996877283 | -1.451644898 | -0.100391384 |
| Row_46 | 0.207804926 | -0.591374391 | -1.152456858 |
| Row_47 | 0.095988520 | -1.042085312 | 0.004032219 |
| Row_48 | -0.693083837 | -0.181814806 | -1.048033255 |
| Row_49 | 2.158581998 | -0.172572207 | 2.976783525 |
| Row_50 | 1.369509641 | 0.687698299 | 1.924718051 |
| Row_51 | 1.257693235 | 0.236987378 | 3.081207129 |
| Row_52 | 0.468620878 | 1.097257885 | 2.029141654 |
| Row_53 | 1.202126687 | 0.337306069 | 2.611928586 |

| | | | |
|---|---|---|---|
| Row_54 | 0.413054330 | 1.197576575 | 1.559863111 |
| Row_55 | 0.301237924 | 0.746865654 | 2.716352189 |
| Row_56 | -0.487834433 | 1.607136161 | 1.664286715 |
| Row_57 | 1.403112263 | -1.940654852 | 2.086127366 |
| Row_58 | 0.614039906 | -1.080384346 | 1.034061891 |
| Row_59 | 0.502223500 | -1.531095267 | 2.190550969 |
| Row_60 | -0.286848857 | -0.670824760 | 1.138485495 |
| Row_61 | 0.446656952 | -1.430776576 | 1.721272426 |
| Row_62 | -0.342415405 | -0.570506070 | 0.669206952 |
| Row_63 | -0.454231811 | -1.021216991 | 1.825696030 |
| Row_64 | -1.243304168 | -0.160946484 | 0.773630555 |

Column Scores

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| V_1C_1 | -0.278099294 | -0.473533152 | -0.002559417 |
| V_1C_2 | 0.591574453 | 1.007302506 | 0.005444406 |
| V_2C_1 | -0.547486807 | 0.015697382 | 1.065345101 |
| V_2C_2 | 0.337859729 | -0.009687016 | -0.657435399 |
| V_3C_1 | -0.132336407 | -0.234135632 | -0.091697480 |
| V_3C_2 | 1.083272097 | 1.916574615 | 0.750612202 |
| V_4C_1 | -0.353179736 | 0.142331258 | -0.079183808 |
| V_4C_2 | 1.185829911 | -0.477888864 | 0.265866125 |
| V_5C_1 | -0.446161501 | 0.153334790 | 0.030395176 |
| V_5C_2 | 1.003437326 | -0.344856853 | -0.068360120 |
| V_6C_1 | -0.243307622 | 0.200528730 | -0.190663127 |
| V_6C_2 | 1.026370067 | -0.845911378 | 0.804294271 |

Variance Matrix of Singular Values

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 2.6534e-005 | -7.4628e-006 | -3.7634e-006 |
| Fac_2 | -7.4628e-006 | 1.2922e-005 | -1.5613e-007 |
| Fac_3 | -3.7634e-006 | -1.5613e-007 | 4.8226e-006 |

Variances of Row Scores

| | Row_01 | Row_02 | Row_03 | Row_04 |
|---|---|---|---|---|
| Fac_1_1 | 0.003567121 | 0.006543601 | 0.004659498 | 0.008479703 |
| Fac_2_1 | -0.005253099 | -0.006999733 | -0.005756734 | -0.005492687 |
| Fac_2_2 | 0.037826187 | 0.017827008 | 0.027885420 | 0.010678172 |
| Fac_3_1 | -0.000931030 | 0.000605487 | -0.000211699 | 0.002189552 |
| Fac_3_2 | -0.003336689 | 0.001379644 | -0.006529049 | -0.001462373 |
| Fac_3_3 | 0.026929930 | 0.030065146 | 0.051835576 | 0.042808961 |

Variances of Row Scores

| | Row_61 | Row_62 | Row_63 | Row_64 |
|---|---|---|---|---|
| Fac_1_1 | 0.004690562 | 0.001142759 | 0.003089313 | 0.000385236 |
| Fac_2_1 | 0.002743134 | -0.000142037 | 0.000731069 | -0.000143421 |
| Fac_2_2 | 0.011358819 | 0.004575386 | 0.009004240 | 0.005012738 |
| Fac_3_1 | -0.000268895 | 0.000970384 | -0.001614912 | 0.000489102 |
| Fac_3_2 | 0.008891565 | 0.000793106 | 0.007881124 | 0.000133007 |
| Fac_3_3 | 0.024509280 | 0.019527303 | 0.020089710 | 0.002945903 |

Variances of Column Scores

| | V_1C_1 | V_1C_2 | V_2C_1 | V_2C_2 |
|---|---|---|---|---|
| Fac_1_1 | 0.000691324 | 0.003102525 | 0.000985744 | 0.000409914 |
| Fac_2_1 | -0.000339554 | -0.001555982 | -0.000114461 | -3.2497e-005 |
| Fac_2_2 | 0.000301235 | 0.001371249 | 0.005536289 | 0.002107693 |
| Fac_3_1 | 0.000117885 | 0.000539141 | 0.000738086 | 0.000263280 |
| Fac_3_2 | -0.000154548 | -0.000689379 | -0.000259709 | -0.000121905 |
| Fac_3_3 | 0.001050755 | 0.004754778 | 0.001980654 | 0.000692861 |

Variances of Column Scores

| | V_5C_1 | V_5C_2 | V_6C_1 | V_6C_2 |
|---|---|---|---|---|
| Fac_1_1 | 0.000166135 | 0.000814020 | 0.000231000 | 0.003721693 |
| Fac_2_1 | 0.000119051 | 0.000662015 | 0.000159863 | 0.003165836 |
| Fac_2_2 | 0.000541762 | 0.002702326 | 0.000460368 | 0.007927174 |
| Fac_3_1 | 4.7187e-005 | 0.000319857 | -0.000130047 | -0.001992905 |
| Fac_3_2 | 9.6731e-005 | 0.000457928 | 0.000193038 | 0.003170719 |
| Fac_3_3 | 0.000997993 | 0.005037112 | 0.000992124 | 0.018397124 |

Correlation Matrix of Singular Values

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | -0.403021977 | -0.332687501 |
| Fac_2 | -0.403021977 | 1.000000000 | -0.019777876 |
| Fac_3 | -0.332687501 | -0.019777876 | 1.000000000 |

Correlations of Row Scores : Row=1

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | -0.452230986 | -0.094991968 |
| Fac_2 | -0.452230986 | 1.000000000 | -0.104544668 |

```
Fac_3  -0.094991968 -0.104544668  1.000000000

     Correlations of Row Scores : Row=64

                Fac_1        Fac_2        Fac_3
Fac_1   1.000000000 -0.103207331  0.459120704
Fac_2  -0.103207331  1.000000000  0.034612132
Fac_3   0.459120704  0.034612132  1.000000000

   Correlations of Column Scores : Column=1

                Fac_1        Fac_2        Fac_3
Fac_1   1.000000000 -0.744072393  0.138313746
Fac_2  -0.744072393  1.000000000 -0.274700920
Fac_3   0.138313746 -0.274700920  1.000000000

  Correlations of Column Scores : Column=12

                Fac_1        Fac_2        Fac_3
Fac_1   1.000000000  0.582852952 -0.240847087
Fac_2   0.582852952  1.000000000  0.262557033
Fac_3  -0.240847087  0.262557033  1.000000000
```

Now, we will not use the `orig` option and obtain slightly different results:

```
optn = [ "freq"        7 ,
         "dim1"        1 ,
         "dim2"        3 ,
         "scale"   "symm" ,
         "print"       3 ,
         "pvars"       2 ,
         "conf"          ];
< scor,vars > = anaprof(a,optn);
print "Scores=", scor;
print "Variances=", vars;
```

```
*******************************
Mapping of Categories to Columns
*******************************

Column Variable Category       Name
    1        1        1          0
```

```
       2                  2           1

       3        2         1           0
       4                  2           1

       5        3         1           0
       6                  2           1

       7        4         1           0
       8                  2           1

       9        5         1           0
      10                  2           1

      11        6         1           0
      12                  2           1
```

```
   *********************************************
   Largest Singular Values and Principal Inertia
   *********************************************

    Singular                             Principal       Chi-
 N     Value Difference Proportion    Inertia     Square
 1 0.51786040            0.21396555 0.26817940 1193563.02
 2 0.45026070 0.06759970 0.18603523 0.20273470 969982.344
 3 0.39701346 0.05324724 0.16403494 0.15761969 412717.253
 4 0.36661718 0.03039628 0.15147604 0.13440816 263846.232
 5 0.34651142 0.02010576 0.14316890 0.12007017 130546.044
 6 0.34203492 0.00447650 0.14131934 0.11698789 1.104e-022
         Total Number of Observations: 4253


       *************************************
       Scree Plot of Largest Singular Values
       *************************************

N      Value +-----------+-----------+-----------+-----------+
1 0.51786040 ************************************************
2 0.45026070 *****************************************
3 0.39701346 ************************************
4 0.36661718 *********************************
5 0.34651142 ********************************
6 0.34203492 *******************************
```

Row Marginals

-------------

| | | | | | |
|---|---|---|---|---|---|
| 1 : | 252 | 198 | 36 | 102 | 72 |
| 6 : | 174 | 48 | 492 | 306 | 414 |
| 11 : | 120 | 324 | 204 | 744 | 162 |
| 16 : | 1902 | 6 | 12 | 0 | 54 |
| 21 : | 6 | 66 | 42 | 354 | 48 |
| 26 : | 138 | 42 | 210 | 60 | 330 |
| 31 : | 78 | 1164 | 66 | 42 | 12 |
| 36 : | 30 | 24 | 48 | 24 | 264 |
| 41 : | 432 | 756 | 270 | 852 | 480 |
| 46 : | 1548 | 822 | 4560 | 0 | 12 |
| 51 : | 0 | 24 | 24 | 78 | 36 |
| 56 : | 180 | 198 | 288 | 228 | 384 |
| 61 : | 252 | 576 | 540 | 4308 | |

Column Marginals
----------------

| | | | | | |
|---|---|---|---|---|---|
| 1 : | 2893 | 1360 | 1623 | 2630 | 3790 |
| 6 : | 463 | 3277 | 976 | 2944 | 1309 |
| 11 : | 3438 | 815 | | | |

Row Scores

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Row_01 | 2.338265452 | 0.687080973 | 0.733162023 |
| Row_02 | 1.770429113 | 1.264335114 | 0.070266043 |
| Row_03 | 1.689963212 | 0.961901469 | 0.798958295 |
| Row_04 | 1.122126874 | 1.539155610 | 0.136062316 |
| Row_05 | 1.649976125 | 1.029216783 | 0.503270558 |
| Row_06 | 1.082139786 | 1.606470924 | -0.159625422 |
| Row_07 | 1.001673885 | 1.304037278 | 0.569066831 |
| Row_08 | 0.433837547 | 1.881291419 | -0.093829149 |
| Row_09 | 1.794610407 | -0.499328441 | 0.171968445 |
| Row_10 | 1.226774068 | 0.077925699 | -0.490927535 |
| Row_11 | 1.146308167 | -0.224507946 | 0.237764718 |
| Row_12 | 0.578471829 | 0.352746195 | -0.425131262 |
| Row_13 | 1.106321079 | -0.157192632 | -0.057923020 |
| Row_14 | 0.538484741 | 0.420061509 | -0.720818999 |
| Row_15 | 0.458018840 | 0.117627863 | 0.007873253 |
| Row_16 | -0.109817498 | 0.694882004 | -0.655022727 |
| Row_17 | 1.942313046 | 0.701083924 | 1.880974251 |
| Row_18 | 1.374476708 | 1.278338064 | 1.218078271 |
| Row_19 | 0.000000000 | 0.000000000 | 0.000000000 |

| | | | |
|---|---|---|---|
| Row_20 | 0.726174468 | 1.553158560 | 1.283874544 |
| Row_21 | 1.254023719 | 1.043219733 | 1.651082786 |
| Row_22 | 0.686187381 | 1.620473874 | 0.988186807 |
| Row_23 | 0.605721480 | 1.318040228 | 1.716879059 |
| Row_24 | 0.037885141 | 1.895294369 | 1.053983079 |
| Row_25 | 1.398658001 | -0.485325491 | 1.319780673 |
| Row_26 | 0.830821663 | 0.091928650 | 0.656884693 |
| Row_27 | 0.750355761 | -0.210504996 | 1.385576946 |
| Row_28 | 0.182519423 | 0.366749145 | 0.722680966 |
| Row_29 | 0.710368674 | -0.143189682 | 1.089889209 |
| Row_30 | 0.142532336 | 0.434064459 | 0.426993229 |
| Row_31 | 0.062066434 | 0.131630814 | 1.155685481 |
| Row_32 | -0.505769904 | 0.708884954 | 0.492789502 |
| Row_33 | 1.949322368 | -0.129801435 | 0.727829431 |
| Row_34 | 1.381486030 | 0.447452706 | 0.064933451 |
| Row_35 | 1.301020128 | 0.145019060 | 0.793625704 |
| Row_36 | 0.733183790 | 0.722273201 | 0.130729724 |
| Row_37 | 1.261033041 | 0.212334374 | 0.497937966 |
| Row_38 | 0.693196703 | 0.789588515 | -0.164958014 |
| Row_39 | 0.612730801 | 0.487154870 | 0.563734239 |
| Row_40 | 0.044894463 | 1.064409011 | -0.099161741 |
| Row_41 | 1.405667323 | -1.316210850 | 0.166635853 |
| Row_42 | 0.837830984 | -0.738956709 | -0.496260127 |
| Row_43 | 0.757365083 | -1.041390355 | 0.232432126 |
| Row_44 | 0.189528745 | -0.464136214 | -0.430463854 |
| Row_45 | 0.717377996 | -0.974075040 | -0.063255611 |
| Row_46 | 0.149541657 | -0.396820900 | -0.726151591 |
| Row_47 | 0.069075756 | -0.699254545 | 0.002540661 |
| Row_48 | -0.498760582 | -0.122000404 | -0.660355319 |
| Row_49 | 0.000000000 | 0.000000000 | 0.000000000 |
| Row_50 | 0.985533624 | 0.461455656 | 1.212745679 |

Row Scores

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Row_51 | 0.000000000 | 0.000000000 | 0.000000000 |
| Row_52 | 0.337231385 | 0.736276151 | 1.278541952 |
| Row_53 | 0.865080635 | 0.226337325 | 1.645750195 |
| Row_54 | 0.297244297 | 0.803591465 | 0.982854215 |
| Row_55 | 0.216778396 | 0.501157820 | 1.711546467 |
| Row_56 | -0.351057943 | 1.078411961 | 1.048650488 |
| Row_57 | 1.009714917 | -1.302207900 | 1.314448081 |
| Row_58 | 0.441878579 | -0.724953759 | 0.651552101 |
| Row_59 | 0.361412678 | -1.027387405 | 1.380244354 |
| Row_60 | -0.206423661 | -0.450133264 | 0.717348374 |

| | | | |
|---|---|---|---|
| Row_61 | 0.321425590 | -0.960072090 | 1.084556617 |
| Row_62 | -0.246410748 | -0.382817949 | 0.421660637 |
| Row_63 | -0.326876649 | -0.685251595 | 1.150352890 |
| Row_64 | -0.894712988 | -0.107997454 | 0.487456910 |

Column Scores

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| V_1C_1 | -0.386450198 | -0.705697154 | -0.004061985 |
| V_1C_2 | 0.822059133 | 1.501163137 | 0.008640679 |
| V_2C_1 | -0.760794398 | 0.023393500 | 1.690782315 |
| V_2C_2 | 0.469494034 | -0.014436369 | -1.043399125 |
| V_3C_1 | -0.183896298 | -0.348927734 | -0.145530756 |
| V_3C_2 | 1.505328225 | 2.856233499 | 1.191277676 |
| V_4C_1 | -0.490782903 | 0.212113477 | -0.125670622 |
| V_4C_2 | 1.647843824 | -0.712188386 | 0.421949415 |
| V_5C_1 | -0.619991507 | 0.228511825 | 0.048239416 |
| V_5C_2 | 1.394388844 | -0.513933394 | -0.108492621 |
| V_6C_1 | -0.338103263 | 0.298844027 | -0.302596636 |
| V_6C_2 | 1.426256464 | -1.260645109 | 1.276475132 |

Variance Matrix of Singular Values

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 4.1519e-005 | 9.4776e-006 | 1.1547e-005 |
| Fac_2 | 9.4776e-006 | 4.1964e-005 | 1.7186e-005 |
| Fac_3 | 1.1547e-005 | 1.7186e-005 | 2.1863e-005 |

Variances of Row Scores

| | Row_01 | Row_02 | Row_03 | Row_04 |
|---|---|---|---|---|
| Fac_1_1 | 0.002504711 | 0.005224071 | 0.003989750 | 0.007121674 |
| Fac_2_1 | -0.002518790 | -0.004506108 | -0.002075981 | -0.002484718 |
| Fac_2_2 | 0.014916640 | 0.007787532 | 0.010867109 | 0.003924380 |
| Fac_3_1 | -0.000379400 | -0.000322509 | 0.000647789 | -0.000185712 |
| Fac_3_2 | -0.001199736 | -0.001171523 | -0.002616200 | -0.001093344 |
| Fac_3_3 | 0.007963457 | 0.009778418 | 0.017848282 | 0.011517780 |

Variances of Row Scores

| | Row_61 | Row_62 | Row_63 | Row_64 |
|---|---|---|---|---|
| Fac_1_1 | 0.003611328 | 0.000895374 | 0.002611392 | 0.000308003 |
| Fac_2_1 | -7.7880e-005 | -0.000348219 | -0.001558652 | -0.000250409 |

| | | | | |
|---|---|---|---|---|
| Fac_2_2 | 0.005106372 | 0.001876779 | 0.007163871 | 0.004120658 |
| Fac_3_1 | -0.000109280 | -0.000269016 | 0.001221966 | 0.000171839 |
| Fac_3_2 | 0.003027509 | 0.000376517 | 0.001538756 | 0.000382407 |
| Fac_3_3 | 0.007279853 | 0.007197412 | 0.009188973 | 0.000961069 |

Variances of Column Scores

| | V_1C_1 | V_1C_2 | V_2C_1 | V_2C_2 |
|---|---|---|---|---|
| Fac_1_1 | 0.001342491 | 0.007436684 | 0.002144407 | 0.000658071 |
| Fac_2_1 | -0.000942592 | -0.003136842 | -0.000182140 | -3.0086e-005 |
| Fac_2_2 | 0.001609890 | 0.006864395 | 0.012029737 | 0.004578960 |
| Fac_3_1 | -0.000165621 | -0.001492020 | 4.8996e-005 | 0.000358051 |
| Fac_3_2 | 0.001326988 | 0.004634310 | -0.000372176 | -0.000228624 |
| Fac_3_3 | 0.003879193 | 0.017537605 | 0.005649501 | 0.001426143 |

Variances of Column Scores

| | V_3C_1 | V_3C_2 | V_4C_1 | V_4C_2 |
|---|---|---|---|---|
| Fac_1_1 | 0.000454969 | 0.046124247 | 0.000186588 | 0.006521224 |
| Fac_2_1 | -0.000158701 | 0.013408796 | 0.000271566 | 0.002096713 |
| Fac_2_2 | 0.000227611 | 0.050188528 | 0.000972943 | 0.010977058 |
| Fac_3_1 | -2.4637e-005 | 0.007545676 | -4.3867e-006 | 0.000983962 |
| Fac_3_2 | -0.000107829 | 0.005769047 | -6.9304e-005 | -0.000986053 |
| Fac_3_3 | 0.000635680 | 0.047356455 | 0.001901904 | 0.021680414 |

Variances of Column Scores

| | V_5C_1 | V_5C_2 | V_6C_1 | V_6C_2 |
|---|---|---|---|---|
| Fac_1_1 | 0.000240125 | 0.002912470 | 0.000391610 | 0.012750425 |
| Fac_2_1 | 0.000334778 | 0.001668909 | 0.000295624 | 0.000566435 |
| Fac_2_2 | 0.001594796 | 0.007854190 | 0.000797063 | 0.017964896 |
| Fac_3_1 | 0.000194136 | -6.0129e-005 | -0.000149999 | 0.005642781 |
| Fac_3_2 | -0.000302187 | -0.001191216 | 0.000247631 | -0.002567746 |
| Fac_3_3 | 0.002549518 | 0.013047869 | 0.002099758 | 0.047612133 |

Correlation Matrix of Singular Values

| | Fac_1 | Fac_2 | Fac_3 |
|---|---|---|---|
| Fac_1 | 1.000000000 | 0.227058714 | 0.383243953 |
| Fac_2 | 0.227058714 | 1.000000000 | 0.567374084 |
| Fac_3 | 0.383243953 | 0.567374084 | 1.000000000 |

Correlations of Row Scores : Row=1

```
              Fac_1        Fac_2        Fac_3
Fac_1   1.000000000 -0.412076283 -0.084950749
Fac_2  -0.412076283  1.000000000 -0.110077718
Fac_3  -0.084950749 -0.110077718  1.000000000


     Correlations of Row Scores : Row=64

              Fac_1        Fac_2        Fac_3
Fac_1   1.000000000 -0.222274343  0.315840176
Fac_2  -0.222274343  1.000000000  0.192161153
Fac_3   0.315840176  0.192161153  1.000000000


   Correlations of Column Scores : Column=1

              Fac_1        Fac_2        Fac_3
Fac_1   1.000000000 -0.641165396 -0.072575227
Fac_2  -0.641165396  1.000000000  0.531004443
Fac_3  -0.072575227  0.531004443  1.000000000


  Correlations of Column Scores : Column=12

              Fac_1        Fac_2        Fac_3
Fac_1   1.000000000  0.037426181  0.229019261
Fac_2   0.037426181  1.000000000 -0.087797259
Fac_3   0.229019261 -0.087797259  1.000000000
```

Equivalent results we will obtain with the following full design specification:

```
f = [ 042 , 033 , 006 , 017 ,
      012 , 029 , 008 , 082 ,
      051 , 069 , 020 , 054 ,
      034 , 124 , 027 , 317 ,
      001 , 002 , 000 , 009 ,
      001 , 011 , 007 , 059 ,
      008 , 023 , 007 , 035 ,
      010 , 055 , 013 , 194 ,
      011 , 007 , 002 , 005 ,
      004 , 008 , 004 , 044 ,
      072 , 126 , 045 , 142 ,
      080 , 258 , 137 , 760 ,
      000 , 002 , 000 , 004 ,
      004 , 013 , 006 , 030 ,
```

```
      033 , 048 , 038 , 064 ,
      042 , 096 , 090 , 718 ];
b = [ 2 2 2 2 2 2 ];

optn = [ "freq"         7 ,
         "dim1"         1 ,
         "dim2"         3 ,
         "scale"   "symm" ,
         "print"        3 ,
         "pvars"        2 ,
         "conf"           ];
< scor,vars > = anaprof(f,optn,b);
print "Scores=", scor;
print "Variances=", vars;
```

## 3.3 Function `cndcov`

---

$<$ ccov,cmu $>$ = cndcov(data,zmu,isel,optn)

**Purpose:** The `cndcov` function implements various algorithms for computing the conditional covariance matrix

$$\mathbf{A}_{1.2} = \mathbf{A}_{11} - \mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{A}_{21}$$

and the conditional mean vector

$$\mu_{1.2} = \mu_1 + \mathbf{A}_{12}\mathbf{A}_{22}^{-1}(z_2 - \mu_2)$$

for given $z_2$ and $\mu$ of the partition

$$\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} \sim N_n\left( \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} A_{11} A_{12} \\ A_{21} A_{22} \end{bmatrix} \right) \qquad \begin{matrix} s \\ n-2 \end{matrix}$$

The problem is similar to that of computing a partial correlation matrix. The algorithms include a specific algorithm for the efficient computation of these for ARMA time series data

$$Z_t - \phi_1 Z_{t-1} - \ldots - \phi_p Z_{t-p} = e_t - \theta_1 e_{t-1} - \ldots - \theta_q e_{t-q}$$

with given coefficients $(\phi_1, \ldots, \phi_p)$ and $(\theta_1, \ldots, \theta_q)$ where the covariance matrix has a diagonal band form with band width $\max(p, q)$ . There are two sets of algorithms implemented depending on whether the input data are the $\phi$ and $\theta$ vector of an ARMA time series or the input data is a $m \times n$ raw data or $n \times n$ covariance matrix.

**Input:** **data** There are three forms of input data depending on the specification of the options vector

1. If `optn[3]` is less than 5 and `optn[2]` is zero then `data` must be an $m \times n$ raw data matrix.
2. If `optn[3]` is less than 5 and `optn[2]` is nonzero then `data` must be a symmetric $n \times n$ covariance matrix.
3. If `optn[3]` is either 5 or 6 then `data` must be a matrix with two rows: the first row must contain the $p$ vector $\phi$ and the second row must contain the $q$ vector $\theta$ of an ARMA time series.

**zmu** The $z$ vector for the conditional mean

**isel** index vector of the columns of the data for the conditional mean and covariance matrix.

**optn** The options vector specifies the form of input data and the algorithm.

**1** amount of printed output (=0: no printed output)

**2** valid only if `optn[3]` is less than 5: If `optn[2]` is zero the first input argument must be a $m \times n$ raw data matrix. If `optn[2]` is nonzero the first input argument must be a $n \times n$ covariance matrix.

**3** specifies the algorithm: the first four algorithms are valid only for raw data or covariance input, the algorithms 5 and 6 are only for ARMA time series data:

1. use sweep operator
2. use common Cholesky method on permuted covariance matrix
3. use common Cholesky method on unpermuted covariance matrix and afterward perform permutation update of Cholesky factor: Monahan's reverse Cholesky
4. use common Cholesky method on unpermuted covariance matrix and afterward perform permutation update of Cholesky factor: use Linpack subroutine `SCHEX`.
5. compare different methods for ARMA data
6. use only Monahan's method for ARMA data

**4** currently unused

**5** currently unused

**6** valid only for `optn[3]` equal 4: specifying 0, 1, or 2 refers to three different strategies of performing the left and right circular shift for the `SCHEX` calls.

**Output:** **ccov** returns the conditional covariance matrix.

**cmu** returns the conditional mean vector.

**Restrictions:** 1. String data are not permitted for *data*.

2. Missing values are not permitted for the first three arguments.

**Relationships:** sweep(), chodec()

**Examples:** 1. Monahan Example 1: Test SWEEP Operator:

```
a = [ 3.  1.  0. ,
      1.  4.  2. ,
      0.  2.  6. ];
zmu = cons(3);
list = [ 2 3 2 1 ];
optn = [ 2 ,   /* ipri */
         1 ,   /* icov */
         1 ];  /* imet */
< ccov,cmu > = cndcov(a,zmu,list,optn);
print "CCOV=", ccov;
print "CMU= ", cmu;
```

```
Input Subset S
--------------

  1 :         0         1         1         2


                  Conditional Covariance Matrix
                  *****************************

                      Step Index       LogDet
                         1     1  1.098612289
                         2     2  2.397895273
                         3     2  1.098612289
                         4     3  2.890371758


Sweep Result Covariance Matrix
------------------------------

   1|      0.3333
   2|      0.3333           0
   3|     -0.3333           3     -0.3333
   4|           0      0.3333      0.1667           2
                     Log Determinant=2.89037


CCOV=
 S |         1         2         3
----------------------------------
```

```
 1 |   0.33333
 2 |  -0.33333   3.00000
 3 |   0.00000   0.33333   0.16667


CMU=  .
```

2. Monahan Example 2: Test Reverse Cholesky Factorization:

```
a = [ 25. 11.  0.  4. ,
      11. 19.  7.  2. ,
       0.  7.  5.  2. ,
       4.  2.  2.  4. ];
zmu = cons(4);


/* (i1) sweep matrix */
list = [ 2 3 ];
optn = [ 2 ,   /* ipri */
         1 ,   /* icov */
         1 ];  /* imet: sweep */
< ccov,cmu > = cndcov(a,zmu,list,optn);
print "CCOV=", ccov;
print "CMU= ", cmu;
```

```
Input Subset S
--------------
  1 :        0        3

Input Subset T
--------------
  1 :        1        2


                  Conditional Covariance Matrix
                  *****************************

                      Step Index       LogDet
                         1     1  3.218875825
                         2     4  4.430816799


Conditional Covariance Matrix
-----------------------------

   1|       14.14
```

```
   2|       6.857        3.81
                    Log Determinant=4.43082


 Conditional Mean Vector
 -----------------------

   1 :         0.5        0.5


CCOV=
 S |         1         2
------------------------
 1 |   14.1429
 2 |    6.8571    3.8095


CMU=
   |         1
--------------
 1 |   0.50000
 2 |   0.50000


  /* (i2) permuted Input */
  list = [ 2 3 ];
  optn = [ 2 ,   /* ipri */
           1 ,   /* icov */
           2 ];  /* imet: Cholesky of permuted */
  < ccov,cmu > = cndcov(a,zmu,list,optn);


 Input Subset S
 --------------
   1 :         1        2

 Input Subset T
 --------------
   1 :         0        3


 Conditional Covariance Matrix
 -----------------------------

   1|        13.1
   2|       6.857        3.81
                    Log Determinant=4.43082
```

```
Conditional Mean Vector
-----------------------

  1 :        0.5        0.5
```

The following input specifications yield the same results:

```
/* (i3) permuted Cholesky: reverse (Monahan) */
list = [ 2 3 ];
optn = [ 2 ,   /* ipri */
         1 ,   /* icov */
         3 ];  /* imet: reverse Cholesky update */
< ccov,cmu > = cndcov(a,zmu,list,optn);
print "CCOV=", ccov;
print "CMU= ", cmu;


/* (i4) permuted Cholesky: Linpack: istr=0 */
list = [ 2 3 ];
optn = [ 2 ,   /* ipri */
         1 ,   /* icov */
         4 ];  /* imet: Linpack Cholesky update */
< ccov,cmu > = cndcov(a,zmu,list,optn);
print "CCOV=", ccov;
print "CMU= ", cmu;


/* (i4) permuted Cholesky: Linpack: istr=1 */
list = [ 2 3 ];
optn = [ 2 ,   /* ipri */
         1 ,   /* icov */
         4 ,   /* imet: Linpack Cholesky update */
         . , . , 1 ]; /* istr=1: */
< ccov,cmu > = cndcov(a,zmu,list,optn);
print "CCOV=", ccov;
print "CMU= ", cmu;


/* (i4) permuted Cholesky: Linpack: istr=2 */
list = [ 2 3 ];
optn = [ 2 ,   /* ipri */
         1 ,   /* icov */
         4 ,   /* imet: Linpack Cholesky update */
         . , . , 1 ]; /* istr=2: */
< ccov,cmu > = cndcov(a,zmu,list,optn);
print "CCOV=", ccov;
print "CMU= ", cmu;
```

3. Monahan Example 3: Comparing Results for ARMA Data:

```
/* zmu[n=8], phi[p=3], theta[q=4] */
phitht= [  .188458   -.152381   -.108700,
         -1.153870   -.374322    .368657    .559246 ];
zmu = [ .609190   .659790   .089071   .011946   .778937 0. 0. 0. ];


list = [ 2  6  8 ];
optn = [ 2 ,   /* ipri */
         1 ,   /* icov */
         5 ];  /* imet */
< ccov,cmu > = cndcov(phitht,zmu,list,optn);
```

```
Input Phi
---------

  1 :      0.1885    -0.1524    -0.1087


Input Theta
-----------

  1 :      -1.154    -0.3743     0.3687     0.5592


Input Subset S
--------------

  1 :         1        5        7


Input Subset T
--------------

  1 :         0        2        3        4        6


Input Conditional Difference (Z2-mu2)
-------------------------------------

  1 :      0.6092     0.6598    0.08907    0.01195     0.7789


Conditional Covariance Matrix (direct)
--------------------------------------
```

```
   1|      0.4963     0.03939     -0.1169
   2|     0.03939      0.4963     -0.5434
   3|     -0.1169     -0.5434       1.792

INV(A22)*A21 (direct)
---------------------

   1|      0.4903       1.049     -0.6561      0.1988    -0.01852
   2|    -0.01852      0.1988     -0.6561       1.049      0.4903
   3|     0.07059      -0.376      0.7947     -0.8571      0.6723

Sweep Result Covariance Matrix
------------------------------

   1|      0.3396      0.4903     -0.1086       0.319    -0.09637
   2|     -0.4903      0.4963      -1.049      0.6561     -0.1988
   3|     -0.1086       1.049       0.774     -0.8486      0.5768
   4|       0.319     -0.6561     -0.8486        1.49     -0.8486
   5|    -0.09637      0.1988      0.5768     -0.8486       0.774
   6|     0.01852     0.03939     -0.1988      0.6561      -1.049
   7|      0.0763    -0.01852    -0.09637       0.319     -0.1086
   8|    -0.07059     -0.1169       0.376     -0.7947      0.8571

   1|    -0.01852      0.0763     0.07059
   2|     0.03939     0.01852     -0.1169
   3|      0.1988    -0.09637      -0.376
   4|     -0.6561       0.319      0.7947
   5|       1.049     -0.1086     -0.8571
   6|      0.4963     -0.4903     -0.5434
   7|      0.4903      0.3396      0.6723
   8|     -0.5434     -0.6723       1.792

            Sweep max abs A11 difference=8.88178e-016
            Sweep max abs A21 difference=5.55112e-016

Inverse of Conditional Covariance Matrix A1.2 (direct)
------------------------------------------------------

   1|       2.047    -0.02429      0.1262
   2|    -0.02429       3.017      0.9137
   3|      0.1262      0.9137      0.8435
```

```
          QCARMA max abs A11 difference=5.77316e-015
          QCARMA max abs A21 difference=2.22045e-016


Inverse of Conditional Covariance Matrix A1.2 (qcarma)
------------------------------------------------------

   1|       2.047    -0.02429      0.1262
   2|    -0.02429       3.017      0.9137
   3|      0.1262      0.9137      0.8435


Conditional Covariance Matrix
-----------------------------

   1|      0.4963
   2|     0.03939      0.4963
   3|     -0.1169     -0.5434       1.792
                    Log Determinant=-1.23731


Conditional Mean Vector
-----------------------

  1 :      0.9202     0.4559     0.3791


 /* zmu[n=8], phi[p=2], theta[q=2] */
 phitht = [ .334927    .031806 ,
            .178533    .682901 ];
 zmu = [ .041551   .354168   .508469   .834386   .522901 0. 0. 0. ];


 list = [ 2  3  4 ];
 optn = [ 2 ,   /* ipri */
          1 ,   /* icov */
          5 ];  /* imet */
 /* options debug="cndco*=4"; */
 < ccov,cmu > = cndcov(phitht,zmu,list,optn);


Input Phi
---------

  1 :      0.3349    0.03181


Input Theta
```

```
-----------

  1 :      0.1785     0.6829


Input Subset S
--------------

  1 :         1        2        3


Input Subset T
--------------

  1 :         0        4        5        6        7


Input Conditional Difference (Z2-mu2)
-------------------------------------

  1 :     0.04155     0.3542     0.5085     0.8344     0.5229


Conditional Covariance Matrix (direct)
--------------------------------------

   1|       1.371      0.1751     -0.5522
   2|      0.1751      0.8495      0.1356
   3|     -0.5522      0.1356       1.049


INV(A22)*A21 (direct)
---------------------

   1|      0.1298       -0.15    -0.03474    -0.07214    -0.03209
   2|     -0.4325     -0.5242    -0.06421     -0.2486    -0.08312
   3|     -0.1308       0.199     -0.5398     0.06003     -0.2545


Sweep Result Covariance Matrix
------------------------------

   1|      0.7026      0.1298     -0.4325     -0.1308     0.04845
   2|     -0.1298       1.371      0.1751     -0.5522        0.15
   3|      0.4325      0.1751      0.8495      0.1356      0.5242
   4|      0.1308     -0.5522      0.1356       1.049      -0.199
   5|     0.04845       -0.15     -0.5242       0.199      0.8758
   6|    0.009905    -0.03474    -0.06421     -0.5398      -0.178
```

```
   7|    0.02322   -0.07214    -0.2486    0.06003     0.3763
   8|   0.009696   -0.03209   -0.08312    -0.2545   -0.01031

   1|   0.009905    0.02322   0.009696
   2|    0.03474    0.07214    0.03209
   3|    0.06421     0.2486    0.08312
   4|     0.5398   -0.06003     0.2545
   5|     -0.178     0.3763   -0.01031
   6|     0.9091    -0.2504     0.3748
   7|    -0.2504     0.9097    -0.1784
   8|     0.3748    -0.1784     0.8726

            Sweep max abs A11 difference=1.11022e-016
            Sweep max abs A21 difference=1.38778e-016

Inverse of Conditional Covariance Matrix A1.2 (direct)
------------------------------------------------------

   1|     0.9934    -0.2943     0.5612
   2|    -0.2943      1.289    -0.3217
   3|     0.5612    -0.3217      1.291

           QCARMA max abs A11 difference=5.55112e-016
           QCARMA max abs A21 difference=1.66533e-016

Inverse of Conditional Covariance Matrix A1.2 (qcarma)
------------------------------------------------------

   1|     0.9934    -0.2943     0.5612
   2|    -0.2943      1.289    -0.3217
   3|     0.5612    -0.3217      1.291

Conditional Covariance Matrix
-----------------------------

   1|      1.371
   2|     0.1751     0.8495
   3|    -0.5522     0.1356      1.049
                    Log Determinant=-0.129842

Conditional Mean Vector
```

```
 ----------------------

  1 :     -0.1424    -0.4871    -0.2924
```

## 3.4 Function `princals`

<gof,eval,load,scor,catqua,single,multi> = princals(data,scale,optn)

**Purpose:** The `princals` function performs principal component analysis for categorical data, like multinomial and multiordinal data. The iterative algorithm is based on the Gifi (1990) suite of programs and the input data can have some missing values. The results are not only variable loadings and observation scores, estimated are also general category quantifications of all variables and the variables category quantifications of all factors.

**Input:** **data** This argument must be a $N \times n$ matrix containing the measured levels of $n$ categorical variables. The levels can be integer or string coded. A sufficiently small number of missing values are permitted.

**scale** This argument must be an $n$ integer vector each entry defining the scale property of a variable. The following scaling options are permitted:

**scale=0** multiple nominal

**scale=1** single nominal

**scale=2** single ordinal

**scale=3** single numerical

Note that the dimension of this vector must be the same as the column number of the data matrix provided for the first argument.

**optn** This argument must be specified in form of a two column matrix where the first column defines the option as string value (in quotes) and the second column can be used for a numeric or string specification of the option. See table below for content.

| Option | Second Column | Meaning |
|---|---|---|
| "maxin" | int | maximum number of iterations for initial phase of iterations (default is 20) |
| "maxit" | int | maximum number of iterations for final phase of iterations (default is 200) |
| "nfac" | int | number of factors (dimension) of decomposition |
| "orig" | | original version of algorithm |
| "phis" | | print optimization history |
| "print" | int | indicates the amount of printed output (=0: no printed output, default is 2) |
| "plot" | int | indicates the amount of plot output (=0: no plot output is default) |
| "seed" | int | seed for random generator (valid only when "orig" is not specified) |
| "tolin" | real | termination tolerance for initial iteration (default is $5.e-2/N$) |
| "tolit" | real | termination tolerance for final iteration (default is $5.e-4$) |

The `orig` option currently refers only to one small difference: it specifies that the original (and obviously not so great) random generator is being used for generating initial values of the object score matrix before the initial iteration can start.

**Output:** There are at most seven results returned:

**gof** a vector of scalar results, like an inidicator for the success of the iterations, the computer time used and the values of differently defined *stress* for the final solution.

**eval** a vector of $nfac$ eigenvalues.

**load** the $n \times nfac$ matrix of factor loadings (defined only for variables with scaling type $> 0$).

**scor** the $N \times nfac$ matrix of object scores

**catqua** the $Ncat$ vector of category quantifications, where $Ncat$ is the total number of categories of all $n$ variables

**single** the $Ncat \times nfac$ matrix of single

**multi** the $Ncat \times nfac$ matrix of multiple (defined only for variables with scaling type $> 0$).

**Restrictions:**

1. String data for levels are permitted. The second input argument can have no missing values.
2. Many missing values or difficult missing value patterns may affect the convergence of the iterative processes.

**Relationships:** homals(), primals(), anacor(), anaprof() canals(), overals(), pca(), svd(),

**Examples:** 1. Small Example by Gifi (1990), Guttman-Bell Data, $N = 7$, $n = 5$:

```
print "Guttman-Bell Data, Gifi, p.179";
data = [  1  1  1  2  2,
          2  2  2  2  2,
          1  1  2  1  1,
          4  2  4  2  3,
          4  4  4  2  3,
          3  3  3  1  2,
          2  3  3  2  2 ];


cnam = [ "Intensity" "Frequency" "FeelBelonging"
         "PhysicalProx" "Formality" ];
rnam = [ "Crowd" "Audience" "Public" "Mob"
         "Primary" "Secondary" "Modern" ];
data = cname(data,cnam);
data = rname(data,rnam);
print "Data=", data;


ctyp = [ 1 1 1 1 1 ];
optn = [ "nfac"      2 ,
         "orig"        ,
         "maxini"   20 ,
         "tolini"  .01 ,
         "maxit"   100 ,
         "tolit" 1.e-4 ,
         "plot"      1 ,
         "print"     3 ,
         "phis"        ];
< gof,eval,load,scors,coord,squant,mquant > = princals(data,ctyp,optn);
```

```
            ***************************************
            Number of Observations for Class Levels
            ***************************************

            Variable       Value    Nobs  Proportion
            C[1]               1       2   28.571429
                               2       2   28.571429
                               3       1   14.285714
                               4       2   28.571429

            C[2]               1       2   28.571429
                               2       2   28.571429
```

| | | | |
|---|---|---|---|
| | 3 | 2 | 28.571429 |
| | 4 | 1 | 14.285714 |
| C[3] | 1 | 1 | 14.285714 |
| | 2 | 2 | 28.571429 |
| | 3 | 2 | 28.571429 |
| | 4 | 2 | 28.571429 |
| C[4] | 1 | 2 | 28.571429 |
| | 2 | 5 | 71.428571 |
| C[5] | 1 | 1 | 14.285714 |
| | 2 | 4 | 57.142857 |
| | 3 | 2 | 28.571429 |

Input Data Matrix: 7 by 5

| | Intensity | Frequency | FeelBelongin | PhysicalProx |
|---|---|---|---|---|
| Crowd | 0.000000000 | 0.000000000 | 0.000000000 | 1.000000000 |
| Audience | 1.000000000 | 1.000000000 | 1.000000000 | 1.000000000 |
| Public | 0.000000000 | 0.000000000 | 1.000000000 | 0.000000000 |
| Mob | 3.000000000 | 1.000000000 | 3.000000000 | 1.000000000 |
| Primary | 3.000000000 | 3.000000000 | 3.000000000 | 1.000000000 |
| Secondary | 2.000000000 | 2.000000000 | 2.000000000 | 0.000000000 |
| Modern | 1.000000000 | 2.000000000 | 2.000000000 | 1.000000000 |

Input Data Matrix: 7 by 5

| | Formality |
|---|---|
| Crowd | 1.000000000 |
| Audience | 1.000000000 |
| Public | 0.000000000 |
| Mob | 2.000000000 |
| Primary | 2.000000000 |
| Secondary | 1.000000000 |
| Modern | 1.000000000 |

Number of Categories per Variable

| | 1 |
|---|---|
| Intensity | 4 |
| Frequency | 4 |
| FeelBelonging | 4 |

| | |
|---|---|
| PhysicalProx | 2 |
| Formality | 3 |

Measurement Level for all Variables: Single Nominal

```
****************************************************
History of Iterations for Initial Metric Configuration
****************************************************
```

| Iter | Total Fit | Total Loss | Multiple Loss | Single Loss | Difference Iterations |
|---|---|---|---|---|---|
| 1 | 0.23193999 | 1.76806001 | 1.36964194 | 0.39841807 | 0.231939993 |
| 2 | 0.86360312 | 1.13639688 | 0.84531222 | 0.29108467 | 0.631663123 |
| 3 | 0.90268845 | 1.09731155 | 0.81401365 | 0.28329790 | 0.039085333 |
| 4 | 0.90305880 | 1.09694120 | 0.81488911 | 0.28205209 | 0.000370347 |

Eigenvalues for Initial Solution

| | 1 |
|---|---|
| Fac_1 | 0.687054858 |
| Fac_2 | 0.216003939 |

```
-----------------------------------------------------------
--- Variable: Intensity Type:  Single Nominal  Missing: 0 ---
-----------------------------------------------------------
```

| Category | Marginal Frequency | Category Quantif. | Single Categ. Coordinates Fac_1 | Fac_2 |
|---|---|---|---|---|
| 1 | 2 | -1.21267813 | 1.15896428 | -0.20938479 |
| 2 | 2 | -0.36380344 | 0.34768928 | -0.06281544 |
| 3 | 1 | 0.48507125 | -0.46358571 | 0.08375392 |
| 4 | 2 | 1.33394594 | -1.27486071 | 0.23032327 |

| Category | Multi. Categ. Coordinates Fac_1 | Fac_2 |
|---|---|---|
| 1 | 1.27346719 | -0.17505512 |
| 2 | 0.08791128 | -0.51521876 |
| 3 | -0.11149114 | 1.68738922 |
| 4 | -1.30563290 | -0.15342072 |

```
-----------------------------------------------------------
--- Variable: Frequency Type:  Single Nominal  Missing: 0 ---
-----------------------------------------------------------
```

```
            Marginal    Category  Single Categ. Coordinates
   Category Frequency    Quantif.       Fac_1       Fac_2
          1         2 -1.24807544  1.01757193 -0.34181670
          2         2 -0.27735010  0.22612709 -0.07595927
          3         2  0.69337525 -0.56531774  0.18989817
          4         1  1.66410059 -1.35676257  0.45575560

                                  Multi. Categ. Coordinates
   Category                                 Fac_1       Fac_2
          1                          1.27346719 -0.17505512
          2                         -0.37393532 -0.59088966
          3                         -0.13287869  0.71947422
          4                         -1.53330635  0.09294113


-----------------------------------------------------------------
--- Variable: FeelBelonging Type:  Single Nominal  Missing: 0 ---
-----------------------------------------------------------------

            Marginal    Category  Single Categ. Coordinates
   Category Frequency    Quantif.       Fac_1       Fac_2
          1         1 -1.66410059  1.50923098 -0.52500490
          2         2 -0.69337525  0.62884624 -0.21875204
          3         2  0.27735010 -0.25153850  0.08750082
          4         2  1.24807544 -1.13192324  0.39375368

                                  Multi. Categ. Coordinates
   Category                                 Fac_1       Fac_2
          1                          1.05171197 -1.44975571
          2                          0.91265560  0.15882436
          3                         -0.13287869  0.71947422
          4                         -1.30563290 -0.15342072


 ----------------------------------------------------------------
 --- Variable: PhysicalProx Type:  Single Nominal  Missing: 0 ---
 ----------------------------------------------------------------

            Marginal    Category  Single Categ. Coordinates
   Category Frequency    Quantif.       Fac_1       Fac_2
          1         2 -1.58113883  0.69186563  1.39351734
          2         5  0.63245553 -0.27674625 -0.55740694

                                  Multi. Categ. Coordinates
   Category                                 Fac_1       Fac_2
          1                          0.69186563  1.39351734
```

```
      2                       -0.27674625 -0.55740694

-------------------------------------------------------------
--- Variable: Formality Type:  Single Nominal  Missing: 0 ---
-------------------------------------------------------------

            Marginal    Category  Single Categ. Coordinates
    Category Frequency    Quantif.        Fac_1       Fac_2
           1         1 -1.78885438  1.64259528  0.56259476
           2         4 -0.22360680  0.20532441  0.07032435
           3         2  1.34164079 -1.23194646 -0.42194607

                                  Multi. Categ. Coordinates
    Category                                  Fac_1       Fac_2
           1                       1.49522241  1.09964546
           2                       0.27901084 -0.19820100
           3                      -1.30563290 -0.15342072


               ********************************
               Summary of Initial Configuration
               ********************************

                     Multiple Fit Stress
                     -------------------

                   Row        Sums     Fac_1     Fac_2
             Intensity 1.452460928    0.9544    0.4981
             Frequency 1.101849798    0.8442    0.2576
         FeelBelonging 1.350178667    0.8881    0.4621
          PhysicalProx 0.968227451    0.1915    0.7768
             Formality 1.052837615    0.8509    0.2019
                  Mean      1.1851    0.7458    0.4393


                      Single Fit Stress
                      -----------------

                   Row        Sums     Fac_1     Fac_2
             Intensity 0.943187331    0.9134    0.0298
             Frequency 0.739742550    0.6647    0.0750
         FeelBelonging 0.922064113    0.8225    0.0995
          PhysicalProx 0.968227451    0.1915    0.7768
             Formality 0.942072539    0.8432    0.0989
                  Mean      0.9031    0.6871    0.2160
```

------------------------
End of Initial Iteration
------------------------

| Iteration Number | Total Fit | Total Loss | Multiple Loss | Single Loss |
|---|---|---|---|---|
| 4 | 0.903058797 | 1.096941203 | 0.814889108 | 0.282052095 |

Correlations After Initialization

| | Intensity | Frequency | FeelBelongin | PhysicalProx |
|---|---|---|---|---|
| Intensity | 1.000000000 | 0.723123254 | 0.924925092 | 0.230089497 |
| Frequency | 0.723123254 | 1.000000000 | 0.750000000 | 0.175411604 |
| FeelBelonging | 0.924925092 | 0.750000000 | 1.000000000 | 0.131558703 |
| PhysicalProx | 0.230089497 | 0.175411604 | 0.131558703 | 1.000000000 |
| Formality | 0.867721831 | 0.589164989 | 0.713199724 | 0.636396103 |

Correlations After Initialization

| | Formality |
|---|---|
| Intensity | 0.867721831 |
| Frequency | 0.589164989 |
| FeelBelonging | 0.713199724 |
| PhysicalProx | 0.636396103 |
| Formality | 1.000000000 |

Object Scores

| | Fac_1 | Fac_2 |
|---|---|---|
| Crowd | 1.051711966 | -1.449755706 |
| Audience | 0.330088794 | -0.781996741 |
| Public | 1.495222413 | 1.099645457 |
| Mob | -1.077959444 | -0.399782573 |
| Primary | -1.533306346 | 0.092941128 |
| Secondary | -0.111491144 | 1.687389219 |
| Modern | -0.154266240 | -0.248440784 |

Component Loadings

| | Fac_1 | Fac_2 |
|---|---|---|
| Intensity | -0.955706429 | 0.172663122 |
| Frequency | -0.815312834 | 0.273875032 |
| FeelBelonging | -0.906934950 | 0.315488683 |

| | | |
|---|---|---|
| PhysicalProx | -0.437574248 | -0.881337749 |
| Formality | -0.918238677 | -0.314500032 |

****************************************************

History of Iterations for Final Metric Configuration

****************************************************

| Iter | Total Fit | Total Loss | Multiple Loss | Single Loss | Difference Iterations |
|---|---|---|---|---|---|
| 1 | 0.92782369 | 1.07217631 | 0.81488911 | 0.25728720 | 0.927823690 |
| 2 | 0.93597230 | 1.06402770 | 0.81920891 | 0.24481879 | 0.008148605 |
| 3 | 0.93914018 | 1.06085982 | 0.82494459 | 0.23591522 | 0.003167889 |
| 4 | 0.94082382 | 1.05917618 | 0.82768069 | 0.23149549 | 0.001683634 |
| 5 | 0.94178269 | 1.05821731 | 0.82857358 | 0.22964373 | 0.000958869 |
| 6 | 0.94230117 | 1.05769883 | 0.82877267 | 0.22892616 | 0.000518482 |
| 7 | 0.94259282 | 1.05740718 | 0.82886112 | 0.22854606 | 0.000291646 |
| 8 | 0.94277449 | 1.05722551 | 0.82901392 | 0.22821160 | 0.000181671 |
| 9 | 0.94289745 | 1.05710255 | 0.82921166 | 0.22789089 | 0.000122963 |
| 10 | 0.94298583 | 1.05701417 | 0.82939040 | 0.22762377 | 8.8381e-005 |

Eigenvalues for Final Solution

| | 1 |
|---|---|
| Fac_1 | 0.748798346 |
| Fac_2 | 0.194187483 |

-------------------------------------------------------------

--- Variable: Intensity Type: Single Nominal Missing: 0 ---

-------------------------------------------------------------

| Category | Marginal Frequency | Category Quantif. | Single Categ. Coordinates Fac_1 | Fac_2 |
|---|---|---|---|---|
| 1 | 2 | -1.31751009 | 1.29950298 | -0.20028575 |
| 2 | 2 | -0.00670580 | 0.00661415 | -0.00101941 |
| 3 | 1 | -0.00794966 | 0.00784101 | -0.00120849 |
| 4 | 2 | 1.32819072 | -1.31003764 | 0.20190940 |

| Category | Multi. Categ. Coordinates Fac_1 | Fac_2 |
|---|---|---|
| 1 | 1.28366924 | -0.30202379 |
| 2 | -0.07766710 | -0.54255987 |
| 3 | 0.23895159 | 1.48376857 |
| 4 | -1.32547793 | 0.10269937 |

```
-------------------------------------------------------------
--- Variable: Frequency Type:  Single Nominal  Missing: 0 ---
-------------------------------------------------------------

              Marginal    Category  Single Categ. Coordinates
    Category Frequency    Quantif.       Fac_1       Fac_2
           1         2 -1.38254826  1.30046587 -0.11988936
           2         2  0.64832208 -0.60983096  0.05622004
           3         2 -0.02976108  0.02799415 -0.00258077
           4         1  1.52797452 -1.43725812  0.13250018

                                    Multi. Categ. Coordinates
    Category                                 Fac_1       Fac_2
           1                            1.28366924 -0.30202379
           2                           -0.64438860 -0.31850620
           3                            0.07889218  0.54933269
           4                           -1.43634564  0.14239459

-----------------------------------------------------------------
--- Variable: FeelBelonging Type:  Single Nominal  Missing: 0 ---
-----------------------------------------------------------------

              Marginal    Category  Single Categ. Coordinates
    Category Frequency    Quantif.       Fac_1       Fac_2
           1         1 -1.81122655  1.53679769 -0.86866170
           2         2 -0.53707959  0.45570372 -0.25758262
           3         2  0.20627685 -0.17502271  0.09893009
           4         2  1.23641602 -1.04907985  0.59298338

                                    Multi. Categ. Coordinates
    Category                                 Fac_1       Fac_2
           1                            1.06363170 -1.70797911
           2                            0.71476990  0.20195749
           3                            0.07889218  0.54933269
           4                           -1.32547793  0.10269937

----------------------------------------------------------------
--- Variable: PhysicalProx Type:  Single Nominal  Missing: 0 ---
----------------------------------------------------------------

              Marginal    Category  Single Categ. Coordinates
    Category Frequency    Quantif.       Fac_1       Fac_2
           1         2 -1.58113883  0.87132919  1.29385005
           2         5  0.63245553 -0.34853167 -0.51754002
```

| | | | Multi. Categ. Coordinates | |
|---|---|---|---|---|
| Category | | | Fac_1 | Fac_2 |
| 1 | | | 0.87132919 | 1.29385005 |
| 2 | | | -0.34853167 | -0.51754002 |

```
------------------------------------------------------------
--- Variable: Formality Type:  Single Nominal  Missing: 0 ---
------------------------------------------------------------
```

| | Marginal | Category | Single Categ. Coordinates | |
|---|---|---|---|---|
| Category | Frequency | Quantif. | Fac_1 | Fac_2 |
| 1 | 1 | -1.79344899 | 1.66566107 | 0.36252918 |
| 2 | 4 | -0.22130527 | 0.20553669 | 0.04473482 |
| 3 | 2 | 1.33933503 | -1.24390391 | -0.27073423 |

| | | | Multi. Categ. Coordinates | |
|---|---|---|---|---|
| Category | | | Fac_1 | Fac_2 |
| 1 | | | 1.50370678 | 1.10393153 |
| 2 | | | 0.28681227 | -0.32733257 |
| 3 | | | -1.32547793 | 0.10269937 |

*******************
Summary of Analysis
*******************

Multiple Fit Stress
-------------------

| Row | Sums | Fac_1 | Fac_2 |
|---|---|---|---|
| Intensity | 1.410343118 | 0.9827 | 0.4277 |
| Frequency | 1.030108816 | 0.8859 | 0.1442 |
| FeelBelonging | 1.328961324 | 0.8113 | 0.5176 |
| PhysicalProx | 0.973305001 | 0.3037 | 0.6696 |
| Formality | 1.110329754 | 0.8720 | 0.2383 |
| Mean | 1.1706 | 0.7711 | 0.3995 |

Single Fit Stress
-----------------

| Row | Sums | Fac_1 | Fac_2 |
|---|---|---|---|
| Intensity | 0.995961326 | 0.9729 | 0.0231 |
| Frequency | 0.892303816 | 0.8848 | 0.0075 |
| FeelBelonging | 0.949940792 | 0.7199 | 0.2300 |
| PhysicalProx | 0.973305001 | 0.3037 | 0.6696 |

|  |  |  |  |
|---|---|---|---|
| Formality | 0.903432608 | 0.8626 | 0.0409 |
| Mean | 0.9430 | 0.7488 | 0.1942 |

----------------------
End of Final Iteration
----------------------

| Iteration Number | Total Fit | Total Loss | Multiple Loss | Single Loss |
|---|---|---|---|---|
| 10 | 0.942988709 | 1.057011291 | 0.829390397 | 0.227620894 |

Correlations Among Optimally Scaled Variables

|  | Intensity | Frequency | FeelBelongin | PhysicalProx |
|---|---|---|---|---|
| Intensity | 1.000000000 | 0.932809737 | 0.911269725 | 0.419147175 |
| Frequency | 0.932809737 | 1.000000000 | 0.796710849 | 0.446611428 |
| FeelBelonging | 0.911269725 | 0.796710849 | 1.000000000 | 0.104609013 |
| PhysicalProx | 0.419147175 | 0.446611428 | 0.104609013 | 1.000000000 |
| Formality | 0.888138735 | 0.795711534 | 0.671938233 | 0.637121237 |

Correlations Among Optimally Scaled Variables

|  | Formality |
|---|---|
| Intensity | 0.888138735 |
| Frequency | 0.795711534 |
| FeelBelonging | 0.671938233 |
| PhysicalProx | 0.637121237 |
| Formality | 1.000000000 |

Object Scores

|  | Fac_1 | Fac_2 |
|---|---|---|
| Crowd | 1.063631698 | -1.707979106 |
| Audience | -0.074166988 | -0.700016547 |
| Public | 1.503706784 | 1.103931527 |
| Mob | -1.214610221 | 0.063004153 |
| Primary | -1.436345644 | 0.142394593 |
| Secondary | 0.238951589 | 1.483768574 |
| Modern | -0.081167219 | -0.385103193 |

Component Loadings

|  | Fac_1 | Fac_2 |
|---|---|---|

| | | |
|---|---|---|
| Intensity | -0.986332469 | 0.152018377 |
| Frequency | -0.940629636 | 0.086716224 |
| FeelBelonging | -0.848484522 | 0.479598591 |
| PhysicalProx | -0.551076964 | -0.818302622 |
| Formality | -0.928747389 | -0.202140780 |

```
                         ************************
                         Object Scores, Unlabeled
                         ************************

       .--+------+------+------+------+------+------+------+---.
 1.484 |                             1                         |
 1.415 |                                                       |
 1.347 |                                                       |
 1.278 |                                                       |
 1.210 |                                                       |
 1.141 |                                                       |
 1.073 |                                                1      |
 1.004 |                                                       |
 0.936 |                                                       |
 0.867 |                                                       |
 0.799 |                                                       |
 0.730 |                                                       |
 0.662 |                                                       |
 0.593 |                                                       |
 0.525 |                                                       |
 0.456 |                                                       |
 0.388 |                                                       |
 0.319 |                                                       |
 0.251 |                                                       |
 0.182 |                                                       |
 0.114 |   1                                                   |
 0.045 |       1                                               |
-0.023 |                                                       |
-0.092 |                                                       |
-0.160 |                                                       |
-0.229 |                                                       |
-0.297 |                                                       |
-0.366 |                        1                              |
-0.435 |                                                       |
-0.503 |                                                       |
-0.572 |                                                       |
-0.640 |                                                       |
-0.709 |                        1                              |
-0.777 |                                                       |
```

```
-0.846 |                                                             |
-0.914 |                                                             |
-0.983 |                                                             |
-1.051 |                                                             |
-1.120 |                                                             |
-1.188 |                                                             |
-1.257 |                                                             |
-1.325 |                                                             |
-1.394 |                                                             |
-1.462 |                                                             |
-1.531 |                                                             |
-1.599 |                                                             |
-1.668 |                                                             |
-1.736 |                                            1                |
       .--+------+------+------+------+------+------+------+---.
         -1.56  -1.10  -0.63  -0.17   0.30   0.77   1.23   1.70
```

```
     ****************************************
     Object Scores, Labeled by Variable Number
     ****************************************
```

```
       .--+------+------+------+------+------+------+------+---.
 0.480 |                         2                                 |
 0.452 |                                                           |
 0.424 |                                                           |
 0.396 |                                                           |
 0.368 |                                                           |
 0.340 |                                                           |
 0.312 |                                                           |
 0.285 |                                                           |
 0.257 |                                                           |
 0.229 |                                                           |
 0.201 |                                                           |
 0.173 |                                                           |
 0.145 |                                                           |
 0.117 |                                                           |
 0.090 |                     1                                     |
 0.062 |                                                           |
 0.034 |                                                           |
 0.006 |                                                           |
-0.022 |                                                           |
-0.050 |                                                           |
-0.078 |                                                           |
-0.105 |                                                           |
-0.133 |                                                           |
```

```
 -0.161 |                                                  |
 -0.189 |                 4                                |
 -0.217 |                                                  |
 -0.245 |                                                  |
 -0.273 |                                                  |
 -0.300 |                                                  |
 -0.328 |                                                  |
 -0.356 |                                                  |
 -0.384 |                                                  |
 -0.412 |                                                  |
 -0.440 |                                                  |
 -0.468 |                                                  |
 -0.495 |                                                  |
 -0.523 |                                                  |
 -0.551 |                                                  |
 -0.579 |                                                  |
 -0.607 |                                                  |
 -0.635 |                                                  |
 -0.663 |                                                  |
 -0.690 |                                                  |
 -0.718 |                                                  |
 -0.746 |                                                  |
 -0.774 |                                                  |
 -0.802 |                                                  |
 -0.830 |                             3                    |
        .--+------+------+------+------+------+------+------+---.
          -1.42  -1.23  -1.04  -0.85  -0.66  -0.47  -0.28  -0.09
```

This is an output of the return arguments:

```
  print "GOF=",  gof;
  print "Eigenvalues=",  eval;
  print "Loadings=", load;
  print "Scores=", scors;
  print "CatCoord=",coord;
  print "Single Quant=",squant;
  print "Multiple Quant=",mquant;
```

```
GOF=
           |         1
----------------------
   Failure |   0.00000
      Time |   0.00000
    TotFit |   0.94299
   TotLoss |   1.05701
```

```
  MultLoss |   0.82939
  SingLoss |   0.22762
    unused |   0.00000
    unused |   0.00000
    unused |   0.00000
    unused |   0.00000


Eigenvalues=
   |     Fac_1     Fac_2
------------------------
 1 |   0.74880   0.19419


Loadings=
                |     Fac_1     Fac_2
-------------------------------------
      Intensity |  -0.98633   0.15202
      Frequency |  -0.94063   0.08672
  FeelBelonging |  -0.84848   0.47960
   PhysicalProx |  -0.55108  -0.81830
      Formality |  -0.92875  -0.20214


Scores=
            |     Fac_1     Fac_2
---------------------------------
      Crowd |   1.06363  -1.70798
   Audience |  -0.07417  -0.70002
     Public |   1.50371   1.10393
        Mob |  -1.21461   0.06300
    Primary |  -1.43635   0.14239
  Secondary |   0.23895   1.48377
     Modern |  -0.08117  -0.38510


CatCoord=
                    |         1
-------------------------------
      Intensity_C_1 |  -1.31751
      Intensity_C_2 |  -0.00671
      Intensity_C_3 |  -0.00795
      Intensity_C_4 |   1.32819
      Frequency_C_1 |  -1.38255
      Frequency_C_2 |   0.64832
      Frequency_C_3 |  -0.02976
      Frequency_C_4 |   1.52797
```

| | |
|---|---|
| FeelBelonging_C_1 | -1.81123 |
| FeelBelonging_C_2 | -0.53708 |
| FeelBelonging_C_3 | 0.20628 |
| FeelBelonging_C_4 | 1.23642 |
| PhysicalProx_C_1 | -1.58114 |
| PhysicalProx_C_2 | 0.63246 |
| Formality_C_1 | -1.79345 |
| Formality_C_2 | -0.22131 |
| Formality_C_3 | 1.33934 |

Single Quant=

| | Fac_1 | Fac_2 |
|---|---|---|
| Intensity_C_1 | 1.29950 | -0.20029 |
| Intensity_C_2 | 0.00661 | -0.00102 |
| Intensity_C_3 | 0.00784 | -0.00121 |
| Intensity_C_4 | -1.31004 | 0.20191 |
| Frequency_C_1 | 1.30047 | -0.11989 |
| Frequency_C_2 | -0.60983 | 0.05622 |
| Frequency_C_3 | 0.02799 | -0.00258 |
| Frequency_C_4 | -1.43726 | 0.13250 |
| FeelBelonging_C_1 | 1.53680 | -0.86866 |
| FeelBelonging_C_2 | 0.45570 | -0.25758 |
| FeelBelonging_C_3 | -0.17502 | 0.09893 |
| FeelBelonging_C_4 | -1.04908 | 0.59298 |
| PhysicalProx_C_1 | 0.87133 | 1.29385 |
| PhysicalProx_C_2 | -0.34853 | -0.51754 |
| Formality_C_1 | 1.66566 | 0.36253 |
| Formality_C_2 | 0.20554 | 0.04473 |
| Formality_C_3 | -1.24390 | -0.27073 |

Multiple Quant=

| | | |
|---|---|---|
| Intensity_C_1 | 1.28367 | -0.30202 |
| Intensity_C_2 | -0.07767 | -0.54256 |
| Intensity_C_3 | 0.23895 | 1.48377 |
| Intensity_C_4 | -1.32548 | 0.10270 |
| Frequency_C_1 | 1.28367 | -0.30202 |
| Frequency_C_2 | -0.64439 | -0.31851 |
| Frequency_C_3 | 0.07889 | 0.54933 |
| Frequency_C_4 | -1.43635 | 0.14239 |
| FeelBelonging_C_1 | 1.06363 | -1.70798 |
| FeelBelonging_C_2 | 0.71477 | 0.20196 |
| FeelBelonging_C_3 | 0.07889 | 0.54933 |

```
FeelBelonging_C_4 |  -1.32548   0.10270
 PhysicalProx_C_1 |   0.87133   1.29385
 PhysicalProx_C_2 |  -0.34853  -0.51754
    Formality_C_1 |   1.50371   1.10393
    Formality_C_2 |   0.28681  -0.32733
    Formality_C_3 |  -1.32548   0.10270
```