

# Automatic Model Improvement using the `cfa(.)` Function in CMAT

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This is a first preliminary version of a paper describing the automatic CFA model improvement algorithm implemented in the `cfa()` function.

## 1 General Remarks

The syntax of the function

```
< gof,est,resi,cov,mod1,mod2,boci> = cfa(data,optn<,patt<,scal<,wgt<,xini<,targ<,wtrg> . >)
```

is explained in detail in the *Reference Manual* of CMAT. (Note, that some of the input and output arguments are lists of matrices.)

The factor model

$$\mathbf{C} = \mathbf{LPL}^T + \text{diag}(\mathbf{U})$$

where  $\mathbf{C}$  is a  $n \times n$  covariance or correlation matrix,  $\mathbf{L}$  is a  $n \times f$  factor loading matrix,  $\mathbf{P}$  the symmetric  $f \times f$  matrix of factor correlations (with unit diagonal), and  $\mathbf{U} = \text{diag}$  is a  $n \times n$  diagonal matrix of unique variances. Currently, the model is restricted to  $\text{diag}(\mathbf{U})$  but an extension is intended for specifying CFA patterns for general symmetric  $\mathbf{U}$ .

The `cfa()` function implements ULS, ML, GLS, WLS, and DWLS estimation for confirmatory (CFA) and exploratory (EFA) factor analysis. From the estimation methods, WLS and DWLS estimation require either raw data input or the specification of a weight matrix with the fifth input argument. The input data may be a  $N \times n$  matrix,  $N$  observations,  $n$  variables, of raw data or a symmetric  $n \times n$  matrix of correlations or covariances. Analyzed is

- the covariance matrix,
- the correlation matrix like a covariance matrix,
- the correlation matrix without fitting the unit diagonal.

The `cfa()` function is able to analyze multiple  $s \geq 1$  data samples with a simultaneous model. For multiple sample analysis the raw data may contain an ID variable specifying the sample number. But the user may also specify lists of input covariance or correlation matrices. Different parameter patterns and even different factor numbers can be specified for the different samples.

If the pattern input argument `patt` is specified as nonmissing, the number  $f$  of factors of the CFA is defined, otherwise it must be specified with the `nfact` option.

One of the larger new developments for the `cfa()` function is an algorithm which tries to find a zero pattern in loading matrix  $\mathbf{F}$  and factor correlations  $\mathbf{P}$  which corresponds to a large  $p$  value of the  $\chi^2$  fit criterion. Since the maximization of  $p$  is numerically very difficult we cannot guarantee that the function will always find a global maximum, but in many applications it found CFA models with surprisingly large  $p$  values.

The CFA model improvement algorithm works within a known number  $f$  of factors. There are two versions:

- if the `patt` input argument is specified as missing: the algorithm start with an EFA and computes a CFA pattern which corresponds to a larger  $p$  value
- if the `patt` input argument is specified as nonmissing: the algorithm start with that CFA specification and tries to improve the CFA pattern for obtaining a larger  $p$  value.

Especially the second version of starting point permits the testing of optimality (w.r.t the  $p$  value) of published CFA specifications. For many well know published model small modifications exist which show larger  $p$  values.

The `cfa()` function is able to analyze ordinal data by computing polychoric correlations. The `scal` input argument should be a binary vector specifying which of the variables of the input raw data are scaled ordinary (set to 1) and which is metric (continuous,interval) (set to 0). This input argument may be a missing value indicating the default, that all variables are continuous. If there are variables of different scaling type (ordinal and continuous) the continuous variables are binned to ordinal (by default into 16 bins, but maybe specified differently using the "`nbin`" option). Note, that nominal variables with  $k$  levels may always be rescaled to  $k - 1$  binary dummy variables.

Except for GLS estimation the `cfa` function is able to perform *genuine* correlation analysis. The difference between the classical covariance analysis (even of correlation matrices) and genuine correlation analysis are:

**covariance analysis** For EFA both, the loadings  $\mathbf{L}$  and unique variances  $\mathbf{U}$  are estimated by fitting the model to the covariance or correlation matrix including its diagonal.

**correlation analysis** The diagonal of the correlation matrix is considered fixed to unity and the model is not fitting the diagonal of the matrix. That means, for EFA only the loadings  $\mathbf{L}$  are estimated, afterward the unique variances  $\mathbf{U}$  are simply set to the difference between the unit diagonal and the communalities  $h_i^2 = \sum_{j=1}^{nvar} l_{ij}^2 \quad i = 1, \dots, n$ .

Genuine ML correlation analysis is computationally expensive and may become very messy for Heywood cases, which occurs when the communalities  $h_i^2$  become larger than 1. And because the degrees of freedom are reduced by  $n$  it is harder to fit applications for small  $n$ . Especially for ordinal data when the polychoric correlation matrix is fitted, the asymptotic covariance matrix becomes indefinite and the ML fit criterion may become negative. For polychoric correlation matrices with negative eigenvalues (indefinite) the `svd()` function will not permit the fit of ML or GLS estimates, therefore ULS, WLS or DWLS estimation maybe used instead.

*Note also, that the ML fit criterion for genuine correlation analysis is changed and does not include the constant term  $\log(\det(\mathbf{S}))$ , since the polychoric correlation matrix may have negative eigenvalues and therefore a negative determinant.* Note, that WLS estimation requires many data observations so that the large

$\binom{n}{2} \times \binom{n}{2}$  weight matrix  $\mathbf{W}$  (based on fourth order moments) becomes nonsingular.

Except for ULS estimation `cfa()` is able to compute standard normal and S-B robust asymptotic standard errors and confidence intervals.

The degrees of freedom are defined as:

- analyzing covariance matrix or correlation as covariance matrix:  $df = s \binom{n}{2} - q$
- genuine correlation analysis:  $df = s \left( \binom{n}{2} - n \right) - q$

using  $s$  for the number of samples and  $q$  for the number of model parameters. For exploratory analysis it also implements a number of popular orthogonal and oblique rotation methods. In addition it incorporates robust (nonnormal, Satorra-Bentler) goodness of fit indices, robust ASEs, and nonnormal (robust) standardized residuals for ULS, ML, and GLS estimates.

The following remarks on rotation apply only to EFA, since CFA solutions are not invariant to rotation:

1. The same rotation methods as in the `frotate` or the older `factor` function are also available in the `cfa()` function.
2. Except for ULS, asymptotic standard errors and confidence intervals are computed for rotated factor loadings, the unique variances, and the factor correlations (when oblique rotation is applied).
3. Also for oblique rotation, the *factor structure* is computed with asymptotic standard errors and confidence intervals.

The asymptotic standard errors and Wald confidence intervals of rotated factor loadings depend on:

1. the kind of rotation method applied and especially the fact whether the rotation is orthogonal or oblique;
2. if the (unstandardized) covariance or the (standardized) correlation matrix is analysed. (Note, you may read in a correlation matrix, but if you specify the analysis option "`anal`" as "`cov`", the results correspond to those of a covariance analysis.)
3. if Kaiser normalization of the loadings is applied for the specific rotation method.

Comparison of `factor()`, `sem`, and `cfa` functions in CMAT:

Feature	factor	cfa	sem
automatic model improvement	-	+	-
multiple sample analysis	-	+	+
standard normal ASEs and CIs	+	+	+
robust CIs and fit measures	-	+	+
profile likelihood CIs	+	+	+
bootstrapping CIs and GOF	+	+	+
Bollen-Stine bootstrap for $p$	+	+	+
pattern specification (*1)	-	+	+
modification indices (*2)	-	+	+
different factor numbers (*3)	-	+	+
ordinal data (polychoric corrs) (*4)	-	+	-
genuine correlation analysis (*5)	-	+	+/-
ULS	+	+	+
GLS (*6)	-	+/-	+
ML (*7)	+	+	+
WLS (*8)	-	+	+
DWLS	-	+	+
standardized factor loadings	-	+	+
determining number of factors (*9)	+	-	-
rotation to simple structure (*10)	+	+/-	+

Of course, the `sem` function is able to analyze models with more general form than that of the exploratory (EFA) or confirmatory (CFA) factor analysis.

- (\*1) there is a default parameter pattern for EFA
- (\*2) for EFA the asymptotic standard errors and the confidence intervals replace the modification indices
- (\*3) pattern definition may specify different number of factors for different samples
- (\*4) for the analysis of ordinal data the polychoric correlation matrix (or  $s$  matrices) may also be computed using the `polychor()` function and provided as input into the `cfa`, `sem`, or `factor` functions
- (\*5) correlation data can be analyzed with all functions, `sem`, `factor` and `cfa`, however the `cfa` function is able to model correlation data with a specific model which results in a model matrix with fixed unit diagonal; the `sem` function permits specifying constraints for models similar to the `cfa` correlation structure model
- (\*6) GLS estimation is not available for polychoric correlations and correlation analysis (polychoric correlation matrices may not be pd)
- (\*7) ML estimation for correlation type analysis is still computational inefficient

- (\*8) note that WLS estimation requires a minimum set of  $\binom{n}{2}$  independent observations (cases) for a nonsingular weight matrix (based on the fourth order moments of the data)
- (\*9) for CFA, usually the pattern definition defines the number of factors; for EFA the `factor` function offers a number of test criteria for the number of factors
- (\*10) only EFA solutions can be rotated, for CFA the specified pattern and goodness of fit is affected by rotation

Flow of computations in `factor()` and `cfa()`:

<b>FACTOR</b>	<b>CFA</b>
1. input data	1. input data (for multiple samples)
2. compute criteria for number of factors	2. for CFA: input parameter pattern (models for all samples) (unnecessary for EFA)
3. obtain suitable starting values for default pattern	3. obtain suitable starting values for default pattern
4. estimate optimal parameters and standard normal ASEs	4. estimate optimal parameters and standard normal ASEs
5. compute goodness-of-fit measures	5. compute goodness-of-fit measures
6. orthogonal or oblique rotation to simple structure with re-computation of standard normal ASEs	6. optional: compute robust goodness-of-fit measures and robust ASEs
7. optional: compute profile likelihood CIs	7. compute modification indices
8. optional: bootstrap for GOFs and confidence intervals	8. optional: compute profile likelihood CIs
	9. optional: perform automatic model improvement starting with EFA or CFA depending on step 2; at the end repeat steps 5, 6, 7 for best solution
	10. optional: bootstrap for GOFs and confidence intervals

Depending on our experience with this new algorithm we may extend it to solve some more complicate models, like

1. optimization w.r.t. other fit criteria, like GFI, RMSEA etc.

2. permit offdiagonal parameter estimates in a symmetric matrix  $\mathbf{U}$

$$\mathbf{C} = \mathbf{LPL}^T + \mathbf{U}$$

3. hierarchical factor analysis models,
4. linear and nonlinear constraints on the parameters.

Some of our findings:

- Most CFA models with a large  $p$  value are not very sparse. Usually they are close to an EFA, but with moderate increase in the  $\chi^2$  value and a substantial increase in the degrees of freedom.
- Not surprisingly, for larger  $f$  values it becomes easier to obtain  $p > 0.01$  values. In all applications we were able to find CFA models with better fit than EFA for the same number  $f$  of factors.
- It should be quite disturbing to researchers in the field that there are normally many very different CFA models structures among those with  $p > 0.01$  values, depending on the value of  $f$  (`cfa` stores the best 20 models found). And it may be hard to explain to researchers who want a foundation of their theory.

## 2 Some CFA Applications Revisited

The following tables show the  $\chi^2$ , degrees of freedom, and the  $p$  value for a number of well known CFA examples. The values are reported for:

**EFA** the exploratory factor analysis ( $f$  factors)

**EFA(imp)** the CFA result of the automatic model improvement algorithm starting at the EFA result

**CFA** the most common CFA model reported in the literature

**CFA(imp)** the CFA result of the automatic model improvement algorithm starting at the (formerly published) CFA result

Our algorithm tries to find a CFA model with corresponding  $p$  value as large as possible. In practice, this must not be the model which is most appropriate for the underlying theory. However, in our experience researchers in Behavioral Sciences are many times hard pressed for developing a CFA model with a  $p$  value large enough to be accepted by others. And this is especially so when the data have more than  $n = 20$  variables. As the example with the *Import/Export Car Data* shows, our algorithm is not always able to find a model with  $p > 0.01$  or even  $p > 0.05$ , especially when the specified number  $f$  of factors is relatively small.

The following examples are in `tcfa11.inp`, `tcfa12.inp`, `tcfa13.inp`:

Lot Data (Maydeu-Olivares)			
N=389, n=8, f=2, ML P.-M. CORR like COV			
	$\chi^2$	<i>df</i>	<i>p</i>
EFA	17.51	13	0.177
EFA(imp)	17.55	15	0.287
CFA	28.66	19	0.072
CFA(imp)	17.55	15	0.287
Lot Data (Maydeu-Olivares)			
N=389, n=8, f=2, ML Poly gen. CORR			
	$\chi^2$	<i>df</i>	<i>p</i>
EFA	21.8634	13	0.0575
EFA(imp)	23.4828	16	0.1014
CFA	35.5829	19	0.0119
CFA(imp)	21.9038	15	0.1104
Lot Data (Maydeu-Olivares)			
N=389, n=8, f=2, WLS Poly gen. CORR			
	$\chi^2$	<i>df</i>	<i>p</i>
EFA	16.0461	13	0.2467
EFA(imp)	17.4932	17	0.4215
CFA	25.2397	19	0.1528
CFA(imp)	17.1661	17	0.4432
Lot Data (Maydeu-Olivares)			
N=389, n=8, f=2, DWLS Poly gen. CORR			
	$\chi^2$	<i>df</i>	<i>p</i>
EFA	5.0867	13	0.9732
EFA(imp)	5.0928	14	0.9845
CFA	12.7332	19	0.8519
CFA(imp)	5.1180	15	0.9911
Import/Export Car Data (Sarle) SEM48:			
N=82, n=10, f=2, ML CORR			
	$\chi^2$	<i>df</i>	<i>p</i>
EFA	68.69	26	1.027e-5
EFA(imp)	69.78	31	8.159e-5
24 Psychological Tests(Harman,1976), SEM28:			
N=145, n=13, f=2, ML CORR			
	$\chi^2$	<i>df</i>	<i>p</i>
EFA	30.22	23	0.143
EFA(imp)	32.92	37	0.6608
EFA(imp)	36.65	43	0.7402

The following examples are in `tcfa14.inp`:

Lawley-Maxwell Data, SEM06: N=100, n=8, f=3, ML CORR			
	$\chi^2$	<i>df</i>	<i>p</i>
EFA	1.78	7	0.9708
EFA(imp)	2.29	12	0.9988
CFA	2.45	10	0.9916
CFA(imp)	2.29	12	0.9988
Simulated CFA (Bentler, 1985) SEM12: N=100, n=8, f=2, ML CORR			
	$\chi^2$	<i>df</i>	<i>p</i>
EFA	9.731	4	0.0452
EFA(imp)	10.88	7	0.1439
CFA	16.69	8	0.0335
CFA(imp)	10.88	7	0.1439
Ability and Aspiration, LISREL VI, SEM13: N=556, n=6, f=2, ML CORR			
	$\chi^2$	<i>df</i>	<i>p</i>
EFA	4.592	4	0.3317
EFA(imp)	5.366	7	0.6154
CFA	9.256	8	0.3212
CFA(imp)	5.366	7	0.6154
Nine Psychological Variables, LISREL VI, SEM16: N=145, n=9, f=3, ML CORR			
	$\chi^2$	<i>df</i>	<i>p</i>
EFA	9.9482	12	0.6205
EFA(imp)	11.251	20	0.9394
CFA	29.008	23	0.1801
CFA(imp)	11.251	20	0.9394

The following examples are in `tcfa15.inp`, `tcfa16.inp`:

Thurstone (1931) Data, SEM09: N=213, n=9, f=3, ML CORR			
	$\chi^2$	<i>df</i>	<i>p</i>
EFA	2.9032	12	0.9962
EFA(imp)	3.1213	16	0.9998
CFA	38.196	24	0.0331
CFA(imp)	3.1065	16	0.9998
Speed factor Data by Lord (1956), SEM33: N=649, n=18, f=6, ML COV			
	$\chi^2$	<i>df</i>	<i>p</i>
EFA	77.248	60	0.0662
EFA(imp)	83.5408	90	0.6713
CFA	174.503	117	0.0005
CFA(imp)	81.1435	94	0.8251
Miller-Lutz Data, SEM34: N=51, n=8, f=4, ML COV			
	$\chi^2$	<i>df</i>	<i>p</i>
EFA	0.7566	2	0.6850
EFA(imp)	0.7819	6	0.9925
CFA (2)	265.34	28	0.0000
CFA (4)	220.93	18	0.0000
Data from LAWLEY & MAXWELL (1971,p.96), SEM43: N=73, n=9, f=3, ML COV			
	$\chi^2$	<i>df</i>	<i>p</i>
EFA	5.5922	12	0.9352
EFA(imp)	6.1829	17	0.9919
CFA	5.5922	12	0.9352
CFA(imp)	6.0717	18	0.9959
JAMES, MULAİK & BRETT (1982), SEM45: N=800, n=12, f=3, ML COV			
	$\chi^2$	<i>df</i>	<i>p</i>
EFA	22.3663	33	0.9191
EFA(imp)	23.4282	43	0.9935
CFA	42.3613	51	0.8001
CFA(imp)	24.7974	46	0.9955

The following examples are in `tcfa16.inp`:

JOERESKOG (Enslein, Ralston & Wilf,1977), SEM46: N=286, n=9, f=3, ML CORR			
	$\chi^2$	<i>df</i>	<i>p</i>
EFA	33.5157	12	0.0008
EFA(imp)	34.4539	16	0.0047
CFA (2)	138.4970	19	0.0000
CFA(2,imp)	138.1978	22	0.0000
CFA (3)	33.5157	12	0.0008
CFA(3,imp)	35.3875	16	0.0035
Attitudes of Morality and Equality, SEM61: N=200, n=8, f=2, ML CORR			
	$\chi^2$	<i>df</i>	<i>p</i>
EFA	21.3073	13	0.0671
EFA(imp)	22.3793	15	0.0982
CFA	41.5842	19	0.0020
CFA(imp)	22.3793	15	0.0982
Sympathy and Anger, BOLLEN (1989), SEM72: N=138, n=6, f=2, ML COV			
	$\chi^2$	<i>df</i>	<i>p</i>
EFA	5.7654	4	0.2174
EFA(imp)	6.0878	6	0.4134
CFA	126.57	8	0.0000
CFA(imp)	6.0878	6	0.4134
Eight Physical Variables (Harman, 1967), SEM80: N=305, n=8, f=2, ML CORR			
	$\chi^2$	<i>df</i>	<i>p</i>
EFA	76.9612	13	4.09e-11
EFA(imp)	78.6508	16	2.91e-10
CFA (1)	169.66	20	1.01e-25
CFA(1,imp)	78.6508	16	2.91e-10
CFA (2)	102.25	19	2.09e-13
CFA(2,imp)	78.6508	16	2.91e-10
Eight Physical Variables (Harman, 1967), SEM80: N=305, n=8, f=3, ML CORR			
	$\chi^2$	<i>df</i>	<i>p</i>
EFA	23.0148	7	0.0017
EFA(imp)	25.2517	13	0.0214

We note, that we were always able to find CFA models with *p* values larger than those obtained from the EFA. Of course we cannot say the same thing for the  $\chi^2$  value, since any constraint will increase the  $\chi^2$  value of the EFA solution. The increase of the *p* value thus is based on the increase of the degrees of freedom associated with a moderate increase of the  $\chi^2$  value.

Also, the  $p$  values of the models found with our algorithm (EFA(imp)) are always larger than those reported for models in the literature (CFA). That means, even when the models found with our algorithm must not be the best, from a theoretical point of view it may be a good starting guess for a model that better describes the theory.

## 2.1 Lot Data (Maydeu-Olivares, 2009)

We analyzed the data in two ways by

1. traditional approach: treating the ordinal data as metric and fitting product-moment correlations including the diagonal of the correlation matrix,
2. new approach: computing polychoric correlations for the ordinal data only fitting the offdiagonal entries of the correlation matrix.

### 2.1.1 Lot Data: ML for Product Moment Correlations

Here we want to show the most common approach, fitting product moment correlations including the diagonal (i.e. equivalent to covariance analysis).

Some univariate moments when treating data as metric:

Name	Mean	Std Dev	Skewness	Kurtosis
i1	2.244215938	1.00744584	-0.12460927	-0.64820145
i4	2.439588689	0.99201754	-0.34919434	-0.35531466
i5	2.596401028	0.98916158	-0.56683881	-0.11488129
i11	2.336760925	0.98557158	-0.56994653	-0.09588667
i3	1.845758355	1.05364549	0.24526977	-0.72410856
i8	1.393316195	1.03149204	0.62842416	-0.14311245
i9	1.321336761	1.00107276	0.68497038	0.00779773
i12	1.398457584	1.06881652	0.70560377	-0.23352418

Variance Divisor = 388

Mardia's Multivariate Kurtosis . . . . .	18.39
Relative Multivariate Kurtosis . . . . .	1.23
Normalized Multivariate Kurtosis . . . . .	14.34
Mardia Based Kappa (Browne, 1982). . . . .	0.2298
Mean Scaled Univariate Kurtosis . . . . .	-0.09613
Adjusted Mean Scaled Univariate Kurtosis . . . . .	-0.08896

Observation numbers with largest contribution to kurtosis

294	94	376	381	335
-----	----	-----	-----	-----

1760.120 904.9803 841.3284 743.3313 528.4092

The matrix of product moment correlations:

Correlation Matrix

	i1	i4	i5	i11	i3
i1	1.0000000				
i4	0.5112302	1.0000000			
i5	0.4405557	0.5332253	1.0000000		
i11	0.2518052	0.3437831	0.2243751	1.0000000	
i3	-0.1562358	-0.2234606	-0.2552427	-0.1086935	1.0000000
i8	-0.2786839	-0.3759395	-0.3315373	-0.1940056	0.5041608
i9	-0.2390110	-0.2905373	-0.3007501	-0.2222896	0.5089277
i12	-0.2150692	-0.3455020	-0.2960516	-0.2549389	0.4369115

Correlation Matrix

	i8	i9	i12
i8	1.0000000		
i9	0.7009527	1.0000000	
i12	0.5377681	0.5183549	1.0000000

Determinant = 7.69133189e-002 (Ln = -2.5651e+000)

This output shows the EFA model fit:

```
*****
Starting Model Improvement (ChiSq=17.5081 Pvalue=0.177104 DF=13)
*****
```

The following table presents a summary of the best 13 solutions. Note, that some models have been found with the same  $\chi^2$  values and degrees of freedom but with different parametrizations in  $\mathbf{L}$  and  $\mathbf{P}$ . In this example we obtain equivalent fit for models with uncorrelated  $\mathbf{P}=\mathbf{I}$  and correlated factors  $p[1,2] \neq 0$ .

```
*****
Table of 20 Best Pattern Solutions
*****
```

N	Npar	DF	Chisquared	P
1	21	15	17.5451923	0.28732998
2	21	15	17.5451923	0.28732998

3	21	15	17.6276394	0.28274721
4	20	16	19.0657624	0.26526770
5	20	16	19.0657624	0.26526770
6	19	17	20.9052521	0.23055711
7	19	17	20.9052521	0.23055711
8	22	14	17.5231748	0.22936779
9	22	14	17.5231748	0.22936779
10	22	14	17.5231748	0.22936779
11	22	14	17.5231749	0.22936779
12	22	14	17.5231749	0.22936779
13	22	14	17.5231749	0.22936779
14	20	16	19.8038723	0.22918436
15	22	14	17.5289063	0.22908589
16	22	14	17.5289063	0.22908589
17	22	14	17.5289063	0.22908589
18	20	16	19.8263737	0.22814273
19	20	16	19.8263737	0.22814273
20	21	15	18.9084259	0.21791361

N	RMSEA	ECVI	SRMR	PGFI	SBC
1	0.02091216	0.15603751	0.02059178	0.52983547	-71.9084978
2	0.02091216	0.15603751	0.02059178	0.52983547	-71.9084978
3	0.02124816	0.15625000	0.02071255	0.52980257	-71.8260507
4	0.02222252	0.15467946	0.02452283	0.56460867	-76.3515071
5	0.02222252	0.15467946	0.02452283	0.56460867	-76.3515071
6	0.02433236	0.15414337	0.02692954	0.59928327	-80.4755967
7	0.02433236	0.15414337	0.02692954	0.59928327	-80.4755967
8	0.02546755	0.16125781	0.02045578	0.49451849	-65.9669360
9	0.02546755	0.16125781	0.02045576	0.49451849	-65.9669360
10	0.02546755	0.16125781	0.02045568	0.49451849	-65.9669360
11	0.02546755	0.16125781	0.02045591	0.49451849	-65.9669359
12	0.02546755	0.16125781	0.02045593	0.49451849	-65.9669359
13	0.02546755	0.16125781	0.02045593	0.49451849	-65.9669359
14	0.02475353	0.15658181	0.02553610	0.56440874	-75.6133972
15	0.02548826	0.16127258	0.02061294	0.49451876	-65.9612045
16	0.02548826	0.16127258	0.02061293	0.49451876	-65.9612045
17	0.02548826	0.16127258	0.02061291	0.49451876	-65.9612045
18	0.02482663	0.15663980	0.02460313	0.56435741	-75.5908958
19	0.02482663	0.15663980	0.02460315	0.56435741	-75.5908958
20	0.02591430	0.15955100	0.02415190	0.52937518	-70.5452643

The following shows details of the best solution found:

Loadings and Uni. Vars. with ASE's and Wald CIs

```

-----
                FAC_1                FAC_2
i1    0.6297975  0.0519471            0            0
      [ 0.527983, 0.731612] [            0,            0]

i4    0.8255722  0.0517764            0            0
      [ 0.724092, 0.927052] [            0,            0]

i5    0.6110768  0.0617574 -0.0887525  0.0576947
      [ 0.490034, 0.732119] [-0.201832, 0.024327]

i11   0.3656166  0.0643032 -0.0765299  0.0617991
      [ 0.239585, 0.491649] [-0.197654, 0.044594]

i3           0            0  0.6179993  0.0492443
      [            0,            0] [ 0.521482, 0.714516]

i8   -0.0972515  0.0510824  0.7888890  0.0528204
      [-0.197371, 2.9e-003] [ 0.685363, 0.892415]

i9           0            0  0.8350369  0.0458291
      [            0,            0] [ 0.745214, 0.924860]

i12  -0.1573878  0.0565717  0.5679202  0.0554314
      [-0.268266,-0.046509] [ 0.459277, 0.676564]

```

```

                U_Var
i1    0.6033552  0.0536629
      [ 0.498178, 0.708532]

i4    0.3184306  0.0565624
      [ 0.207570, 0.429291]

i5    0.5712696  0.0520766
      [ 0.469201, 0.673338]

i11   0.8359933  0.0628656
      [ 0.712779, 0.959208]

i3    0.6180768  0.0498835
      [ 0.520307, 0.715847]

i8    0.3010894  0.0378062
      [ 0.226991, 0.375188]

```

i9 0.3027134 0.0405970  
[ 0.223145, 0.382282]

i12 0.5745127 0.0462982  
[ 0.483770, 0.665256]

Factor Correlations Phi

	FAC_1	FAC_2
FAC_1	1.0000000	0 -0.4373454 0.0606098
	[ 1.000000, 1.000000]	[-0.556138,-0.318552]
FAC_2	-0.4373454 0.0606098 1.0000000	0
	[-0.556138,-0.318552]	[ 1.000000, 1.000000]

(1) Standalone Fit Measures: -----  
Fit criterion . . . . . 0.0452  
Normal Th. Chi-square (df = 15) 17.5452 Prob>chi\*\*2 = 0.2873  
Elliptic cor. Chi-square. . . . 14.2661 Prob>chi\*\*2 = 0.5054  
Normal Theory Reweighted LS Chi-square . . . . . 17.2203  
Probability of Close Fit . . . . . 0.9158  
Z-Test of Wilson & Hilferty (1931). . . . . 0.5623  
(2) Incremental Fit Measures: -----  
Null Model Chi-square (df = 28) . . . . . 995.2496  
RMSEA Estimate . . . . . 0.0209 90%C.I.[ . , 0.0544]  
ECVI Estimate . . . . . 0.1560 90%C.I.[ . , 0.1943]  
McDonald's (1989) Centrality. . . . . 0.9967  
Tucker-Lewis Coefficient TLI. . . . . 0.9951  
Bentler & Bonett's (1980) NFI . . . . . 0.9824  
Bentler's Comparative Fit Index CFI . . . . . 0.9974  
Parsimonious NFI (James, Mulaik, & Brett,1982). . . . . 0.5263  
Bollen (1986) Normed Index Rho1 . . . . . 0.9671  
Bollen (1988) Non-normed Index Delta2 . . . . . 0.9974  
(3) Information Criteria: -----  
Akaike's Information Criterion. . . . . -12.4548  
Bozdogan's (1987) CAIC. . . . . -86.9085  
Schwarz's Bayesian Criterion. . . . . -71.9085  
(4) Other Fit Measures: -----  
Goodness of Fit Index (GFI) . . . . . 0.9890  
Parsimonious GFI (Mulaik, 1989) . . . . . 0.5298  
GFI Adjusted for Degrees of Freedom (AGFI). . . . . 0.9737  
Root Mean Square Residual (RMR) . . . . . 0.0206

Standardized Factor Loadings

	FAC_1	FAC_2
i1	0.6297975	0
i4	0.8255722	0
i5	0.6110768	-0.0887525
i11	0.3656166	-0.0765299
i3	0	0.6179993
i8	-0.0972515	0.7888890
i9	0	0.8350369
i12	-0.1573878	0.5679202

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	i1	0.60335517	1.00000000	0.39664483
2	i4	0.31843058	1.00000000	0.68156942
3	i5	0.57126962	1.00000000	0.42873038
4	i11	0.83599328	1.00000000	0.16400672
5	i3	0.61807681	1.00000000	0.38192319
6	i8	0.30108944	1.00000000	0.69891056
7	i9	0.30271343	1.00000000	0.69728657
8	i12	0.57451268	1.00000000	0.42548732

Maydeu-Olivares found the CFA model with zero loadings in  $L[1, 2], L[2, 2], L[3, 2], L[4, 2], L[5, 1], L[6, 1], L[7, 1]$ , is better suitable for his purpose. The following shows some of the results for this model:

Loadings and Uni. Vars. with ASE's and Wald CIs

	FAC_1		FAC_2
i1	0.6358694	0.0516265	0
	[ 0.534683, 0.737055]	[	0, 0]
i4	0.7990097	0.0501158	0
	[ 0.700785, 0.897235]	[	0, 0]
i5	0.6728991	0.0512115	0
	[ 0.572526, 0.773272]	[	0, 0]
i11	0.4062557	0.0547096	0
			0

```

[ 0.299027, 0.513485] [      0,      0]
i3      0      0 0.6130510 0.0490786
[      0,      0] [ 0.516859, 0.709243]
i8      0      0 0.8473233 0.0445086
[      0,      0] [ 0.760088, 0.934559]
i9      0      0 0.8158311 0.0451235
[      0,      0] [ 0.727391, 0.904271]
i12     0      0 0.6521216 0.0483176
[      0,      0] [ 0.557421, 0.746822]

```

```

                                U_Var
i1    0.5956700 0.0530467
      [ 0.491700, 0.699640]
i4    0.3615835 0.0510587
      [ 0.261510, 0.461657]
i5    0.5472068 0.0517912
      [ 0.445698, 0.648716]
i11   0.8349563 0.0633126
      [ 0.710866, 0.959047]
i3    0.6241685 0.0498275
      [ 0.526508, 0.721829]
i8    0.2820432 0.0368046
      [ 0.209908, 0.354179]
i9    0.3344197 0.0376715
      [ 0.260585, 0.408255]
i12   0.5747374 0.0471383
      [ 0.482348, 0.667127]

```

Factor Correlations Phi

```

-----
                                FAC_1                                FAC_2
FAC_1  1.0000000                                0 -0.5365535 0.0484893

```

[ 1.000000, 1.000000] [-0.631591,-0.441516]  
 FAC\_2 -0.5365535 0.0484893 1.0000000 0  
 [-0.631591,-0.441516] [ 1.000000, 1.000000]

Note, that the  $p$  value for this model is still sufficiently large ( $p > 0.05$ ):

(1) Standalone Fit Measures: -----  
 Fit criterion . . . . . 0.0739  
 Normal Th. Chi-square (df = 19) 28.6588 Prob>chi\*\*2 = 0.0715  
 Elliptic cor. Chi-square. . . . 23.3027 Prob>chi\*\*2 = 0.2243  
 Normal Theory Reweighted LS Chi-square . . . . . 27.8669  
 Probability of Close Fit . . . . . 0.7905  
 Z-Test of Wilson & Hilferty (1931). . . . . 1.4659  
 (2) Incremental Fit Measures: -----  
 Null Model Chi-square (df = 28) . . . . . 995.2496  
 RMSEA Estimate . . . . . 0.0362 90%C.I.[ . , 0.0618]  
 ECVI Estimate . . . . . 0.1636 90%C.I.[ . , 0.2118]  
 McDonald's (1989) Centrality. . . . . 0.9877  
 Tucker-Lewis Coefficient TLI. . . . . 0.9853  
 Bentler & Bonett's (1980) NFI . . . . . 0.9712  
 Bentler's Comparative Fit Index CFI . . . . . 0.9900  
 Parsimonious NFI (James, Mulaik, & Brett,1982). . . . . 0.6590  
 Bollen (1986) Normed Index Rho1 . . . . . 0.9576  
 Bollen (1988) Non-normed Index Delta2 . . . . . 0.9901  
 (3) Information Criteria: -----  
 Akaike's Information Criterion. . . . . -9.3412  
 Bozdogan's (1987) CAIC. . . . . -103.6492  
 Schwarz's Bayesian Criterion. . . . . -84.6492  
 (4) Other Fit Measures: -----  
 Goodness of Fit Index (GFI) . . . . . 0.9824  
 Parsimonious GFI (Mulaik, 1989) . . . . . 0.6666  
 GFI Adjusted for Degrees of Freedom (AGFI). . . . . 0.9666  
 Root Mean Square Residual (RMR) . . . . . 0.0334  
 Hoelter's (1983) Critical N . . . . . 410

Standardized Factor Loadings

	FAC_1	FAC_2
i1	0.6358694	0
i4	0.7990097	0
i5	0.6728991	0
i11	0.4062557	0
i3	0	0.6130510

i8	0	0.8473233
i9	0	0.8158311
i12	0	0.6521216

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	i1	0.59567005	1.00000000	0.40432995
2	i4	0.36158355	1.00000000	0.63841645
3	i5	0.54720684	1.00000000	0.45279316
4	i11	0.83495631	1.00000000	0.16504369
5	i3	0.62416848	1.00000000	0.37583152
6	i8	0.28204318	1.00000000	0.71795682
7	i9	0.33441968	1.00000000	0.66558032
8	i12	0.57473739	1.00000000	0.42526261

2.1.2 Lot Data: ML Fit of Polychoric Correlations

The following shows some of the results when fitting only the offdiagonal entries of the polychoric correlation matrix:

\*\*\*\*\*  
Sums of Weights and Frequencies for Class Levels  
\*\*\*\*\*

Variable	Value	Nobs	Proportion	Freq	Weight
	0	13	0.03341902	13	13.0000000
	1	84	0.21593830	84	84.0000000
	2	125	0.32133676	125	125.0000000
	3	129	0.33161954	129	129.0000000
	4	38	0.09768638	38	38.0000000
	0	12	0.03084833	12	12.0000000
	1	56	0.14395887	56	56.0000000
	2	121	0.31105398	121	121.0000000
	3	149	0.38303342	149	149.0000000
	4	51	0.13110540	51	51.0000000
	.....				
	0	69	0.17737789	69	69.0000000
	1	186	0.47814910	186	186.0000000
	2	60	0.15424165	60	60.0000000
	3	58	0.14910026	58	58.0000000
	4	16	0.04113111	16	16.0000000

\*\*\*\*\*  
 Thresholds  
 \*\*\*\*\*

1	-1.83276143	-0.67651357	0.17814153	1.29484771
2	-1.86846944	-0.93533747	-0.03544818	1.12118112
3	-1.90673221	-1.05143443	-0.27068619	0.97603151
4	-1.60738861	-0.88654975	-0.00322189	1.42470796
5	-1.44271677	-0.17159847	0.55195841	1.54068570
6	-0.91556101	0.31102177	0.99699679	1.79924808
7	-0.86761683	0.40028580	1.07410343	1.90673221
8	-0.92540400	0.40028580	0.87704399	1.73770831

Polychoric Correlation(s)

1.0000000				
0.5577219	1.0000000			
0.4859851	0.5877967	1.0000000		
0.2803616	0.3723243	0.2652058	1.0000000	
-0.1636465	-0.2297360	-0.2780625	-0.1216389	1.0000000
-0.3080950	-0.3925849	-0.3718812	-0.1979306	0.5536118
-0.2654568	-0.3005806	-0.3364019	-0.2294538	0.5587520
-0.2585184	-0.3656555	-0.3312995	-0.3016573	0.4891194

Polychoric Correlation(s)

1.0000000			
0.7644719	1.0000000		
0.5980033	0.5932275	1.0000000	

Matrix is Positive Definite: Determinant=0.0414436

S-B Mean-Corr. MVN Test: Chi<sup>2</sup>=1070.92 df=420 pval= 0.0000  
 (See A. Maydeu-Olivares, 2003)

\*\*\*\*\*  
 Correlation ML Factor Analysis  
 (Exploratory Factor Analysis)  
 \*\*\*\*\*

Input Data . . . . . Raw Data  
 Analysis of . . . . . Polychoric W/O Diag  
 Number of Items . . . . . 8  
 Rotation Method . . . . . 2

```

Number of Subjects. . . . . 389
Number of Parameters. . . . . 15
Version . . . . . SEM-Type Matrix Model
Orthogonal Rotation Method. . . . .Varimax (Kaiser, 1958)
Unnormed Rotation: Parameter. . . . . 1.0000000
ASE for Rotated Solution. . . . .Wald (Analyt.)

```

This output shows the EFA model fit:

```

*****
Starting Model Improvement (ChiSq=21.8634 Pvalue=0.0575151 DF=13)
*****

```

```

*****
Table of 20 Best Pattern Solutions
*****

```

N	Npar	DF	Chisquared	P
1	13	15	21.9038083	0.11035108
2	12	16	23.4828056	0.10142491
3	14	14	21.9014747	0.08066159
4	14	14	21.9014747	0.08066159
5	13	15	23.4803746	0.07445955
6	11	17	26.2344729	0.07029331
7	13	15	24.0281274	0.06461737
8	15	13	21.8633824	0.05751505
9	15	13	21.8633824	0.05751505
10	12	16	26.2339567	0.05082391
11	10	18	30.3830713	0.03388456
12	9	19	35.5829340	0.01187108
13	10	18	34.5596562	0.01073329
14	11	17	33.8939594	0.00866469
15	13	15	43.2016837	1.465e-004
16	13	15	43.2016837	1.465e-004
17	13	15	43.3605522	1.384e-004
18	12	16	46.1921741	9.092e-005
19	12	16	46.1921741	9.092e-005
20	11	17	52.6930884	1.598e-005

N	RMSEA	ECVI	SRMR	PGFI	SBC
1	0.03444156	0.12505470	0.03870009	0.53177191	-67.5498819
2	0.03471813	0.12384723	0.04194483	0.56699868	-71.9344639

3	0.03813943	0.13032573	0.03865676	0.49631866	-61.5886361
4	0.03813943	0.13032573	0.03865681	0.49631866	-61.5886361
5	0.03817210	0.12911801	0.04189964	0.53155910	-65.9733156
6	0.03741672	0.12566211	0.04518281	0.60188640	-75.1463759
7	0.03938559	0.13052975	0.04151720	0.53140157	-65.4255628
8	0	0	0	0	0
9	0.04191912	0.13550460	0.03848420	0.46086941	-55.6631490
10	0.04060186	0.13093783	0.04516349	0.56648006	-69.1833128
11	0.04210775	0.13107733	0.04974364	0.63646480	-76.9613569
12	0.04742835	0.13920200	0.05649097	0.67059668	-77.7250735
13	0.04869379	0.14184173	0.05589841	0.63549228	-72.7847720
14	0.05060873	0.14540306	0.05465688	0.60025733	-67.4868895
15	0.06961070	0.17994613	0.08948907	0.52921459	-46.2520065
16	0.06961070	0.17994613	0.08948917	0.52921460	-46.2520065
17	0.06980650	0.18035558	0.08932140	0.52943257	-46.0931379
18	0.06973830	0.18237653	0.09455658	0.56441320	-49.2250954
19	0.06973830	0.18237653	0.09455668	0.56441321	-49.2250954
20	0.07356170	0.19385442	0.10245884	0.59905854	-48.6877604

Loadings and Uni. Vars. with ASE's and Wald CIs

```

-----
                                FAC_1                                FAC_2
i1      0.6477315  0.0484128                                0      0
      [ 0.552844, 0.742619] [      0,      0]

i4      0.8284240  0.0480574                                0      0
      [ 0.734233, 0.922615] [      0,      0]

i5      0.6294115  0.0566221 -0.1003851  0.0525502
      [ 0.518434, 0.740389] [-0.203382, 2.6e-003]

i11     0.3718424  0.0594592 -0.0712707  0.0564553
      [ 0.255304, 0.488380] [-0.181921, 0.039380]

i3              0      0  0.6208688  0.0437439
      [      0,      0] [ 0.535132, 0.706605]

i8     -0.1024494  0.0473546  0.7837677  0.0453847
      [-0.195263, -9.6e-003] [ 0.694815, 0.872720]

i9              0      0  0.8453638  0.0396660
      [      0,      0] [ 0.767620, 0.923108]

i12    -0.1597340  0.0525165  0.5814684  0.0488632

```

[-0.262664,-0.056804] [ 0.485698, 0.677239]

		U_Var
i1	0.5804440	0
	[ 0.580444, 0.580444]	
i4	0.3137136	0
	[ 0.313714, 0.313714]	
i5	0.5427439	0
	[ 0.542744, 0.542744]	
i11	0.8352540	0
	[ 0.835254, 0.835254]	
i3	0.6145219	0
	[ 0.614522, 0.614522]	
i8	0.3103736	0
	[ 0.310374, 0.310374]	
i9	0.2853600	0
	[ 0.285360, 0.285360]	
i12	0.5613796	0
	[ 0.561380, 0.561380]	

Factor Correlations Phi

	FAC_1	FAC_2
FAC_1	1.0000000	0 -0.4037454 0.0568514
	[ 1.000000, 1.000000]	[-0.515172,-0.292319]
FAC_2	-0.4037454 0.0568514	1.0000000 0
	[-0.515172,-0.292319]	[ 1.000000, 1.000000]

(1) Standalone Fit Measures: -----  
Fit criterion . . . . . 0.0565  
Normal Th. Chi-square (df = 15) 21.9038 Prob>chi\*\*2 = 0.1104  
Normal Theory Reweighted LS Chi-square . . . . . 10.9519  
Probability of Close Fit . . . . . 0.7844  
Z-Test of Wilson & Hilferty (1931). . . . . 1.2269

(2) Incremental Fit Measures: -----

Null Model Chi-square (df = 28)	1235.1674
RMSEA Estimate	0.0344 90%C.I.[ . , 0.0636]
ECVI Estimate	0.1251 90%C.I.[ . , 0.1895]
McDonald's (1989) Centrality	0.9912
Tucker-Lewis Coefficient TLI	0.9893
Bentler & Bonett's (1980) NFI	0.9823
Bentler's Comparative Fit Index CFI	0.9943
Parsimonious NFI (James, Mulaik, & Brett,1982)	0.5262
Bollen (1986) Normed Index Rho1	0.9669
Bollen (1988) Non-normed Index Delta2	0.9943

(3) Information Criteria: -----

Akaike's Information Criterion	-8.0962
Bozdogan's (1987) CAIC	-82.5499
Schwarz's Bayesian Criterion	-67.5499

(4) Other Fit Measures: -----

Goodness of Fit Index (GFI)	0.9934
Parsimonious GFI (Mulaik, 1989)	0.5322
GFI Adjusted for Degrees of Freedom (AGFI)	0.9841
Root Mean Square Residual (RMR)	0.0387
Hoelter's (1983) Critical N	444

Standardized Factor Loadings

	FAC_1	FAC_2
i1	0.6477315	0
i4	0.8284240	0
i5	0.6294115	-0.1003851
i11	0.3718424	-0.0712707
i3	0	0.6208688
i8	-0.1024494	0.7837677
i9	0	0.8453638
i12	-0.1597340	0.5814684

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	i1	0.58044397	1.00000000	0.41955603
2	i4	0.31371361	1.00000000	0.68628639
3	i5	0.54274387	1.00000000	0.45725613
4	i11	0.83525399	1.00000000	0.16474601
5	i3	0.61452187	1.00000000	0.38547813
6	i8	0.31037364	1.00000000	0.68962636
7	i9	0.28536004	1.00000000	0.71463996

8 i12            0.56137964   1.00000000   0.43862036

Again, we are fitting only the offdiagonal entries of the polychoric correlation matrix however now with EFA.

\*\*\*\*\*  
Correlation ML Factor Analysis  
(Exploratory Factor Analysis)  
\*\*\*\*\*

Input Data. . . . . Raw Data  
Analysis of . . . . . Polychoric W/O Diag  
Number of Items . . . . . 8  
Rotation Method . . . . . 2  
Number of Subjects. . . . . 389  
Number of Parameters. . . . . 15  
Version . . . . . SEM-Type Matrix Model  
Orthogonal Rotation Method. . . . .Varimax (Kaiser, 1958)  
Unnormed Rotation: Parameter. . . . . 1.0000000  
ASE for Rotated Solution. . . . .Wald (Analyt.)

(1) Standalone Fit Measures: -----  
Fit criterion . . . . . 0.0563  
Normal Th. Chi-square (df = 13) 21.8634 Prob>chi\*\*2 = 0.0575  
Bartlett Corrected Chi-square (df = 13) 21.5910 p= 0.0575  
Normal Theory Reweighted LS Chi-square . . . . . 10.9317  
Probability of Close Fit . . . . . 0.6344  
Z-Test of Wilson & Hilferty (1931). . . . . 1.5779  
(2) Incremental Fit Measures: -----  
Null Model Chi-square (df = 28) . . . . . -388.0000  
RMSEA Estimate . . . . . 0.0419 90%C.I.[ . , 0.0716]  
ECVI Estimate . . . . . 0.1355 90%C.I.[ . , 0.2009]  
McDonald's (1989) Centrality. . . . . 0.9887  
Tucker-Lewis Coefficient TLI. . . . . 1.0459  
Bentler & Bonett's (1980) NFI . . . . . 1.0563  
Bentler's Comparative Fit Index CFI . . . . . 0.0000  
Parsimonious NFI (James, Mulaik, & Brett,1982). . . . . 0.4904  
Bollen (1986) Normed Index Rho1 . . . . . 1.1214  
Bollen (1988) Non-normed Index Delta2 . . . . . 1.0221  
(3) Information Criteria: -----  
Akaike's Information Criterion. . . . . -4.1366  
Bozdogan's (1987) CAIC. . . . . -68.6631  
Schwarz's Bayesian Criterion. . . . . -55.6631  
(4) Other Fit Measures: -----

Goodness of Fit Index (GFI) . . . . .	0.9934
Parsimonious GFI (Mulaik, 1989) . . . . .	0.4612
GFI Adjusted for Degrees of Freedom (AGFI). . . . .	0.9817
Root Mean Square Residual (RMR) . . . . .	0.0385
Hoelter's (1983) Critical N . . . . .	398

Rotated Factor Loadings with Standard Errors

	FAC_1		FAC_2	
i1	0.6235542	0.0422227	-0.1644182	0.0428581
	[ 0.540799,	0.706309]	[-0.248419,	-0.080418]
i4	0.8077756	0.0399853	-0.1989762	0.0347872
	[ 0.729406,	0.886145]	[-0.267158,	-0.130795]
i5	0.6259097	0.0413365	-0.2527542	0.0420524
	[ 0.544892,	0.706928]	[-0.335175,	-0.170333]
i11	0.3725920	0.0511884	-0.1611601	0.0509114
	[ 0.272265,	0.472919]	[-0.260945,	-0.061376]
i3	-0.1064721	0.0457239	0.6115087	0.0374094
	[-0.196089,	-0.016855]	[ 0.538188,	0.684830]
i8	-0.2315572	0.0348744	0.7975712	0.0274638
	[-0.299910,	-0.163205]	[ 0.743743,	0.851399]
i9	-0.1414585	0.0325585	0.8342607	0.0266517
	[-0.205272,	-0.077645]	[ 0.782024,	0.886497]
i12	-0.2527785	0.0443307	0.6122940	0.0363948
	[-0.339665,	-0.165892]	[ 0.540962,	0.683627]

	U_Var	
i1	0.5841468	0
	[ 0.584147,	0.584147]
i4	0.3079070	0
	[ 0.307907,	0.307907]
i5	0.5443523	0
	[ 0.544352,	0.544352]

i11	0.8352026	0
	[ 0.835203, 0.835203]	
i3	0.6147208	0
	[ 0.614721, 0.614721]	
i8	0.3102614	0
	[ 0.310261, 0.310261]	
i9	0.2839986	0
	[ 0.283999, 0.283999]	
i12	0.5611990	0
	[ 0.561199, 0.561199]	

Standardized Factor Loadings

	FAC_1	FAC_2
i1	0.6235542	-0.1644182
i4	0.8077756	-0.1989762
i5	0.6259097	-0.2527542
i11	0.3725920	-0.1611601
i3	-0.1064721	0.6115087
i8	-0.2315572	0.7975712
i9	-0.1414585	0.8342607
i12	-0.2527785	0.6122940

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	i1	0.58414683	1.00000000	0.41585317
2	i4	0.30790700	1.00000000	0.69209300
3	i5	0.54435232	1.00000000	0.45564768
4	i11	0.83520259	1.00000000	0.16479741
5	i3	0.61472076	1.00000000	0.38527924
6	i8	0.31026142	1.00000000	0.68973858
7	i9	0.28399858	1.00000000	0.71600142
8	i12	0.56119902	1.00000000	0.43880098

Total Determination of All Equations = 0.998  
Total Determination of Manifest Variables = 0.940

### 2.1.3 Lot Data: WLS Fit of Polychoric Correlations

Again, only the off-diagonal entries of the polychoric correlation matrix are fitted.

```
*****
Starting Model Improvement (ChiSq=16.0461 p=0.246651 DF=13)
*****
```

```
*****
Table of 16 Best Pattern Solutions
*****
```

N	Npar	DF	Chisquared	P
1	11	17	17.4932261	0.42146361
2	12	16	16.7221159	0.40379538
3	13	15	16.0839664	0.37649325
4	12	16	17.1562985	0.37555349
5	10	18	19.9382364	0.33630766
6	14	14	16.0747095	0.30883317
7	11	17	19.3922544	0.30648710
8	11	17	19.5929426	0.29557704
9	12	16	18.9872319	0.26932586
10	10	18	22.6323496	0.20511575
11	9	19	25.2396995	0.15280258
12	9	19	25.2396995	0.15280258
13	9	19	25.2396995	0.15280258
14	10	18	24.3006715	0.14542201
15	8	20	110.269653	1.754e-014
16	8	20	110.269653	1.754e-014

N	RMSEA	ECVI	SRMR	PGFI	SBC
1	0.00864734	0.10178667	0.04035342	0.59710684	-83.8876227
2	0.01078518	0.10495391	0.03710830	0.56239927	-78.6951536
3	0.01364729	0.10846383	0.03419656	0.52757236	-73.3697237
4	0.01364769	0.10607293	0.03886014	0.56216483	-78.2609710
5	0.01665909	0.10293360	0.04400652	0.63074553	-87.4061918
6	0.01954334	0.11359461	0.03407470	0.49240524	-67.4154013
7	0.01904423	0.10668107	0.04121360	0.59601735	-81.9885944
8	0.01982696	0.10719831	0.04323028	0.59590221	-81.7879063
9	0.02193606	0.11079183	0.04113782	0.56117620	-76.4300376
10	0.02575423	0.10987719	0.04705782	0.62910897	-84.7120786
11	0	0	0	0	0
12	0.02909304	0.11144252	0.04960816	0.66238763	-88.0683081

13	0.02909304	0.11144252	0.04960712	0.66238763	-88.0683081
14	0.03003595	0.11417699	0.04760267	0.62809554	-83.0437567
15	0.10785494	0.32543725	0.19892470	0.63985901	-9.00193377
16	0.10785494	0.32543725	0.19892470	0.63985901	-9.00193377

Loadings and Uni. Vars. with ASE's and Wald CIs

```

-----
                                FAC_1                                FAC_2
i1    0.6757371  0.0442865                                0          0
      [ 0.588937, 0.762537] [          0,          0]

i4    0.8518933  0.0351081                                0          0
      [ 0.783083, 0.920704] [          0,          0]

i5    0.6460490  0.0606375 -0.1093036  0.0663641
      [ 0.527202, 0.764896] [-0.239375, 0.020768]

i11   0.4604004  0.0558383                                0          0
      [ 0.350959, 0.569842] [          0,          0]

i3           0          0  0.6724844  0.0459350
      [          0,          0] [ 0.582453, 0.762515]

i8           0          0  0.9178680  0.0257476
      [          0,          0] [ 0.867404, 0.968332]

i9           0          0  0.8988828  0.0242453
      [          0,          0] [ 0.851363, 0.946403]

i12  -0.1536135  0.0640301  0.6577492  0.0572753
      [-0.279110,-0.028117] [ 0.545492, 0.770007]

```

```

                                U_Var
i1    0.5433794                                0
      [ 0.543379, 0.543379]

i4    0.2742779                                0
      [ 0.274278, 0.274278]

i5    0.5011459                                0
      [ 0.501146, 0.501146]

i11   0.7880315                                0

```

```

[ 0.788031, 0.788031]

i3    0.5477648      0
      [ 0.547765, 0.547765]

i8    0.1575183      0
      [ 0.157518, 0.157518]

i9    0.1920096      0
      [ 0.192010, 0.192010]

i12   0.4442864      0
      [ 0.444286, 0.444286]

```

Factor Correlations Phi  
-----

	FAC_1	FAC_2
FAC_1	1.0000000	0 -0.4922966 0.0557069
	[ 1.000000, 1.000000]	[-0.601480,-0.383113]
FAC_2	-0.4922966 0.0557069	1.0000000 0
	[-0.601480,-0.383113]	[ 1.000000, 1.000000]

```

(1) Standalone Fit Measures: -----
Fit criterion . . . . . 0.0451
Normal Th. Chi-square (df = 17) 17.4932 Prob>chi**2 = 0.4215
Probability of Close Fit . . . . . 0.9656
Z-Test of Wilson & Hilferty (1931). . . . . 0.1981
(2) Incremental Fit Measures: -----
Null Model Chi-square (df = 28) . . . . . 11415.9663
RMSEA Estimate . . . . . 0.0086 90%C.I.[ . , 0.0472]
ECVI Estimate . . . . . 0.1018 90%C.I.[ . , 0.1590]
McDonald's (1989) Centrality. . . . . 0.9994
Tucker-Lewis Coefficient TLI. . . . . 0.9999
Bentler & Bonett's (1980) NFI . . . . . 0.9985
Bentler's Comparative Fit Index CFI . . . . . 1.0000
Parsimonious NFI (James, Mulaik, & Brett,1982). . . . . 0.6062
Bollen (1986) Normed Index Rho1 . . . . . 0.9975
Bollen (1988) Non-normed Index Delta2 . . . . . 1.0000
(3) Information Criteria: -----
Akaike's Information Criterion. . . . . -16.5068
Bozdogan's (1987) CAIC. . . . . -100.8876
Schwarz's Bayesian Criterion. . . . . -83.8876

```

(4) Other Fit Measures: -----  
 Goodness of Fit Index (GFI) . . . . . 0.9835  
 Parsimonious GFI (Mulaik, 1989) . . . . . 0.5971  
 GFI Adjusted for Degrees of Freedom (AGFI). . . . . 0.9650  
 Root Mean Square Residual (RMR) . . . . . 0.0404  
 Hoelter's (1983) Critical N . . . . . 613

Standardized Factor Loadings

	FAC_1	FAC_2
i1	0.6757371	0
i4	0.8518933	0
i5	0.6460490	-0.1093036
i11	0.4604004	0
i3	0	0.6724844
i8	0	0.9178680
i9	0	0.8988828
i12	-0.1536135	0.6577492

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	i1	0.54337942	1.00000000	0.45662058
2	i4	0.27427788	1.00000000	0.72572212
3	i5	0.50114592	1.00000000	0.49885408
4	i11	0.78803144	1.00000000	0.21196856
5	i3	0.54776477	1.00000000	0.45223523
6	i8	0.15751825	1.00000000	0.84248175
7	i9	0.19200963	1.00000000	0.80799037
8	i12	0.44428644	1.00000000	0.55571356

Total Determination of All Equations = 0.983  
 Total Determination of Manifest Variables = 1.000

The following shows the EFA fit of the polychoric correlation matrix:

(1) Standalone Fit Measures: -----  
 Fit criterion . . . . . 0.0414  
 Normal Th. Chi-square (df = 13) 16.0461 Prob>chi\*\*2 = 0.2467  
 Probability of Close Fit . . . . . 0.8725  
 Z-Test of Wilson & Hilferty (1931). . . . . 0.6867  
 (2) Incremental Fit Measures: -----  
 Null Model Chi-square (df = 28) . . . . . 11415.9663

RMSEA Estimate . . . . .	0.0246	90%C.I.[ . ,	0.0588]
ECVI Estimate . . . . .	0.1187	90%C.I.[ . ,	0.1764]
McDonald's (1989) Centrality. . . . .			0.9961
Tucker-Lewis Coefficient TLI. . . . .			0.9994
Bentler & Bonett's (1980) NFI . . . . .			0.9986
Bentler's Comparative Fit Index CFI . . . . .			0.9997
Parsimonious NFI (James, Mulaik, & Brett,1982). . . . .			0.4636
Bollen (1986) Normed Index Rho1 . . . . .			0.9970
Bollen (1988) Non-normed Index Delta2 . . . . .			0.9997
(3) Information Criteria: -----			
Akaike's Information Criterion. . . . .			-9.9539
Bozdogan's (1987) CAIC. . . . .			-74.4805
Schwarz's Bayesian Criterion. . . . .			-61.4805
(4) Other Fit Measures: -----			
Goodness of Fit Index (GFI) . . . . .			0.9848
Parsimonious GFI (Mulaik, 1989) . . . . .			0.4572
GFI Adjusted for Degrees of Freedom (AGFI). . . . .			0.9580
Root Mean Square Residual (RMR) . . . . .			0.0342
Hoelter's (1983) Critical N . . . . .			542

Rotated Factor Loadings with Standard Errors

	FAC_1		FAC_2	
i1	0.6562847	0.0382584	-0.1974561	0.0400465
	[ 0.581300,	0.731270]	[-0.275946,	-0.118966]
i4	0.8261019	0.0351926	-0.2338132	0.0333772
	[ 0.757126,	0.895078]	[-0.299231,	-0.168395]
i5	0.6406256	0.0378389	-0.2873865	0.0394141
	[ 0.566463,	0.714789]	[-0.364637,	-0.210136]
i11	0.4077281	0.0484299	-0.1766353	0.0480273
	[ 0.312807,	0.502649]	[-0.270767,	-0.082504]
i3	-0.1239008	0.0421997	0.6635098	0.0312584
	[-0.206611,	-0.041191]	[ 0.602245,	0.724775]
i8	-0.2252224	0.0288431	0.8715148	0.0177197
	[-0.281754,	-0.168691]	[ 0.836785,	0.906245]
i9	-0.1720307	0.0284790	0.8860567	0.0173419
	[-0.227849,	-0.116213]	[ 0.852067,	0.920046]

i12 -0.2582669 0.0387080 0.6991847 0.0278767  
 [-0.334133,-0.182401] [ 0.644547, 0.753822]

i1 0.5303015 0  
 [ 0.530302, 0.530302]

i4 0.2628870 0  
 [ 0.262887, 0.262887]

i5 0.5070078 0  
 [ 0.507008, 0.507008]

i11 0.8025578 0  
 [ 0.802558, 0.802558]

i3 0.5444033 0  
 [ 0.544403, 0.544403]

i8 0.1897369 0  
 [ 0.189737, 0.189737]

i9 0.1853089 0  
 [ 0.185309, 0.185309]

i12 0.4444390 0  
 [ 0.444439, 0.444439]

The following shows the CFA fit of the polychoric correlation matrix using the model by Maydeu-Olivares:

Loadings and Uni. Vars. with ASE's and Wald CIs

```

-----
              FAC_1              FAC_2
i1  0.6818860  0.0432233      0      0
     [ 0.597170, 0.766602] [ 0,      0]

i4  0.8388110  0.0341808      0      0
     [ 0.771818, 0.905804] [ 0,      0]

i5  0.7215944  0.0396250      0      0
     [ 0.643931, 0.799258] [ 0,      0]

i11 0.4302503  0.0554110      0      0

```

```

[ 0.321647, 0.538854] [ 0, 0]
i3      0      0 0.6736862 0.0464487
[ 0, 0] [ 0.582648, 0.764724]
i8      0      0 0.9153590 0.0252889
[ 0, 0] [ 0.865794, 0.964924]
i9      0      0 0.8965837 0.0240553
[ 0, 0] [ 0.849436, 0.943731]
i12     0      0 0.7567778 0.0423568
[ 0, 0] [ 0.673760, 0.839796]

```

```

U_Var
i1      0.5350315      0
[ 0.535032, 0.535032]
i4      0.2963962      0
[ 0.296396, 0.296396]
i5      0.4793015      0
[ 0.479302, 0.479302]
i11     0.8148847      0
[ 0.814885, 0.814885]
i3      0.5461469      0
[ 0.546147, 0.546147]
i8      0.1621178      0
[ 0.162118, 0.162118]
i9      0.1961377      0
[ 0.196138, 0.196138]
i12     0.4272873      0
[ 0.427287, 0.427287]

```

Factor Correlations Phi

```

-----
FAC_1      FAC_1      FAC_2
FAC_1  1.0000000      0 -0.5618570  0.0488902

```

```

[ 1.000000, 1.000000] [-0.657680,-0.466034]
FAC_2 -0.5618570 0.0488902 1.0000000 0
[-0.657680,-0.466034] [ 1.000000, 1.000000]

```

```

(1) Standalone Fit Measures: -----
Fit criterion . . . . . 0.0651
Normal Th. Chi-square (df = 19) 25.2397 Prob>chi**2 = 0.1528
Probability of Close Fit . . . . . 0.8850
Z-Test of Wilson & Hilferty (1931). . . . . 1.0262
(2) Incremental Fit Measures: -----
Null Model Chi-square (df = 28) . . . . . 11415.9663
RMSEA Estimate . . . . . 0.0291 90%C.I.[ . , 0.0564]
ECVI Estimate . . . . . 0.1114 90%C.I.[ . , 0.1764]
McDonald's (1989) Centrality. . . . . 0.9920
Tucker-Lewis Coefficient TLI. . . . . 0.9992
Bentler & Bonett's (1980) NFI . . . . . 0.9978
Bentler's Comparative Fit Index CFI . . . . . 0.9995
Parsimonious NFI (James, Mulaik, & Brett,1982). . . . . 0.6771
Bollen (1986) Normed Index Rho1 . . . . . 0.9967
Bollen (1988) Non-normed Index Delta2 . . . . . 0.9995
(3) Information Criteria: -----
Akaike's Information Criterion. . . . . -12.7603
Bozdogan's (1987) CAIC. . . . . -107.0683
Schwarz's Bayesian Criterion. . . . . -88.0683
(4) Other Fit Measures: -----
Goodness of Fit Index (GFI) . . . . . 0.9762
Parsimonious GFI (Mulaik, 1989) . . . . . 0.6624
GFI Adjusted for Degrees of Freedom (AGFI). . . . . 0.9548
Root Mean Square Residual (RMR) . . . . . 0.0496
Hoelter's (1983) Critical N . . . . . 465

```

### 2.1.4 Lot Data: DWLS Fit of Polychoric Correlations

Again, only the off-diagonal entries of the polychoric correlaton matrix are fitted.

```

*****
Table of 15 Best Pattern Solutions
*****

```

N	Npar	DF	Chisquared	P
1	13	15	5.11806027	0.99107875
2	14	14	5.09283788	0.98447823

3	12	16	6.35525440	0.98378366
4	11	17	8.14378935	0.96337469
5	10	18	9.14447449	0.95630138
6	11	17	10.3320077	0.88911341
7	10	18	11.3720935	0.87790496
8	10	18	11.5809997	0.86809803
9	12	16	10.2630614	0.85253419
10	9	19	12.7331886	0.85194528
11	9	19	12.7331886	0.85194528
12	9	19	12.7331886	0.85194528
13	11	17	11.2644477	0.84249104
14	8	20	374.505125	3.896e-067
15	8	20	374.505125	3.896e-067

N	RMSEA	ECVI	SRMR	PGFI	SBC
1	0	0.08020119	0.02200114	0.53410883	-84.3356299
2	0	0.08529082	0.02191276	0.49850896	-78.3972729
3	0	0.07823519	0.02461146	0.56930212	-89.0620151
4	0	0.07769018	0.02791070	0.60424766	-93.2370595
5	0	0.07511462	0.02965252	0.63941496	-98.1999537
6	0	0.08332992	0.03151421	0.60346973	-91.0488411
7	0	0.08085591	0.03291566	0.63857643	-95.9723346
8	0	0.08139433	0.03320311	0.63849780	-95.7634285
9	0	0.08830686	0.03148998	0.56799458	-85.1542081
10	0	0.07920925	0.03478164	0.67351209	-100.574819
11	0	0	0	0	0
12	0	0.07920925	0.03478150	0.67351209	-100.574819
13	0	0.08573311	0.03270651	0.60313824	-90.1164012
14	0.21373734	1.00645651	0.19215615	0.55765013	255.233538
15	0.21373734	1.00645651	0.19215615	0.55765013	255.233538

Loadings and Uni. Vars. with ASE's and Wald CIs

	FAC_1		FAC_2	
i1	0.6703500	0.0397637	0	0
	[ 0.592415,	0.748285] [	0,	0]
i4	0.8447774	0.0488002	0	0
	[ 0.749131,	0.940424] [	0,	0]
i5	0.6526541	0.0678055	-0.1030619	0.0685362
	[ 0.519758,	0.785550] [-0.237390,	0.031267]	

i11	0.3873697	0.0621310	-0.0804273	0.0615312
	[ 0.265595,	0.509144]	[-0.201026,	0.040172]
i3	0	0	0.6529447	0.0387571
	[ 0,	0]	[ 0.576982,	0.728907]
i8	-0.0947586	0.0813314	0.8198499	0.0717772
	[-0.254165,	0.064648]	[ 0.679169,	0.960531]
i9	0	0	0.8736692	0.0460049
	[ 0,	0]	[ 0.783501,	0.963837]
i12	-0.1640964	0.0737977	0.6259643	0.0623590
	[-0.308737,-0.019456]	[ 0.503743,	0.748186]	

		U_Var
i1	0.5506309	0
	[ 0.550631,	0.550631]
i4	0.2863512	0
	[ 0.286351,	0.286351]
i5	0.5060607	0
	[ 0.506061,	0.506061]
i11	0.8169081	0
	[ 0.816908,	0.816908]
i3	0.5736632	0
	[ 0.573663,	0.573663]
i8	0.2526176	0
	[ 0.252618,	0.252618]
i9	0.2367022	0
	[ 0.236702,	0.236702]
i12	0.4936464	0
	[ 0.493646,	0.493646]

Factor Correlations Phi  
-----

	FAC_1	FAC_2
FAC_1	1.0000000	0 -0.4263825 0.0545373
	[ 1.000000, 1.000000]	[-0.533274,-0.319491]
FAC_2	-0.4263825 0.0545373 1.0000000	0
	[-0.533274,-0.319491]	[ 1.000000, 1.000000]

(1) Standalone Fit Measures: -----  
Fit criterion . . . . . 0.0132  
Normal Th. Chi-square (df = 15) 5.1181 Prob>chi\*\*2 = 0.9911  
Probability of Close Fit . . . . . 1.0000  
Z-Test of Wilson & Hilferty (1931). . . . . -2.3531  
(2) Incremental Fit Measures: -----  
Null Model Chi-square (df = 28) . . . . . 4648.2583  
RMSEA Estimate . . . . . 0.0000  
ECVI Estimate . . . . . 0.0802  
McDonald's (1989) Centrality. . . . . 1.0128  
Tucker-Lewis Coefficient TLI. . . . . 1.0040  
Bentler & Bonett's (1980) NFI . . . . . 0.9989  
Bentler's Comparative Fit Index CFI . . . . . 1.0000  
Parsimonious NFI (James, Mulaik, & Brett,1982). . . . . 0.5351  
Bollen (1986) Normed Index Rho1 . . . . . 0.9979  
Bollen (1988) Non-normed Index Delta2 . . . . . 1.0021  
(3) Information Criteria: -----  
Akaike's Information Criterion. . . . . -24.8819  
Bozdogan's (1987) CAIC. . . . . -99.3356  
Schwarz's Bayesian Criterion. . . . . -84.3356  
(4) Other Fit Measures: -----  
Goodness of Fit Index (GFI) . . . . . 0.9970  
Parsimonious GFI (Mulaik, 1989) . . . . . 0.5341  
GFI Adjusted for Degrees of Freedom (AGFI). . . . . 0.9928  
Root Mean Square Residual (RMR) . . . . . 0.0220  
Hoelter's (1983) Critical N . . . . . 1896

Standardized Factor Loadings

	FAC_1	FAC_2
i1	0.6703500	0
i4	0.8447774	0
i5	0.6526541	-0.1030619
i11	0.3873697	-0.0804273
i3	0	0.6529447
i8	-0.0947586	0.8198499
i9	0	0.8736692

i12 -0.1640964 0.6259643

N	Variable	ErrVariance	TotVariance	R-squared
1	i1	0.55063085	1.00000000	0.44936915
2	i4	0.28635120	1.00000000	0.71364880
3	i5	0.50606066	1.00000000	0.49393934
4	i11	0.81690815	1.00000000	0.18309185
5	i3	0.57366321	1.00000000	0.42633679
6	i8	0.25261760	1.00000000	0.74738240
7	i9	0.23670217	1.00000000	0.76329783
8	i12	0.49364638	1.00000000	0.50635362

Total Determination of All Equations = 0.975  
Total Determination of Manifest Variables = 0.973

The following shows the EFA fit of the polychoric correlation matrix:

(1) Standalone Fit Measures: -----  
Fit criterion . . . . . 0.0131  
Normal Th. Chi-square (df = 13) 5.0867 Prob>chi\*\*2 = 0.9732  
Probability of Close Fit . . . . . 0.9997  
Z-Test of Wilson & Hilferty (1931). . . . . -1.9235  
(2) Incremental Fit Measures: -----  
Null Model Chi-square (df = 28) . . . . . 4587.5178  
RMSEA Estimate . . . . . 0.0000  
ECVI Estimate . . . . . 0.0904  
McDonald's (1989) Centrality. . . . . 1.0102  
Tucker-Lewis Coefficient TLI. . . . . 1.0037  
Bentler & Bonett's (1980) NFI . . . . . 0.9989  
Bentler's Comparative Fit Index CFI . . . . . 1.0000  
Parsimonious NFI (James, Mulaik, & Brett,1982). . . . . 0.4638  
Bollen (1986) Normed Index Rho1 . . . . . 0.9976  
Bollen (1988) Non-normed Index Delta2 . . . . . 1.0017  
(3) Information Criteria: -----  
Akaike's Information Criterion. . . . . -20.9133  
Bozdogan's (1987) CAIC. . . . . -85.4398  
Schwarz's Bayesian Criterion. . . . . -72.4398  
(4) Other Fit Measures: -----  
Goodness of Fit Index (GFI) . . . . . 0.9970  
Parsimonious GFI (Mulaik, 1989) . . . . . 0.4629  
GFI Adjusted for Degrees of Freedom (AGFI). . . . . 0.9918  
Root Mean Square Residual (RMR) . . . . . 0.0219  
Hoelter's (1983) Critical N . . . . . 1707

Rotated Factor Loadings with Standard Errors

	FAC_1		FAC_2	
i1	0.6503989	0.0394191	-0.1719797	0.0410431
	[ 0.573139,	0.727659]	[-0.252423,	-0.091537]
i4	0.8120786	0.0363689	-0.2225821	0.0341954
	[ 0.740797,	0.883360]	[-0.289604,	-0.155560]
i5	0.6484993	0.0385200	-0.2715475	0.0401367
	[ 0.573001,	0.723997]	[-0.350214,	-0.192881]
i11	0.3885002	0.0496874	-0.1799930	0.0493350
	[ 0.291115,	0.485886]	[-0.276688,	-0.083298]
i3	-0.1077119	0.0435602	0.6473818	0.0338529
	[-0.193088,	-0.022335]	[ 0.581031,	0.713732]
i8	-0.2355034	0.0320577	0.8325272	0.0226060
	[-0.298335,	-0.172671]	[ 0.788220,	0.876834]
i9	-0.1592354	0.0309847	0.8554850	0.0220635
	[-0.219964,	-0.098506]	[ 0.812241,	0.898729]
i12	-0.2679428	0.0412792	0.6595682	0.0319459
	[-0.348849,	-0.187037]	[ 0.596955,	0.722181]

	U_Var	
i1	0.5474043	0
	[ 0.547404,	0.547404]
i4	0.2909855	0
	[ 0.290985,	0.290985]
i5	0.5057106	0
	[ 0.505711,	0.505711]
i11	0.8166701	0
	[ 0.816670,	0.816670]
i3	0.5692950	0
	[ 0.569295,	0.569295]
i8	0.2514367	0

```

[ 0.251437, 0.251437]
i9  0.2427895      0
    [ 0.242789, 0.242789]
i12 0.4931764      0
    [ 0.493176, 0.493176]

```

The following shows the CFA fit of the polychoric correlation matrix using the model by Maydeu-Olivares:

Loadings and Uni. Vars. with ASE's and Wald CIs

```

-----
                FAC_1                FAC_2
i1  0.6554355  0.0376231      0      0
    [ 0.581696, 0.729175] [ 0,      0]
i4  0.8123717  0.0423662      0      0
    [ 0.729335, 0.895408] [ 0,      0]
i5  0.7339822  0.0405931      0      0
    [ 0.654421, 0.813543] [ 0,      0]
i11 0.4450190  0.0379459      0      0
    [ 0.370646, 0.519392] [ 0,      0]
i3   0          0  0.6361935  0.0365814
    [ 0,          0] [ 0.564495, 0.707892]
i8   0          0  0.8796857  0.0402388
    [ 0,          0] [ 0.800819, 0.958552]
i9   0          0  0.8398419  0.0384565
    [ 0,          0] [ 0.764468, 0.915215]
i12  0          0  0.7362067  0.0382401
    [ 0,          0] [ 0.661257, 0.811156]

                U_Var
i1  0.5704043      0
    [ 0.570404, 0.570404]
i4  0.3400523      0

```

```

[ 0.340052, 0.340052]
i5  0.4612701      0
    [ 0.461270, 0.461270]
i11 0.8019581      0
    [ 0.801958, 0.801958]
i13  0.5952578      0
    [ 0.595258, 0.595258]
i18  0.2261530      0
    [ 0.226153, 0.226153]
i19  0.2946657      0
    [ 0.294666, 0.294666]
i12  0.4579997      0
    [ 0.458000, 0.458000]

```

Factor Correlations Phi

```

-----
                FAC_1                FAC_2
FAC_1  1.0000000                0 -0.5408324  0.0322304
      [ 1.000000, 1.000000] [-0.604003,-0.477662]
FAC_2 -0.5408324  0.0322304  1.0000000                0
      [-0.604003,-0.477662] [ 1.000000, 1.000000]

```

```

(1) Standalone Fit Measures: -----
Fit criterion . . . . . 0.0328
Normal Th. Chi-square (df = 19) 12.7332 Prob>chi**2 = 0.8519
Probability of Close Fit . . . . . 0.9987
Z-Test of Wilson & Hilferty (1931). . . . . -1.0467
(2) Incremental Fit Measures: -----
Null Model Chi-square (df = 28) . . . . . 4174.0836
RMSEA Estimate . . . . . 0.0000 90%C.I.[ . , 0.0251]
ECVI Estimate . . . . . 0.0792 90%C.I.[ . , 0.1279]
McDonald's (1989) Centrality. . . . . 1.0081
Tucker-Lewis Coefficient TLI. . . . . 1.0022
Bentler & Bonett's (1980) NFI . . . . . 0.9969
Bentler's Comparative Fit Index CFI . . . . . 1.0000
Parsimonious NFI (James, Mulaik, & Brett,1982). . 0.6765

```

```

Bollen (1986) Normed Index Rho1 . . . . . 0.9955
Bollen (1988) Non-normed Index Delta2 . . . . . 1.0015
(3) Information Criteria: -----
Akaike's Information Criterion. . . . . -25.2668
Bozdogan's (1987) CAIC. . . . . -119.5748
Schwarz's Bayesian Criterion. . . . . -100.5748
(4) Other Fit Measures: -----
Goodness of Fit Index (GFI) . . . . . 0.9925
Parsimonious GFI (Mulaik, 1989) . . . . . 0.6735
GFI Adjusted for Degrees of Freedom (AGFI). . . . . 0.9859
Root Mean Square Residual (RMR) . . . . . 0.0348
Hoelter's (1983) Critical N . . . . . 920

```

## 2.2 Nine Psychological Variables (H & S, 1937): SEM16

```

print "SEM16: Nine Psychological Variables, LISREL VI, p. III.106";
print "Confirmatory Factor Analysis, Data of Holzinger \& Swineford (1937)";
print "EFA: p= 0.6205, chi= 9.95, df=12";

```

```

c16 = [ 1. ,
        .318 1. ,
        .436 .419 1. ,
        .335 .234 .323 1. ,
        .304 .157 .283 .722 1. ,
        .326 .195 .350 .714 .685 1. ,
        .116 .057 .056 .203 .246 .170 1. ,
        .314 .145 .229 .095 .181 .113 .585 1. ,
        .489 .239 .361 .309 .345 .280 .408 .512 1. ];

```

This output shows the EFA model fit:

```

*****
Starting Model Improvement (ChiSq=9.94823 Pvalue=0.620503 DF=12)
*****

```

The following table presents a summary of the best 15 solutions.

```

*****
Table of 20 Best Pattern Solutions
*****

```

```

N  Npar  DF  Chisquared  P

```

1	25	20	11.2513458	0.93944202
2	26	19	10.7247316	0.93275083
3	26	19	10.7263835	0.93269941
4	25	20	11.5091596	0.93193552
5	24	21	12.3712894	0.92904775
6	26	19	10.9716580	0.92478811
7	26	19	10.9826233	0.92442157
8	25	20	11.8806178	0.92012372
9	27	18	10.3471365	0.91998450
10	27	18	10.3785326	0.91884778
11	26	19	11.1772709	0.91773130
12	27	18	10.4235965	0.91719925
13	27	18	10.5682819	0.91177126
14	27	18	10.5805499	0.91130155
15	24	21	12.9248614	0.91123297
16	25	20	12.1851493	0.90955583
17	27	18	10.7111720	0.90620882
18	25	20	12.4203547	0.90084840
19	27	18	10.8867485	0.89910059
20	26	19	11.7020850	0.89796157

N	RMSEA	ECVI	SRMR	PGFI	SBC
1	0	0.45126867	0.02525887	0.54589698	-88.2833290
2	0	0.46253700	0.02167876	0.51911584	-83.8332095
3	0	0.46254848	0.02210502	0.51902253	-83.8315576
4	0	0.45305905	0.02650475	0.54573851	-88.0255153
5	0	0.44412069	0.02785032	0.57213963	-92.1401191
6	0	0.46425177	0.02411269	0.51881167	-83.5862831
7	0	0.46432792	0.02432238	0.51885331	-83.5753178
8	0	0.45563862	0.03146928	0.54548695	-87.6540570
9	0	0.47484019	0.02137929	0.49195998	-79.2340709
10	0	0.47505822	0.02066109	0.49198665	-79.2026748
11	0	0.46567964	0.02510962	0.51866077	-83.3806702
12	0	0.47537116	0.02127910	0.49195562	-79.1576109
13	0	0.47637592	0.02154743	0.49178819	-79.0129255
14	0	0.47646112	0.02341660	0.49182332	-79.0006575
15	0	0.44796494	0.03152857	0.57172483	-91.5865472
16	0	0.45775342	0.02736283	0.54508076	-87.3495256
17	0	0.47736821	0.02289020	0.49166657	-78.8700354
18	0	0.45938679	0.02754853	0.54484200	-87.1143201
19	0	0.47858749	0.02522464	0.49153255	-78.6944589
20	0	0.46932418	0.03001944	0.51834954	-82.8558561

The following shows details of the best solution found:

Loadings and Uni. Vars. with ASE's and Wald CIs

-----

	FAC_1		FAC_2	
V1	0.6987112	0.0857264	0.0937473	0.0842428
	[ 0.530690,	0.866732]	[-0.071366,	0.258860]
V2	0.4881935	0.0903112	0	0
	[ 0.311187,	0.665200]	[ 0,	0]
V3	0.6718046	0.0869520	0	0
	[ 0.501382,	0.842227]	[ 0,	0]
V4	0	0	0	0
	[ 0,	0]	[ 0,	0]
V5	0	0	0.1143820	0.0630808
	[ 0,	0]	[-9.3e-003,	0.238018]
V6	0	0	0	0
	[ 0,	0]	[ 0,	0]
V7	0	0	0.7712926	0.1010557
	[ 0,	0]	[ 0.573227,	0.969358]
V8	0.4197159	0.1058744	0.7332916	0.0970976
	[ 0.212206,	0.627226]	[ 0.542984,	0.923599]
V9	0.5893710	0.0858883	0.4441484	0.0825194
	[ 0.421033,	0.757709]	[ 0.282413,	0.605883]
	FAC_3		U_Var	
V1	0	0	0.5046143	0.0853445
	[ 0,	0]	[ 0.337342,	0.671886]
V2	0	0	0.7616671	0.1000075
	[ 0,	0]	[ 0.565656,	0.957678]
V3	0	0	0.5486786	0.0901259
	[ 0,	0]	[ 0.372035,	0.725322]
V4	0.8738382	0.0698783	0.2364069	0.0507738
	[ 0.736879,	1.010797]	[ 0.136892,	0.335922]
V5	0.8219073	0.0714973	0.3088296	0.0522597

```

[ 0.681775, 0.962039] [ 0.206402, 0.411257]
V6  0.8242760  0.0716267  0.3205690  0.0539320
    [ 0.683890, 0.964662] [ 0.214864, 0.426274]
V7  0.1991920  0.0852976  0.3612541  0.1222980
    [ 0.032012, 0.366372] [ 0.121554, 0.600954]
V8 -0.1246921  0.0967887  0.3383583  0.0955084
    [-0.314394, 0.065010] [ 0.151165, 0.525551]
V9      0      0  0.4617680  0.0719642
    [      0,      0] [ 0.320721, 0.602815]

```

Factor Correlations Phi

```

-----
FAC_1      FAC_1      FAC_2
FAC_1  1.0000000      0      0
    [ 1.000000, 1.000000] [      0,      0]
FAC_2      0      0  1.0000000      0
    [      0,      0] [ 1.000000, 1.000000]
FAC_3  0.5520284  0.0801574      0      0
    [ 0.394923, 0.709134] [      0,      0]
FAC_1      FAC_3
FAC_1  0.5520284  0.0801574
    [ 0.394923, 0.709134]
FAC_2      0      0
    [      0,      0]
FAC_3  1.0000000      0
    [ 1.000000, 1.000000]

```

```

(1) Standalone Fit Measures: -----
Fit criterion . . . . . 0.0781
Normal Th. Chi-square (df = 20) 11.2513 Prob>chi**2 = 0.9394
Normal Theory Reweighted LS Chi-square . . . . . 11.4651
Probability of Close Fit . . . . . 0.9916
Z-Test of Wilson & Hilferty (1931). . . . . -1.5499
(2) Incremental Fit Measures: -----

```

Null Model Chi-square (df = 36)	496.6694
RMSEA Estimate	0.0000 90%C.I.[ . , 0.0159]
ECVI Estimate	0.4513
McDonald's (1989) Centrality	1.0306
Tucker-Lewis Coefficient TLI	1.0342
Bentler & Bonett's (1980) NFI	0.9773
Bentler's Comparative Fit Index CFI	1.0000
Parsimonious NFI (James, Mulaik, & Brett,1982)	0.5430
Bollen (1986) Normed Index Rho1	0.9592
Bollen (1988) Non-normed Index Delta2	1.0184
(3) Information Criteria: -----	
Akaike's Information Criterion	-28.7487
Bozdogan's (1987) CAIC	-108.2833
Schwarz's Bayesian Criterion	-88.2833
(4) Other Fit Measures: -----	
Goodness of Fit Index (GFI)	0.9826
Parsimonious GFI (Mulaik, 1989)	0.5459
GFI Adjusted for Degrees of Freedom (AGFI)	0.9609
Root Mean Square Residual (RMR)	0.0253
Hoelter's (1983) Critical N	404

Standardized Factor Loadings

	FAC_1	FAC_2	FAC_3
V1	0.6981528	0.0936724	0
V2	0.4881935	0	0
V3	0.6718046	0	0
V4	0	0	0.8738382
V5	0	0.1145285	0.8229595
V6	0	0	0.8242760
V7	0	0.7729082	0.1996092
V8	0.4176322	0.7296512	-0.1240731
V9	0.5874958	0.4427352	0

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	V1	0.50461428	1.00160011	0.49619186
2	V2	0.76166711	1.00000000	0.23833289
3	V3	0.54867857	1.00000000	0.45132143
4	V4	0.23640687	1.00000000	0.76359313
5	V5	0.30882955	0.99744437	0.69037917
6	V6	0.32056901	1.00000000	0.67943099
7	V7	0.36125406	0.99582374	0.63723093

```

      8 V8      0.33835828  1.01000332  0.66499291
      9 V9      0.46176796  1.00639398  0.54116582
Total Determination of All Equations = 0.992
Total Determination of Manifest Variables = 0.991

```

The CFA model is reported in the LISREL VI Manual and also shown in sem16.inp:

```

l = [ . 0. 0., . 0. 0., . 0. 0.,
      0. . 0., 0. . 0., 0. . 0.,
      0. 0. ., 0. 0. ., . 0. . ];
p = [ 1. , . 1., . . 1. ];
u = [ 9 # . ];
list patt;
patt[1] = 1; patt[2] = (tri2sym)p; patt[3] = u;

```

Loadings and Uni. Vars. with ASE's and Wald CIs

	FAC_1		FAC_2	
V1	0.7081902	0.0868147	0	0
	[ 0.538036,	0.878344]	[ 0,	0]
V2	0.4833306	0.0907553	0	0
	[ 0.305453,	0.661208]	[ 0,	0]
V3	0.6498856	0.0874669	0	0
	[ 0.478454,	0.821318]	[ 0,	0]
V4	0	0	0.8679084	0.0701579
	[ 0,	0]	[ 0.730402,	1.005415]
V5	0	0	0.8298926	0.0714842
	[ 0,	0]	[ 0.689786,	0.969999]
V6	0	0	0.8250112	0.0716510
	[ 0,	0]	[ 0.684578,	0.965445]
V7	0	0	0	0
	[ 0,	0]	[ 0,	0]
V8	0	0	0	0
	[ 0,	0]	[ 0,	0]

V9 0.4566634 0.0886175 0 0  
 [ 0.282976, 0.630350] [ 0, 0]

		FAC_3	U_Var
V1	0	0 0.4984667	0.0901823
	[ 0,	0] [ 0.321713,	0.675221]
V2	0	0 0.7663915	0.1006133
	[ 0,	0] [ 0.569193,	0.963590]
V3	0	0 0.5776487	0.0911299
	[ 0,	0] [ 0.399037,	0.756260]
V4	0	0 0.2467350	0.0513112
	[ 0,	0] [ 0.146167,	0.347303]
V5	0	0 0.3112783	0.0536661
	[ 0,	0] [ 0.206095,	0.416462]
V6	0	0 0.3193565	0.0540504
	[ 0,	0] [ 0.213420,	0.425293]
V7	0.6808743	0.0886616 0.5364102	0.0931701
	[ 0.507101,	0.854648] [ 0.353800,	0.719020]
V8	0.8591342	0.0916707 0.2618885	0.1132573
	[ 0.679463,	1.038805] [ 0.039908,	0.483869]
V9	0.4188227	0.0884722 0.4669924	0.0726962
	[ 0.245420,	0.592225] [ 0.324511,	0.609474]

Factor Correlations Phi

-----

	FAC_1	FAC_2	
FAC_1	1.0000000	0 0.5574397 0.0811712	
	[ 1.000000,	1.000000] [ 0.398347,	0.716532]
FAC_2	0.5574397	0.0811712 1.0000000	
	[ 0.398347,	0.716532] [ 1.000000,	1.000000]
FAC_3	0.3896605	0.1045577 0.2229733 0.0960244	
	[ 0.184731,	0.594590] [ 0.034769,	0.411178]

		FAC_3
FAC_1	0.3896605	0.1045577
	[ 0.184731, 0.594590]	
FAC_2	0.2229733	0.0960244
	[ 0.034769, 0.411178]	
FAC_3	1.0000000	0
	[ 1.000000, 1.000000]	

(1) Standalone Fit Measures: -----  
Fit criterion . . . . . 0.2014  
Normal Th. Chi-square (df = 23) 29.0077 Prob>chi\*\*2 = 0.1801  
Normal Theory Reweighted LS Chi-square . . . . . 28.7441  
Probability of Close Fit . . . . . 0.5674  
Z-Test of Wilson & Hilferty (1931). . . . . 0.9165

(2) Incremental Fit Measures: -----  
Null Model Chi-square (df = 36) . . . . . 496.6694  
RMSEA Estimate . . . . . 0.0426 90%C.I.[ . , 0.0851]  
ECVI Estimate . . . . . 0.5298 90%C.I.[ . , 0.6580]  
McDonald's (1989) Centrality. . . . . 0.9795  
Tucker-Lewis Coefficient TLI. . . . . 0.9796  
Bentler & Bonett's (1980) NFI . . . . . 0.9416  
Bentler's Comparative Fit Index CFI . . . . . 0.9870  
Parsimonious NFI (James, Mulaik, & Brett,1982). . . . . 0.6016  
Bollen (1986) Normed Index Rho1 . . . . . 0.9086  
Bollen (1988) Non-normed Index Delta2 . . . . . 0.9873

(3) Information Criteria: -----  
Akaike's Information Criterion. . . . . -16.9923  
Bozdogan's (1987) CAIC. . . . . -108.4571  
Schwarz's Bayesian Criterion. . . . . -85.4571

(4) Other Fit Measures: -----  
Goodness of Fit Index (GFI) . . . . . 0.9575  
Parsimonious GFI (Mulaik, 1989) . . . . . 0.6118  
GFI Adjusted for Degrees of Freedom (AGFI). . . . . 0.9169  
Root Mean Square Residual (RMR) . . . . . 0.0451  
Hoelter's (1983) Critical N . . . . . 176

Standardized Factor Loadings

	FAC_1	FAC_2	FAC_3
V1	0.7081902	0	0
V2	0.4833306	0	0

V3	0.6498856	0	0
V4	0	0.8679084	0
V5	0	0.8298926	0
V6	0	0.8250112	0
V7	0	0	0.6808743
V8	0	0	0.8591342
V9	0.4566634	0	0.4188227

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	V1	0.49846668	1.00000000	0.50153332
2	V2	0.76639154	1.00000000	0.23360846
3	V3	0.57764865	1.00000000	0.42235135
4	V4	0.24673495	1.00000000	0.75326505
5	V5	0.31127826	1.00000000	0.68872174
6	V6	0.31935650	1.00000000	0.68064350
7	V7	0.53641025	1.00000000	0.46358975
8	V8	0.26188849	1.00000000	0.73811151
9	V9	0.46699238	1.00000000	0.53300762

Total Determination of All Equations = 0.991  
Total Determination of Manifest Variables = 0.989

### 2.3 Data by Thurstone (1931): SEM09

```

thur = [ 1. ,
        .828 1. ,
        .776 .779 1. ,
        .439 .493 .460 1. ,
        .432 .464 .425 .674 1. ,
        .447 .489 .443 .590 .541 1. ,
        .447 .432 .401 .381 .402 .288 1. ,
        .541 .537 .534 .350 .367 .320 .555 1. ,
        .380 .358 .359 .424 .446 .325 .598 .452 1. ];

```

This output shows the EFA model fit:

```

*****
Starting Model Improvement (ChiSq=2.9032 Pvalue=0.99619 DF=12)
*****

```

\*\*\*\*\*  
 Table of 20 Best Pattern Solutions  
 \*\*\*\*\*

N	Npar	DF	Chisquared	P
1	29	16	3.10652944	0.99978601
2	28	17	3.65139986	0.99972255
3	29	16	3.23210757	0.99972182
4	30	15	2.97690527	0.99961664
5	30	15	2.98291727	0.99961181
6	30	15	2.98343774	0.99961139
7	27	18	4.30652327	0.99959675
8	29	16	3.48509077	0.99954465
9	29	16	3.48509077	0.99954465
10	30	15	3.06858017	0.99953741
11	30	15	3.07668316	0.99952981
12	29	16	3.50299596	0.99952927
13	30	15	3.09050625	0.99951663
14	30	15	3.09050627	0.99951663
15	29	16	3.53120039	0.99950420
16	30	15	3.11376352	0.99949378
17	28	17	4.02829078	0.99945776
18	29	16	3.59905747	0.99943942
19	29	16	3.59905748	0.99943942
20	30	15	3.20987961	0.99938997

N	RMSEA	ECVI	SRMR	PGFI	SBC
1	0	0.30178215	0.00624752	0.44301711	-82.6741452
2	0	0.29445131	0.00831949	0.47042501	-87.4905670
3	0	0.30237450	0.00662794	0.44295734	-82.5485671
4	0	0.31107171	0.00573816	0.41538856	-77.4424772
5	0	0.31110007	0.00579723	0.41538446	-77.4364652
6	0	0.31110252	0.00581218	0.41538478	-77.4359447
7	0	0.28764052	0.00986447	0.49774912	-92.1967357
8	0	0.30356782	0.00716500	0.44284805	-82.2955839
9	0	0.30356782	0.00716498	0.44284805	-82.2955839
10	0	0.31150414	0.00614370	0.41534420	-77.3508023
11	0	0.31154236	0.00571654	0.41534617	-77.3426993
12	0	0.30365228	0.00914108	0.44283839	-82.2776787
13	0	0.31160756	0.00627418	0.41533361	-77.3288762
14	0	0.31160756	0.00627426	0.41533362	-77.3288762
15	0	0.30378532	0.00790744	0.44281245	-82.2494743
16	0	0.31171727	0.00620186	0.41533461	-77.3056190

17	0	0.29622909	0.00992466	0.47025539	-87.1136760
18	0	0.30410540	0.00812188	0.44277769	-82.1816172
19	0	0.30410540	0.00812215	0.44277769	-82.1816172
20	0	0.31217064	0.00668120	0.41528353	-77.2095029

The following shows details of the best solution found:

Loadings and Uni. Vars. with ASE's and Wald CIs

```

-----
                                FAC_1                                FAC_2
OBS1  0.9471675  0.0617547                                0          0
       [ 0.826131, 1.068204] [          0,          0]

OBS2  0.9130025  0.0538786                                0          0
       [ 0.807402, 1.018603] [          0,          0]

OBS3  0.8560433  0.0559666                                0          0
       [ 0.746351, 0.965736] [          0,          0]

OBS4           0          0 -0.4002501  0.0935147
       [          0,          0] [-0.583536,-0.216965]

OBS5           0          0 -0.4411033  0.0888934
       [          0,          0] [-0.615331,-0.266876]

OBS6  0.1814241  0.0824835 -0.2078905  0.0880482
       [ 0.019759, 0.343089] [-0.380462,-0.035319]

OBS7  0.1135365  0.1196842 -0.7938235  0.1086546
       [-0.121040, 0.348113] [-1.006783,-0.580864]

OBS8  0.4334076  0.0940989 -0.4145607  0.0758386
       [ 0.248977, 0.617838] [-0.563202,-0.265920]

OBS9           0          0 -0.6962829  0.0770788
       [          0,          0] [-0.847354,-0.545211]

OBS1  -0.0868570  0.0521968  0.1730363  0.0289007
       [-0.189161, 0.015447] [ 0.116392, 0.229681]

OBS2           0          0  0.1669560  0.0270921
       [          0,          0] [ 0.113856, 0.220056]

```

OBS3	0	0	0.2676555	0.0329201
	[	0,	0]	[ 0.203133, 0.332178]
OBS4	0.7571450	0.0701888	0.2674337	0.0583942
	[	0.619577, 0.894713]	[	0.152983, 0.381884]
OBS5	0.6587992	0.0737304	0.3722769	0.0532866
	[	0.514290, 0.803308]	[	0.267837, 0.476717]
OBS6	0.5417772	0.0829298	0.5043147	0.0596782
	[	0.379238, 0.704317]	[	0.387348, 0.621282]
OBS7	0	0	0.2751764	0.0987070
	[	0,	0]	[ 0.081714, 0.468639]
OBS8	-0.0569989	0.0775350	0.4972177	0.0581030
	[-	0.208965, 0.094967]	[	0.383338, 0.611098]
OBS9	0.1983391	0.0878593	0.4762630	0.0714034
	[	0.026138, 0.370540]	[	0.336315, 0.616211]

Factor Correlations Phi

```

-----
                FAC_1                FAC_2
FAC_1  1.0000000                0 -0.4544176  0.1157565
      [ 1.000000, 1.000000] [-0.681296,-0.227539]

FAC_2 -0.4544176  0.1157565  1.0000000                0
      [-0.681296,-0.227539] [ 1.000000, 1.000000]

FAC_3  0.4694901  0.1007843                0                0
      [ 0.271957, 0.667024] [                0,                0]

                FAC_3
      FAC_1  0.4694901  0.1007843
            [ 0.271957, 0.667024]

      FAC_2                0                0
            [                0,                0]

      FAC_3  1.0000000                0
            [ 1.000000, 1.000000]

```

(1) Standalone Fit Measures: -----

Fit criterion . . . . .	0.0147
Normal Th. Chi-square (df = 16) 3.1065 Prob>chi**2 = 0.9998	
Normal Theory Reweighted LS Chi-square . . . . .	3.0736
Probability of Close Fit . . . . .	1.0000
Z-Test of Wilson & Hilferty (1931). . . . .	-3.4540
(2) Incremental Fit Measures: -----	
Null Model Chi-square (df = 36) . . . . .	1101.8924
RMSEA Estimate . . . . .	0.0000
ECVI Estimate . . . . .	0.3018
McDonald's (1989) Centrality. . . . .	1.0307
Tucker-Lewis Coefficient TLI. . . . .	1.0272
Bentler & Bonett's (1980) NFI . . . . .	0.9972
Bentler's Comparative Fit Index CFI . . . . .	1.0000
Parsimonious NFI (James, Mulaik, & Brett,1982). . . . .	0.4432
Bollen (1986) Normed Index Rho1 . . . . .	0.9937
Bollen (1988) Non-normed Index Delta2 . . . . .	1.0119
(3) Information Criteria: -----	
Akaike's Information Criterion. . . . .	-28.8935
Bozdogan's (1987) CAIC. . . . .	-98.6741
Schwarz's Bayesian Criterion. . . . .	-82.6741
(4) Other Fit Measures: -----	
Goodness of Fit Index (GFI) . . . . .	0.9968
Parsimonious GFI (Mulaik, 1989) . . . . .	0.4430
GFI Adjusted for Degrees of Freedom (AGFI). . . . .	0.9910
Root Mean Square Residual (RMR) . . . . .	0.0062
Hoelter's (1983) Critical N . . . . .	1796

Standardized Factor Loadings

	FAC_1	FAC_2	FAC_3
OBS1	0.9469504	0	-0.0868371
OBS2	0.9127608	0	0
OBS3	0.8558441	0	0
OBS4	0	-0.4000696	0.7568036
OBS5	0	-0.4409126	0.6585143
OBS6	0.1813749	-0.2078342	0.5416304
OBS7	0.1135289	-0.7937702	0
OBS8	0.4333498	-0.4145054	-0.0569913
OBS9	0	-0.6961397	0.1982984

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	OBS1	0.17303634	1.00045863	0.82704299

2	OBS2	0.16695601	1.00052958	0.83313236
3	OBS3	0.26765547	1.00046555	0.73246907
4	OBS4	0.26743374	1.00090241	0.73280738
5	OBS5	0.37227686	1.00086534	0.62804501
6	OBS6	0.50431474	1.00054204	0.49595848
7	OBS7	0.27517642	1.00013413	0.72486049
8	OBS8	0.49721772	1.00026677	0.50291489
9	OBS9	0.47626301	1.00041126	0.52393278

Total Determination of All Equations = 0.995  
Total Determination of Manifest Variables = 0.995

McDONALD (1985, p.57) is using the following CFA Model where all entries of **L** are zeroed except  $L[1, 1]$ ,  $L[2, 1]$ ,  $L[3, 1]$ ,  $L[4, 2]$ ,  $L[5, 2]$ ,  $L[6, 2]$ ,  $L[7, 3]$ ,  $L[8, 3]$ ,  $L[9, 3]$  and all offdiagonal entries of **P** are parameters. This model is also specified in sem09.inp

Loadings and Uni. Vars. with ASE's and Wald CIs

	FAC_1		FAC_2	
OBS1	0.9047110	0.0542156	0	0
	[ 0.798450,	1.010972]	[ 0,	0]
OBS2	0.9138216	0.0538657	0	0
	[ 0.808247,	1.019396]	[ 0,	0]
OBS3	0.8560762	0.0560231	0	0
	[ 0.746273,	0.965879]	[ 0,	0]
OBS4	0	0	0.8357615	0.0607813
	[ 0,	0]	[ 0.716632,	0.954891]
OBS5	0	0	0.7971823	0.0617654
	[ 0,	0]	[ 0.676124,	0.918240]
OBS6	0	0	0.7025557	0.0642509
	[ 0,	0]	[ 0.576626,	0.828485]
OBS7	0	0	0	0
	[ 0,	0]	[ 0,	0]
OBS8	0	0	0	0
	[ 0,	0]	[ 0,	0]
OBS9	0	0	0	0

	[	0,	0]	[	0,	0]
			FAC_3		U_Var	
OBS1	0	0	0.1814980	0.0284775		
	[	0,	0]	[	0.125683,	0.237313]
OBS2	0	0	0.1649300	0.0277694		
	[	0,	0]	[	0.110503,	0.219357]
OBS3	0	0	0.2671335	0.0333634		
	[	0,	0]	[	0.201742,	0.332525]
OBS4	0	0	0.3015026	0.0510219		
	[	0,	0]	[	0.201502,	0.401504]
OBS5	0	0	0.3645004	0.0526354		
	[	0,	0]	[	0.261337,	0.467664]
OBS6	0	0	0.5064156	0.0596261		
	[	0,	0]	[	0.389551,	0.623281]
OBS7	0.7808184	0.0642916	0.3903225	0.0593363		
	[	0.654809,	0.906828]	[	0.274026,	0.506620]
OBS8	0.7201523	0.0656347	0.4813806	0.0622489		
	[	0.591511,	0.848794]	[	0.359375,	0.603386]
OBS9	0.7034975	0.0660291	0.5050912	0.0633281		
	[	0.574083,	0.832912]	[	0.380970,	0.629212]

Factor Correlations Phi

-----

			FAC_1		FAC_2	
FAC_1	1.0000000	0	0.6426983	0.0505065		
	[	1.000000,	1.000000]	[	0.543707,	0.741689]
FAC_2	0.6426983	0.0505065	1.0000000	0		
	[	0.543707,	0.741689]	[	1.000000,	1.000000]
FAC_3	0.6699934	0.0510583	0.6371772	0.0585959		
	[	0.569921,	0.770066]	[	0.522331,	0.752023]

FAC\_3

FAC_1	0.6699934	0.0510583
	[ 0.569921,	0.770066]
FAC_2	0.6371772	0.0585959
	[ 0.522331,	0.752023]
FAC_3	1.0000000	0
	[ 1.000000,	1.000000]

(1) Standalone Fit Measures: -----  
Fit criterion . . . . . 0.1802  
Normal Th. Chi-square (df = 24) 38.1963 Prob>chi\*\*2 = 0.0331  
Normal Theory Reweighted LS Chi-square . . . . . 40.1947  
Probability of Close Fit . . . . . 0.4088  
Z-Test of Wilson & Hilferty (1931). . . . . 1.8373

(2) Incremental Fit Measures: -----  
Null Model Chi-square (df = 36) . . . . . 1101.8924  
RMSEA Estimate . . . . . 0.0528 90%C.I.[ 0.0153, 0.0831]  
ECVI Estimate . . . . . 0.3881 90%C.I.[ . , 0.4888]  
McDonald's (1989) Centrality. . . . . 0.9672  
Tucker-Lewis Coefficient TLI. . . . . 0.9800  
Bentler & Bonett's (1980) NFI . . . . . 0.9653  
Bentler's Comparative Fit Index CFI . . . . . 0.9867  
Parsimonious NFI (James, Mulaik, & Brett,1982). . . . . 0.6436  
Bollen (1986) Normed Index Rho1 . . . . . 0.9480  
Bollen (1988) Non-normed Index Delta2 . . . . . 0.9868

(3) Information Criteria: -----  
Akaike's Information Criterion. . . . . -9.8037  
Bozdogan's (1987) CAIC. . . . . -114.4747  
Schwarz's Bayesian Criterion. . . . . -90.4747

(4) Other Fit Measures: -----  
Goodness of Fit Index (GFI) . . . . . 0.9596  
Parsimonious GFI (Mulaik, 1989) . . . . . 0.6397  
GFI Adjusted for Degrees of Freedom (AGFI). . . . . 0.9242  
Root Mean Square Residual (RMR) . . . . . 0.0436  
Hoelter's (1983) Critical N . . . . . 204

Standardized Factor Loadings

	FAC_1	FAC_2	FAC_3
OBS1	0.9047110	0	0
OBS2	0.9138216	0	0
OBS3	0.8560762	0	0
OBS4	0	0.8357615	0

OBS5	0	0.7971823	0
OBS6	0	0.7025557	0
OBS7	0	0	0.7808184
OBS8	0	0	0.7201523
OBS9	0	0	0.7034975

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	OBS1	0.18149802	1.00000000	0.81850198
2	OBS2	0.16493005	1.00000000	0.83506995
3	OBS3	0.26713349	1.00000000	0.73286651
4	OBS4	0.30150265	1.00000000	0.69849735
5	OBS5	0.36450044	1.00000000	0.63549956
6	OBS6	0.50641556	1.00000000	0.49358444
7	OBS7	0.39032255	1.00000000	0.60967745
8	OBS8	0.48138061	1.00000000	0.51861939
9	OBS9	0.50509124	1.00000000	0.49490876

Total Determination of All Equations = 0.994  
Total Determination of Manifest Variables = 0.992

## 2.4 Speed factor Data by Lord(1956): SEM33

The automatic model improvement algorithm starting at the  $f = 6$  factor EFA obtains:

```
*****
Starting Model Improvement (ChiSq=77.2481 Pvalue=0.0662436 DF=60)
*****
```

```
*****
Table of 20 Best Pattern Solutions
*****
```

N	Npar	DF	Chisquared	P
1	81	90	83.5408048	0.67127810
2	80	91	84.5127706	0.67113528
3	82	89	82.5913392	0.67078703
4	79	92	85.6235961	0.66710531
5	82	89	82.7220320	0.66705714
6	80	91	84.6690951	0.66672585

7	83	88	81.8036686	0.66564614
8	78	93	86.8054509	0.66111865
9	81	90	83.9222530	0.66042589
10	83	88	82.0392250	0.65884309
11	83	88	82.0392426	0.65884258
12	84	87	81.1057439	0.65781014
13	80	91	85.0045103	0.65720598
14	82	89	83.2192018	0.65275594
15	80	91	85.2134775	0.65123653
16	82	89	83.3072553	0.65020554
17	82	89	83.3196657	0.64984568
18	83	88	82.3715996	0.64917838
19	81	90	84.3317432	0.64866368
20	81	90	84.3727857	0.64747878

N	RMSEA	ECVI	SRMR	PGFI	SBC
1	0	0.38647266	0.00966930	0.57988505	-499.248140
2	0	0.38479296	0.01033847	0.58623987	-504.751607
3	0	0.38818709	0.00951834	0.57350654	-493.722173
4	0	0.38332755	0.00976527	0.59250184	-510.116214
5	0	0.38838878	0.00972279	0.57352962	-493.591480
6	0	0.38503420	0.00926660	0.58612022	-504.595282
7	0	0.39015120	0.00942374	0.56714585	-488.034411
8	0	0.38197175	0.01060256	0.59883830	-515.409792
9	0	0.38706132	0.01055991	0.57982539	-498.866692
10	0	0.39051471	0.00956565	0.56714384	-487.798854
11	0	0.39051474	0.00956506	0.56714371	-487.798836
12	0	0.39225381	0.00939487	0.56075812	-482.256902
13	0	0.38555182	0.00993579	0.58613672	-504.259867
14	0	0.38915601	0.00918856	0.57344398	-493.094310
15	0	0.38587430	0.00954506	0.58610865	-504.050900
16	0	0.38929190	0.00994625	0.57344592	-493.006256
17	0	0.38931105	0.00995461	0.57343891	-492.993846
18	0	0.39102764	0.00893986	0.56708063	-487.466480
19	0	0.38769325	0.00993271	0.57978047	-498.457201
20	0	0.38775659	0.00936871	0.57975575	-498.416159

The following shows details of the best solution found:

Loadings and Uni. Vars. with ASE's and Wald CIs

-----

FAC\_1

FAC\_2

V1	6.0719262	0.3200259	0	0
	[ 5.444687,	6.699166]	[ 0,	0]
V2	6.2437766	0.3100025	2.1423596	0.9009982
	[ 5.636183,	6.851370]	[ 0.376436,	3.908284]
V3	6.8310094	0.3172840	0	0
	[ 6.209144,	7.452875]	[ 0,	0]
V4	6.9658948	0.3072605	0	0
	[ 6.363675,	7.568114]	[ 0,	0]
V5	7.6929515	0.2810391	-1.2485381	0.6242692
	[ 7.142125,	8.243778]	[-2.472083,	-0.024993]
V6	7.7870139	0.2891260	0	0
	[ 7.220337,	8.353690]	[ 0,	0]
V7	0	0	0	0
	[ 0,	0]	[ 0,	0]
V8	0	0	1.8166981	0.5691888
	[ 0,	0]	[ 0.701109,	2.932288]
V9	0.2220296	0.2247509	2.4150871	0.5772850
	[-0.218474,	0.662533]	[ 1.283629,	3.546545]
V10	0.2766660	0.2281057	0	0
	[-0.170413,	0.723745]	[ 0,	0]
V11	0	0	0	0
	[ 0,	0]	[ 0,	0]
V12	-0.2980340	0.2101544	0	0
	[-0.709929,	0.113861]	[ 0,	0]
V13	0	0	4.5067282	1.1895548
	[ 0,	0]	[ 2.175244,	6.838213]
V14	0	0	2.9739798	1.0806625
	[ 0,	0]	[ 0.855920,	5.092039]
V15	0	0	8.1041119	0.3947306
	[ 0,	0]	[ 7.330454,	8.877770]
V16	0	0	1.8274981	0.9398649

	[	0,	0]	[-0.014603, 3.669599]
V17	0	0	0	0
	[	0,	0]	[ 0, 0]
V18	0	0	0	0
	[	0,	0]	[ 0, 0]
			FAC_3	FAC_4
V1	3.3150185	0.3912797	0	0
	[ 2.548124,	4.081913]	[ 0,	0]
V2	2.6437782	0.3597777	0.7592340	0.2513204
	[ 1.938627,	3.348930]	[ 0.266655,	1.251813]
V3	2.3283132	0.3517641	0	0
	[ 1.638868,	3.017758]	[ 0,	0]
V4	0	0	0.8004526	0.2404981
	[ 0,	0]	[ 0.329085,	1.271820]
V5	0	0	0	0
	[ 0,	0]	[ 0,	0]
V6	-1.6157835	0.3859872	0	0
	[-2.372305,	-0.859262]	[ 0,	0]
V7	1.1626474	0.3260419	7.4778976	0.3350734
	[ 0.523617,	1.801678]	[ 6.821166,	8.134629]
V8	0	0	8.2038849	0.3240872
	[ 0,	0]	[ 7.568686,	8.839084]
V9	0	0	8.3908953	0.3321450
	[ 0,	0]	[ 7.739903,	9.041888]
V10	0	0	7.9178492	0.3382666
	[ 0,	0]	[ 7.254859,	8.580840]
V11	0	0	8.2027800	0.3341874
	[ 0,	0]	[ 7.547785,	8.857775]
V12	-0.3922719	0.2638650	8.1946402	0.3479574
	[-0.909438,	0.124894]	[ 7.512656,	8.876624]

V13	1.3362929	0.3819892	1.8252620	0.4760826
	[ 0.587608,	2.084978]	[ 0.892157,	2.758367]
V14	0.5979773	0.4200843	2.2408969	0.5030036
	[-0.225373,	1.421327]	[ 1.255028,	3.226766]
V15	0	0	1.5797407	0.5364307
	[ 0,	0]	[ 0.528356,	2.631126]
V16	-0.9700643	0.4003435	1.6302612	0.5414816
	[-1.754723,-0.185405]	[ 0.568977,	2.691546]	
V17	0	0	2.0031391	0.5203115
	[ 0,	0]	[ 0.983347,	3.022931]
V18	-0.6202871	0.4172748	1.0923294	0.5254678
	[-1.438131, 0.197556]	[ 0.062431,	2.122227]	
		FAC_5		FAC_6
V1	0	0	3.5556992	0.3902137
	[ 0,	0]	[ 2.790894,	4.320504]
V2	0.6916515	0.4076770	1.5668374	0.9287374
	[-0.107381, 1.490684]	[-0.253454,	3.387129]	
V3	0.5847585	0.3823806	3.8687194	0.4010005
	[-0.164694, 1.334211]	[ 3.082773,	4.654666]	
V4	0	0	4.2910363	0.3961712
	[ 0,	0]	[ 3.514555,	5.067518]
V5	0	0	5.5029723	0.7064593
	[ 0,	0]	[ 4.118338,	6.887607]
V6	0	0	4.6685783	0.3955291
	[ 0,	0]	[ 3.893356,	5.443801]
V7	0	0	2.2613639	0.5377250
	[ 0,	0]	[ 1.207442,	3.315285]
V8	0	0	0	0
	[ 0,	0]	[ 0,	0]
V9	0	0	0	0
	[ 0,	0]	[ 0,	0]

V10	-2.3262888	0.3672806	2.3140842	0.5590919
	[-3.046146, -1.606432] [ 1.218284, 3.409884]			
V11	-3.0770671	0.3761633	2.0841849	0.5693505
	[-3.814334, -2.339801] [ 0.968278, 3.200091]			
V12	-2.5674990	0.3557420	2.6479295	0.5699106
	[-3.264741, -1.870258] [ 1.530925, 3.764934]			
V13	0	0	2.0032229	1.1527076
	[ 0, 0] [-0.256042, 4.262488]			
V14	1.5996717	0.5018659	3.7016316	1.0390729
	[ 0.616033, 2.583311] [ 1.665086, 5.738177]			
V15	0	0	0	0
	[ 0, 0] [ 0, 0]			
V16	0	0	6.1955204	0.9294210
	[ 0, 0] [ 4.373889, 8.017152]			
V17	0	0	7.4938882	0.3582164
	[ 0, 0] [ 6.791797, 8.195980]			
V18	0	0	7.6414969	0.3471791
	[ 0, 0] [ 6.961038, 8.321955]			

	U_Var	
V1	26.222160	2.4930495
	[ 21.33587, 31.10845]	
V2	26.641908	2.1451506
	[ 22.43749, 30.84633]	
V3	27.575886	2.0059081
	[ 23.64438, 31.50739]	
V4	29.619543	1.8696129
	[ 25.95517, 33.28392]	
V5	13.165456	1.2414845
	[ 10.73219, 15.59872]	
V6	12.608255	2.1187183

[ 8.455643, 16.76087]

V7 28.655486 1.9498993  
[ 24.83375, 32.47722]

V8 22.878885 1.6740535  
[ 19.59780, 26.15997]

V9 19.905897 1.5660904  
[ 16.83642, 22.97538]

V10 22.515727 1.5842072  
[ 19.41074, 25.62072]

V11 15.581128 1.7782928  
[ 12.09574, 19.06652]

V12 16.743815 1.4472403  
[ 13.90728, 19.58035]

V13 36.842973 2.6206218  
[ 31.70665, 41.97930]

V14 44.048633 3.0554245  
[ 38.06011, 50.03715]

V15 26.478451 4.5841282  
[ 17.49373, 35.46318]

V16 33.447490 2.4565987  
[ 28.63264, 38.26233]

V17 36.840295 2.6370810  
[ 31.67171, 42.00888]

V18 36.663896 2.7163183  
[ 31.34001, 41.98778]

Factor Correlations Phi

-----

	FAC_1	FAC_2
FAC_1	1.0000000	0
	[ 1.000000, 1.000000]	[ 0, 0]

FAC_2	0	0	1.0000000	0
[	0,	0]	[ 1.000000, 1.000000]	
FAC_3	0	0	0	0
[	0,	0]	[ 0, 0]	
FAC_4	0	0	0	0
[	0,	0]	[ 0, 0]	
FAC_5	0	0	0	0
[	0,	0]	[ 0, 0]	
FAC_6	0	0	0.8595380	0.0375430
[	0,	0]	[ 0.785955, 0.933121]	
		FAC_3		FAC_4
FAC_1	0	0	0	0
[	0,	0]	[ 0, 0]	
FAC_2	0	0	0	0
[	0,	0]	[ 0, 0]	
FAC_3	1.0000000	0	0	0
[	1.000000, 1.000000]	[ 0,	0]	
FAC_4	0	0	1.0000000	0
[	0,	0]	[ 1.000000, 1.000000]	
FAC_5	0	0	0	0
[	0,	0]	[ 0, 0]	
FAC_6	0	0	0	0
[	0,	0]	[ 0, 0]	
		FAC_5		FAC_6
FAC_1	0	0	0	0
[	0,	0]	[ 0, 0]	
FAC_2	0	0	0.8595380	0.0375430
[	0,	0]	[ 0.785955, 0.933121]	
FAC_3	0	0	0	0
[	0,	0]	[ 0, 0]	

FAC_4	0	0	0	0
	[ 0,	0]	[ 0,	0]
FAC_5	1.000000	0	0	0
	[ 1.000000,	1.000000]	[ 0,	0]
FAC_6	0	0	1.000000	0
	[ 0,	0]	[ 1.000000,	1.000000]

(1) Standalone Fit Measures: -----  
Fit criterion . . . . . 0.1289  
Normal Th. Chi-square (df = 90) 83.5408 Prob>chi\*\*2 = 0.6713  
Normal Theory Reweighted LS Chi-square . . . . . 83.9798  
Probability of Close Fit . . . . . 1.0000  
Z-Test of Wilson & Hilferty (1931). . . . . -0.4437  
(2) Incremental Fit Measures: -----  
Null Model Chi-square (df = 153). . . . . 9151.0889  
RMSEA Estimate . . . . . 0.0000 90%C.I.[ . , 0.0177]  
ECVI Estimate . . . . . 0.3865 90%C.I.[ . , 0.4249]  
McDonald's (1989) Centrality. . . . . 1.0050  
Tucker-Lewis Coefficient TLI. . . . . 1.0012  
Bentler & Bonett's (1980) NFI . . . . . 0.9909  
Bentler's Comparative Fit Index CFI . . . . . 1.0000  
Parsimonious NFI (James, Mulaik, & Brett,1982). . . . . 0.5829  
Bollen (1986) Normed Index Rho1 . . . . . 0.9845  
Bollen (1988) Non-normed Index Delta2 . . . . . 1.0007  
(3) Information Criteria: -----  
Akaike's Information Criterion. . . . . -96.4592  
Bozdogan's (1987) CAIC. . . . . -589.2481  
Schwarz's Bayesian Criterion. . . . . -499.2481  
(4) Other Fit Measures: -----  
Goodness of Fit Index (GFI) . . . . . 0.9858  
Parsimonious GFI (Mulaik, 1989) . . . . . 0.5799  
GFI Adjusted for Degrees of Freedom (AGFI). . . . . 0.9730  
Root Mean Square Residual (RMR) . . . . . 0.9015  
Hoelter's (1983) Critical N . . . . . 879

Standardized Factor Loadings

	FAC_1	FAC_2	FAC_3	FAC_4	FAC_5
V1	0.6520184	0	0.3559749	0	0
V2	0.6713886	0.2303663	0.2842835	0.0816399	0.0743728

V3	0.7009628	0	0.2389194	0	0.0600049
V4	0.7065625	0	0	0.0811913	0
V5	0.8004060	-0.1299030	0	0	0
V6	0.7880067	0	-0.1635092	0	0
V7	0	0	0.1218518	0.7837246	0
V8	0	0.1878955	0	0.8485027	0
V9	0.0226378	0.2462389	0	0.8555239	0
V10	0.0282296	0	0	0.8078966	-0.2373626
V11	0	0	0	0.8342481	-0.3129472
V12	-0.0301457	0	-0.0396777	0.8288745	-0.2596983
V13	0	0.4982822	0.1477460	0.2018084	0
V14	0	0.3076308	0.0618552	0.2318001	0.1654713
V15	0	0.8329973	0	0.1623768	0
V16	0	0.1843848	-0.0978743	0.1644847	0
V17	0	0	0	0.2033762	0
V18	0	0	-0.0630997	0.1111189	0

Standardized Factor Loadings

	FAC_6
V1	0.3818197
V2	0.1684808
V3	0.3969880
V4	0.4352471
V5	0.5725517
V6	0.4724367
V7	0.2370033
V8	0
V9	0
V10	0.2361173
V11	0.2119681
V12	0.2678337
V13	0.2214845
V14	0.3828996
V15	0
V16	0.6250950
V17	0.7608451
V18	0.7773431

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	V1	26.2221597	86.7227919	0.69763243
2	V2	26.6419081	86.4861926	0.69195189

3	V3	27.5758859	94.9685501	0.70963139
4	V4	29.6195433	97.1969511	0.69526263
5	V5	13.1654562	92.3773057	0.85748170
6	V6	12.6082549	97.6522196	0.87088614
7	V7	28.6554864	91.0399549	0.68524274
8	V8	22.8788851	93.4830054	0.75526156
9	V9	19.9058973	96.1949634	0.79306716
10	V10	22.5157266	96.0512122	0.76558623
11	V11	15.5811277	96.6788957	0.83883631
12	V12	16.7438146	97.7422254	0.82869415
13	V13	36.8429733	81.8035200	0.54961628
14	V14	44.0486326	93.4579918	0.52867987
15	V15	26.4784513	94.6506617	0.72025075
16	V16	33.4474896	98.2343848	0.65951342
17	V17	36.8402954	97.0112226	0.62024708
18	V18	36.6638962	96.6343101	0.62059132

In `sem33.inp` we specified a model developed by Joereskog (1978, p. 454-5):

```

print "*** CFA Model in SEM33: ***";
l = [" Z1  5 # 0. ,
      Z2  5 # 0. ,
      Z3  0.  0. Z19 0. 0. ,
      Z4  0.  0. Z20 0. 0. ,
      Z5  0.  0. Z21 0. 0. ,
      Z6  0.  0. Z22 0. 0. ,
      0. Z7  4 # 0. ,
      0. Z8  4 # 0. ,
      0. Z9  0. 0. Z23 0. ,
      0. Z10 0. 0. Z24 0. ,
      0. Z11 0. 0. Z25 0. ,
      0. Z12 0. 0. Z26 0. ,
      0. 0. Z13  3 # 0. ,
      0. 0. Z14  3 # 0. ,
      0. 0. Z15  0. 0. Z27 ,
      0. 0. Z16  0. 0. Z28 ,
      0. 0. Z17  0. 0. Z29 ,
      0. 0. Z18  0. 0. Z30 "];
print "L=",l;

p = [" 1., C1 1., C2 C3 1., 0. 0. 0. 1.,
      0. 0. 0. C4 1., 0. 0. 0. C5 C6 1. "];
p = (tri2sym)p; print "P=",p;
u = [" u1:u18 "];
print "U=",u;

```

```

list patt;
patt[1] = 1; patt[2] = p; patt[3] = u;
print "Patt=",patt;

print "CFA with VARMAX rotation: p=0.0005, chi=174.50, df=117";
optn = [ "data"      "cor"  ,
         "nobs"      649  ,
         "anal"      "cov"  ,
         "hey"       ,
         "meth"      "ml"   ,
         "frot"      "varmax",
         "cl"        "wald" ,
         "nfac"      6      ,
         "tech"      "nrridg",
         "pmod"     ,
         "phis"     ,
         "pini"      2      ,
         "prin"     3      ];
< gof,parm,resi,covm,mod1,mod2 > = cfa(lord,optn,patt);

```

Loadings and Uni. Vars. with ASE's and Wald CIs

```

-----
                FAC_1                FAC_2
V1    7.6283657  0.3109221            0            0
      [ 7.018970, 8.237762] [ 0,            0]
V2    7.7238755  0.3087426            0            0
      [ 7.118751, 8.329000] [ 0,            0]
V3    8.1770373  0.3255776            0            0
      [ 7.538917, 8.815158] [ 0,            0]
V4    7.5301310  0.3434640            0            0
      [ 6.856954, 8.203308] [ 0,            0]
V5    8.0861675  0.3178589            0            0
      [ 7.463176, 8.709159] [ 0,            0]
V6    7.7649167  0.3411981            0            0
      [ 7.096181, 8.433653] [ 0,            0]
V7            0            0  7.8722749  0.3138617

```

	[	0,	0]	[	7.257117,	8.487433]
V8	0	0	8.4189221	0.3092419		
	[	0,	0]	[	7.812819,	9.025025]
V9	0	0	8.6224021	0.3160839		
	[	0,	0]	[	8.002889,	9.241915]
V10	0	0	8.1935822	0.3249554		
	[	0,	0]	[	7.556681,	8.830483]
V11	0	0	8.4023056	0.3228630		
	[	0,	0]	[	7.769506,	9.035105]
V12	0	0	8.4909755	0.3236832		
	[	0,	0]	[	7.856568,	9.125383]
V13	0	0	0	0		
	[	0,	0]	[	0,	0]
V14	0	0	0	0		
	[	0,	0]	[	0,	0]
V15	0	0	0	0		
	[	0,	0]	[	0,	0]
V16	0	0	0	0		
	[	0,	0]	[	0,	0]
V17	0	0	0	0		
	[	0,	0]	[	0,	0]
V18	0	0	0	0		
	[	0,	0]	[	0,	0]
			FAC_3		FAC_4	
V1	0	0	0	0		
	[	0,	0]	[	0,	0]
V2	0	0	0	0		
	[	0,	0]	[	0,	0]
V3	0	0	0.8296144	0.3850475		
	[	0,	0]	[	0.074935,	1.584294]

V4	0	0	3.2916481	0.3633609
	[	0,	0]	[ 2.579474, 4.003822]
V5	0	0	3.7419424	0.3163946
	[	0,	0]	[ 3.121820, 4.362064]
V6	0	0	4.8221642	0.3458560
	[	0,	0]	[ 4.144299, 5.500030]
V7	0	0	0	0
	[	0,	0]	[ 0, 0]
V8	0	0	0	0
	[	0,	0]	[ 0, 0]
V9	0	0	0	0
	[	0,	0]	[ 0, 0]
V10	0	0	0	0
	[	0,	0]	[ 0, 0]
V11	0	0	0	0
	[	0,	0]	[ 0, 0]
V12	0	0	0	0
	[	0,	0]	[ 0, 0]
V13	6.8307132	0.3228920	0	0
	[	6.197856,	7.463570]	[ 0, 0]
V14	7.0978273	0.3481031	0	0
	[	6.415558,	7.780097]	[ 0, 0]
V15	7.6032641	0.3460626	0	0
	[	6.924994,	8.281534]	[ 0, 0]
V16	7.3072750	0.3606371	0	0
	[	6.600439,	8.014111]	[ 0, 0]
V17	7.0930379	0.3610033	0	0
	[	6.385485,	7.800591]	[ 0, 0]
V18	6.7901582	0.3674533	0	0
	[	6.069963,	7.510353]	[ 0, 0]

		FAC_5		FAC_6		
V1	0	0	0	0		
	[	0,	0]	[	0,	0]
V2	0	0	0	0		
	[	0,	0]	[	0,	0]
V3	0	0	0	0		
	[	0,	0]	[	0,	0]
V4	0	0	0	0		
	[	0,	0]	[	0,	0]
V5	0	0	0	0		
	[	0,	0]	[	0,	0]
V6	0	0	0	0		
	[	0,	0]	[	0,	0]
V7	0	0	0	0		
	[	0,	0]	[	0,	0]
V8	0	0	0	0		
	[	0,	0]	[	0,	0]
V9	0.5456933	0.4436800	0	0		
	[	-0.323903,	1.415290]	[	0,	0]
V10	2.6128046	0.3885210	0	0		
	[	1.851317,	3.374292]	[	0,	0]
V11	3.1089389	0.3852227	0	0		
	[	2.353916,	3.863961]	[	0,	0]
V12	3.2239759	0.3860668	0	0		
	[	2.467299,	3.980653]	[	0,	0]
V13	0	0	0	0		
	[	0,	0]	[	0,	0]
V14	0	0	0	0		
	[	0,	0]	[	0,	0]
V15	0	0	1.0898998	0.4623232		
	[	0,	0]	[	0.183763,	1.996037]

V16	0	0	3.5812933	0.4683548
[	0,	0]	[ 2.663335,	4.499252]
V17	0	0	2.9350545	0.4641535
[	0,	0]	[ 2.025330,	3.844779]
V18	0	0	3.6317776	0.4822470
[	0,	0]	[ 2.686591,	4.576964]

		U_Var
V1	28.298036	2.0874177
	[ 24.20677,	32.38930]
V2	26.645847	2.0348994
	[ 22.65752,	30.63418]
V3	27.151259	2.0413703
	[ 23.15025,	31.15227]
V4	29.793435	1.8983181
	[ 26.07280,	33.51407]
V5	13.100164	1.2644546
	[ 10.62188,	15.57845]
V6	14.433571	1.8961901
	[ 10.71711,	18.15004]
V7	29.229787	2.0052323
	[ 25.29960,	33.15997]
V8	22.630650	1.8128745
	[ 19.07748,	26.18382]
V9	21.732031	1.7513687
	[ 18.29941,	25.16465]
V10	22.713296	1.5844860
	[ 19.60776,	25.81883]
V11	17.139269	1.5802589
	[ 14.04202,	20.23652]
V12	15.935406	1.6149322
	[ 12.77020,	19.10062]

V13	35.243857	2.5341835
	[ 30.27695,	40.21077]
V14	43.129748	2.9856493
	[ 37.27798,	48.98151]
V15	35.788956	2.5487612
	[ 30.79348,	40.78444]
V16	32.344787	2.6248803
	[ 27.20012,	37.48946]
V17	38.382053	2.6422283
	[ 33.20338,	43.56073]
V18	37.670778	2.8821251
	[ 32.02192,	43.31964]

Factor Correlations Phi

-----

	FAC_1		FAC_2	
FAC_1	1.0000000	0	0.1618146	0.0419585
	[ 1.000000,	1.000000]	[ 0.079578,	0.244052]
FAC_2	0.1618146	0.0419585	1.0000000	0
	[ 0.079578,	0.244052]	[ 1.000000,	1.000000]
FAC_3	0.4701773	0.0359095	0.4827407	0.0349155
	[ 0.399796,	0.540559]	[ 0.414308,	0.551174]
FAC_4	0	0	0	0
	[ 0,	0]	[ 0,	0]
FAC_5	0	0	0	0
	[ 0,	0]	[ 0,	0]
FAC_6	0	0	0	0
	[ 0,	0]	[ 0,	0]
		FAC_3		FAC_4
FAC_1	0.4701773	0.0359095	0	0

		[ 0.399796, 0.540559]	[	0,	0]	
FAC_2	0.4827407	0.0349155		0	0	
		[ 0.414308, 0.551174]	[	0,	0]	
FAC_3	1.0000000	0		0	0	
		[ 1.000000, 1.000000]	[	0,	0]	
FAC_4	0	0	1.0000000		0	
	[	0,	0]	[	1.000000, 1.000000]	
FAC_5	0	0	0.2627967	0.0850491		
	[	0,	0]	[	0.096104, 0.429490]	
FAC_6	0	0	0.6321221	0.0885425		
	[	0,	0]	[	0.458582, 0.805662]	
			FAC_5		FAC_6	
FAC_1	0	0	0	0	0	
	[	0,	0]	[	0,	0]
FAC_2	0	0	0	0	0	
	[	0,	0]	[	0,	0]
FAC_3	0	0	0	0	0	
	[	0,	0]	[	0,	0]
FAC_4	0.2627967	0.0850491	0.6321221	0.0885425		
	[	0.096104, 0.429490]	[	0.458582, 0.805662]		
FAC_5	1.0000000	0	0.4208009	0.1070164		
	[	1.000000, 1.000000]	[	0.211053, 0.630549]		
FAC_6	0.4208009	0.1070164	1.0000000	0		
	[	0.211053, 0.630549]	[	1.000000, 1.000000]		

(1) Standalone Fit Measures: -----  
Fit criterion . . . . . 0.2693  
Normal Th. Chi-square (df = 117) . . . . . 174.5032 prob= 0.0005  
Normal Theory Reweighted LS Chi-square . . . . . 174.6905  
Probability of Close Fit . . . . . 1.0000  
Z-Test of Wilson & Hilferty (1931). . . . . 3.3143  
(2) Incremental Fit Measures: -----  
Null Model Chi-square (df = 153). . . . . 9151.0889

RMSEA Estimate . . . . .	0.0275	90%C.I.[ 0.0185,	0.0358]
ECVI Estimate . . . . .	0.4410	90%C.I.[ 0.3922,	0.5025]
McDonald's (1989) Centrality. . . . .			0.9567
Tucker-Lewis Coefficient TLI. . . . .			0.9916
Bentler & Bonett's (1980) NFI . . . . .			0.9809
Bentler's Comparative Fit Index CFI . . . . .			0.9936
Parsimonious NFI (James, Mulaik, & Brett,1982). . . . .			0.7501
Bollen (1986) Normed Index Rho1 . . . . .			0.9751
Bollen (1988) Non-normed Index Delta2 . . . . .			0.9936
(3) Information Criteria: -----			
Akaike's Information Criterion. . . . .			-59.4968
Bozdogan's (1987) CAIC. . . . .			-700.1224
Schwarz's Bayesian Criterion. . . . .			-583.1224
(4) Other Fit Measures: -----			
Goodness of Fit Index (GFI) . . . . .			0.9709
Parsimonious GFI (Mulaik, 1989) . . . . .			0.7425
GFI Adjusted for Degrees of Freedom (AGFI). . . . .			0.9575
Root Mean Square Residual (RMR) . . . . .			2.4779
Hoelter's (1983) Critical N . . . . .			533

Standardized Factor Loadings

	FAC_1	FAC_2	FAC_3	FAC_4	FAC_5
V1	0.8202544	0	0	0	0
V2	0.8314182	0	0	0	0
V3	0.8402588	0	0	0.0852498	0
V4	0.7632668	0	0	0.3336470	0
V5	0.8408124	0	0	0.3890930	0
V6	0.7844520	0	0	0.4871599	0
V7	0	0.8243220	0	0	0
V8	0	0.8706228	0	0	0
V9	0	0.8783036	0	0	0.0555859
V10	0	0.8333302	0	0	0.2657359
V11	0	0.8513559	0	0	0.3150104
V12	0	0.8558595	0	0	0.3249651
V13	0	0	0.7547749	0	0
V14	0	0	0.7340049	0	0
V15	0	0	0.7809566	0	0
V16	0	0	0.7360212	0	0
V17	0	0	0.7190490	0	0
V18	0	0	0.6895540	0	0

Standardized Factor Loadings

	FAC_6
V1	0
V2	0
V3	0
V4	0
V5	0
V6	0
V7	0
V8	0
V9	0
V10	0
V11	0
V12	0
V13	0
V14	0
V15	0.1119472
V16	0.3607238
V17	0.2975380
V18	0.3688142

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	V1	28.2980362	86.4900000	0.67281725
2	V2	26.6458468	86.3041000	0.69125630
3	V3	27.1512593	94.7034583	0.71330235
4	V4	29.7934352	97.3312549	0.69389653
5	V5	13.1001637	92.4884014	0.85835885
6	V6	14.4335714	97.9807707	0.85268975
7	V7	29.2297872	91.2025000	0.67950673
8	V8	22.6306500	93.5089000	0.75798400
9	V9	21.7320310	96.3756294	0.77450699
10	V10	22.7132957	96.6748325	0.76505472
11	V11	17.1392691	97.4035089	0.82403848
12	V12	15.9354063	98.4260915	0.83809774
13	V13	35.2438570	81.9025000	0.56968521
14	V14	43.1297479	93.5089000	0.53876318
15	V15	35.7889559	94.7864618	0.62242545
16	V16	32.3447871	98.5667172	0.67184879
17	V17	38.3820530	97.3077852	0.60556031
18	V18	37.6707783	96.9668346	0.61150863

## 2.5 Eight Physical Variables (Harman, 1967): SEM47

```

phys8 = [ 1.0    .846    .805    .859    .473    .398    .301    .382 ,
          .846    1.0    .881    .826    .376    .326    .277    .415 ,
          .805    .881    1.0    .801    .380    .319    .237    .345 ,
          .859    .826    .801    1.0    .436    .329    .327    .365 ,
          .473    .376    .380    .436    1.0    .762    .730    .629 ,
          .398    .326    .319    .329    .762    1.0    .583    .577 ,
          .301    .277    .237    .327    .730    .583    1.0    .539 ,
          .382    .415    .345    .365    .629    .577    .539    1.0 ];

```

The data were analyzed before and we obtain for the EFA solution for  $f = 2$ :

```

*****
Starting Model Improvement (ChiSq=76.9612 Pvalue=4.09863e-011 DF=13)
*****

```

The best CFA result found by our algorithm is only slightly better than that of the EFA:

```

*****
Table of 20 Best Pattern Solutions
*****

```

N	Npar	DF	Chisquared	P
1	20	16	78.6507507	2.910e-010
2	20	16	78.6507507	2.910e-010
3	21	15	76.9807295	2.479e-010
4	21	15	76.9807295	2.479e-010
5	21	15	78.0383980	1.592e-010
6	21	15	78.6105180	1.253e-010
7	21	15	78.6136805	1.251e-010
8	22	14	76.9613310	1.032e-010
9	22	14	76.9613310	1.032e-010
10	22	14	76.9613310	1.032e-010
11	22	14	76.9613310	1.032e-010
12	22	14	76.9803629	1.023e-010
13	22	14	76.9803630	1.023e-010
14	22	14	76.9803630	1.023e-010
15	22	14	76.9803630	1.023e-010
16	22	14	78.0164924	6.589e-011
17	22	14	78.0164925	6.589e-011
18	22	14	78.5709244	5.205e-011

19	22	14	78.5709245	5.205e-011
20	23	13	76.9611549	4.099e-011

N	RMSEA	ECVI	SRMR	PGFI	SBC
1	0.11349229	0.39431280	0.02402992	0.53687346	-12.8742377
2	0.11349229	0.39431280	0.02402992	0.53687346	-12.8742377
3	0.11658587	0.39559897	0.02012035	0.50394621	-8.82394714
4	0.11658587	0.39559897	0.02012037	0.50394622	-8.82394714
5	0.11757640	0.39907814	0.02258945	0.50352476	-7.76627866
6	0.11810874	0.40096011	0.02389540	0.50334229	-7.19415866
7	0.11811168	0.40097051	0.02419009	0.50331156	-7.19099610
8	0.12162872	0.40231482	0.02025498	0.47034959	-3.12303388
9	0.12162872	0.40231482	0.02025498	0.47034959	-3.12303388
10	0.12162872	0.40231482	0.02025508	0.47034963	-3.12303386
11	0.12162872	0.40231482	0.02025512	0.47034962	-3.12303385
12	0.12164710	0.40237742	0.02011283	0.47035014	-3.10400197
13	0.12164710	0.40237742	0.02011292	0.47035017	-3.10400190
14	0.12164710	0.40237742	0.02011299	0.47035013	-3.10400189
15	0.12164710	0.40237742	0.02011240	0.47034994	-3.10400185
16	0.12264367	0.40578574	0.02271931	0.46995350	-2.06787245
17	0.12264367	0.40578574	0.02271901	0.46995330	-2.06787242
18	0.12317362	0.40760953	0.02406193	0.46977930	-1.51344043
19	0.12317362	0.40760953	0.02406194	0.46977923	-1.51344041
20	0.12721833	0.40909390	0.02024873	0.43675329	2.59710176

The following shows details of the best solution found:

Loadings and Uni. Vars. with ASE's and Wald CIs

	FAC_1		FAC_2	
X1	0.8501964	0.0446153	0.1270203	0.0319009
	[ 0.762752,	0.937641]	[ 0.064496,	0.189545]
X2	0.9453263	0.0431220	0	0
	[ 0.860809,	1.029844]	[ 0,	0]
X3	0.9130442	0.0442776	0	0
	[ 0.826262,	0.999827]	[ 0,	0]
X4	0.8534984	0.0460557	0.0895713	0.0336441
	[ 0.763231,	0.943766]	[ 0.023630,	0.155513]

X5	0	0	0.9579197	0.0445318
	[	0,	0]	[ 0.870639, 1.045200]
X6	0	0	0.7961673	0.0490769
	[	0,	0]	[ 0.699978, 0.892356]
X7	0	0	0.7586894	0.0500183
	[	0,	0]	[ 0.660655, 0.856724]
X8	0.1370728	0.0500064	0.6096575	0.0554413
	[ 0.039062,	0.235084]	[ 0.500994,	0.718320]

		U_Var
X1	0.1704068	0.0175540
	[ 0.136002,	0.204812]
X2	0.1063582	0.0147247
	[ 0.077498,	0.135218]
X3	0.1663503	0.0179632
	[ 0.131143,	0.201558]
X4	0.1993628	0.0197228
	[ 0.160707,	0.238019]
X5	0.0823898	0.0280919
	[ 0.027331,	0.137449]
X6	0.3661176	0.0359400
	[ 0.295676,	0.436559]
X7	0.4243904	0.0393853
	[ 0.347197,	0.501584]
X8	0.5394005	0.0463537
	[ 0.448549,	0.630252]

Factor Correlations Phi

	FAC_1	FAC_2	
FAC_1	1.0000000	0 0.4195913 0.0515996	
	[ 1.000000,	1.000000]	[ 0.318458, 0.520725]

FAC\_2 0.4195913 0.0515996 1.0000000 0  
 [ 0.318458, 0.520725] [ 1.000000, 1.000000]

(1) Standalone Fit Measures: -----  
 Fit criterion . . . . . 0.2587  
 Normal Th. Chi-square (df = 16) 78.6508 Prob>chi\*\*2 = 0.0000  
 Normal Theory Reweighted LS Chi-square . . . . . 78.2660  
 Probability of Close Fit . . . . . 0.0000  
 Z-Test of Wilson & Hilferty (1931). . . . . 6.0602  
 (2) Incremental Fit Measures: -----  
 Null Model Chi-square (df = 28) . . . . . 2110.0336  
 RMSEA Estimate . . . . . 0.1135 90%C.I.[ 0.0892, 0.1391]  
 ECVI Estimate . . . . . 0.3943 90%C.I.[ 0.3146, 0.4996]  
 McDonald's (1989) Centrality. . . . . 0.9024  
 Tucker-Lewis Coefficient TLI. . . . . 0.9473  
 Bentler & Bonett's (1980) NFI . . . . . 0.9627  
 Bentler's Comparative Fit Index CFI . . . . . 0.9699  
 Parsimonious NFI (James, Mulaik, & Brett,1982). . . . . 0.5501  
 Bollen (1986) Normed Index Rho1 . . . . . 0.9348  
 Bollen (1988) Non-normed Index Delta2 . . . . . 0.9701  
 (3) Information Criteria: -----  
 Akaike's Information Criterion. . . . . 46.6508  
 Bozdogan's (1987) CAIC. . . . . -28.8742  
 Schwarz's Bayesian Criterion. . . . . -12.8742  
 (4) Other Fit Measures: -----  
 Goodness of Fit Index (GFI) . . . . . 0.9395  
 Parsimonious GFI (Mulaik, 1989) . . . . . 0.5369  
 GFI Adjusted for Degrees of Freedom (AGFI). . . . . 0.8639  
 Root Mean Square Residual (RMR) . . . . . 0.0240  
 Hoelter's (1983) Critical N . . . . . 103

Standardized Factor Loadings

	FAC_1	FAC_2
X1	0.8501961	0.1270205
X2	0.9453265	0
X3	0.9130442	0
X4	0.8534982	0.0895714
X5	0	0.9579197
X6	0	0.7961673
X7	0	0.7586894
X8	0.1370728	0.6096574

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	X1	0.17040709	1.00000000	0.82959291
2	X2	0.10635785	1.00000000	0.89364215
3	X3	0.16635024	1.00000000	0.83364976
4	X4	0.19936302	1.00000000	0.80063698
5	X5	0.08238983	1.00000000	0.91761017
6	X6	0.36611759	1.00000000	0.63388241
7	X7	0.42439039	1.00000000	0.57560961
8	X8	0.53940050	1.00000000	0.46059950

Similar to the model by Maydeu-Olivares for the Lot data, we used in our sem80.inp application zero loadings in  $L[1, 1], L[2, 1], L[3, 1], L[4, 1], L[5, 2], L[6, 2], L[7, 2], L[8, 2]$ :

Loadings and Uni. Vars. with ASE's and Wald CIs

	FAC_1		FAC_2	
X1	0.9138431	0.0442390	0	0
	[ 0.827136,	1.000550]	[ 0,	0]
X2	0.9367505	0.0434153	0	0
	[ 0.851658,	1.021843]	[ 0,	0]
X3	0.9079356	0.0444483	0	0
	[ 0.820818,	0.995053]	[ 0,	0]
X4	0.9002127	0.0447201	0	0
	[ 0.812563,	0.987863]	[ 0,	0]
X5	0	0	0.9454962	0.0448809
	[ 0,	0]	[ 0.857531,	1.033461]
X6	0	0	0.8031657	0.0489491
	[ 0,	0]	[ 0.707227,	0.899104]
X7	0	0	0.7631068	0.0499931
	[ 0,	0]	[ 0.665122,	0.861091]
X8	0	0	0.6829877	0.0519718
	[ 0,	0]	[ 0.581125,	0.784851]

	U_Var	
X1	0.1648907	0.0177858

```

[ 0.130031, 0.199750]
X2  0.1224985  0.0153121
[ 0.092487, 0.152510]
X3  0.1756529  0.0184803
[ 0.139432, 0.211874]
X4  0.1896171  0.0194127
[ 0.151569, 0.227665]
X5  0.1060370  0.0277814
[ 0.051586, 0.160488]
X6  0.3549249  0.0355108
[ 0.285325, 0.424525]
X7  0.4176681  0.0392187
[ 0.340801, 0.494535]
X8  0.5335278  0.0469720
[ 0.441464, 0.625591]

```

Factor Correlations Phi  
-----

		FAC_1		FAC_2
FAC_1	1.0000000	0	0.4755122	0.0481644
	[ 1.000000, 1.000000]	[ 0.381112,	0.569913]	
FAC_2	0.4755122	0.0481644	1.0000000	0
	[ 0.381112, 0.569913]	[ 1.000000,	1.000000]	

```

(1) Standalone Fit Measures: -----
Fit criterion . . . . . 0.3364
Normal Th. Chi-square (df = 19) . . . . . 102.2519 prob= 0.0000
Normal Theory Reweighted LS Chi-square . . . . . 105.6593
Probability of Close Fit . . . . . 0.0000
Z-Test of Wilson & Hilferty (1931). . . . . 7.0655
(2) Incremental Fit Measures: -----
Null Model Chi-square (df = 28) . . . . . 2110.0336
RMSEA Estimate . . . . . 0.1201 90%C.I.[ 0.0978, 0.1434]
ECVI Estimate . . . . . 0.4516 90%C.I.[ 0.3585, 0.5703]
McDonald's (1989) Centrality. . . . . 0.8724

```

Tucker-Lewis Coefficient TLI. . . . .	0.9411
Bentler & Bonett's (1980) NFI . . . . .	0.9515
Bentler's Comparative Fit Index CFI . . . . .	0.9600
Parsimonious NFI (James, Mulaik, & Brett,1982). . . . .	0.6457
Bollen (1986) Normed Index Rho1 . . . . .	0.9286
Bollen (1988) Non-normed Index Delta2 . . . . .	0.9602
(3) Information Criteria: -----	
Akaike's Information Criterion. . . . .	64.2519
Bozdogan's (1987) CAIC. . . . .	-25.4340
Schwarz's Bayesian Criterion. . . . .	-6.4340
(4) Other Fit Measures: -----	
Goodness of Fit Index (GFI) . . . . .	0.9201
Parsimonious GFI (Mulaik, 1989) . . . . .	0.6243
GFI Adjusted for Degrees of Freedom (AGFI). . . . .	0.8485
Root Mean Square Residual (RMR) . . . . .	0.0404
Hoelter's (1983) Critical N . . . . .	91

Standardized Factor Loadings

	FAC_1	FAC_2
X1	0.9138431	0
X2	0.9367505	0
X3	0.9079356	0
X4	0.9002127	0
X5	0	0.9454962
X6	0	0.8031657
X7	0	0.7631068
X8	0	0.6829877

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	X1	0.16489073	1.00000000	0.83510927
2	X2	0.12249846	1.00000000	0.87750154
3	X3	0.17565293	1.00000000	0.82434707
4	X4	0.18961707	1.00000000	0.81038293
5	X5	0.10603700	1.00000000	0.89396300
6	X6	0.35492486	1.00000000	0.64507514
7	X7	0.41766807	1.00000000	0.58233193
8	X8	0.53352777	1.00000000	0.46647223

Finally, for  $f = 3$  we obtain slightly better results. This output shows the EFA model fit:

\*\*\*\*\*  
 Starting Model Improvement (ChiSq=23.0148 Pvalue=0.00169455 DF=7)  
 \*\*\*\*\*

The following shows the result of the optimization:

\*\*\*\*\*  
 Table of 20 Best Pattern Solutions  
 \*\*\*\*\*

N	Npar	DF	Chisquared	P
1	23	13	25.2517652	0.02138621
2	24	12	24.1218575	0.01957841
3	22	14	27.2238927	0.01800267
4	24	12	24.6786237	0.01642138
5	25	11	23.5660773	0.01469193
6	23	13	26.4985822	0.01456105
7	24	12	25.1172959	0.01427672
8	25	11	23.7363159	0.01389186
9	25	11	23.7363159	0.01389186
10	25	11	23.8729910	0.01327942
11	23	13	26.9282636	0.01272552
12	25	11	24.2106974	0.01187358
13	25	11	24.6428060	0.01027925
14	22	14	29.0722357	0.01021769
15	26	10	23.2429488	0.00988448
16	26	10	23.2429488	0.00988448
17	25	11	24.9076075	0.00940482
18	26	10	23.4142151	0.00931670
19	26	10	23.4142151	0.00931670
20	24	12	26.4870590	0.00915264

N	RMSEA	ECVI	SRMR	PGFI	SBC
1	0.05567893	0.23899722	0.01835913	0.45508251	-49.1122879
2	0.05764441	0.24206008	0.01510296	0.42042403	-44.5218839
3	0.05574153	0.23870482	0.02009596	0.48921722	-52.8604722
4	0.05895337	0.24389155	0.01715112	0.42017008	-43.9651176
5	0.06130088	0.24701152	0.01335737	0.38556998	-39.3573522
6	0.05844342	0.24309859	0.01936837	0.45463393	-47.8654709
7	0.05996457	0.24533455	0.01641867	0.42015391	-43.5264454
8	0.06171472	0.24757151	0.01387635	0.38548115	-39.1871136
9	0.06171472	0.24757151	0.01387629	0.38548116	-39.1871136

10	0.06204497	0.24802110	0.01493086	0.38541288	-39.0504386
11	0.05936631	0.24451202	0.01976952	0.45441674	-47.4357895
12	0.06285354	0.24913198	0.01607712	0.38533984	-38.7127322
13	0.06387321	0.25055339	0.01467080	0.38521030	-38.2806236
14	0.05950974	0.24478490	0.02445962	0.48880577	-51.0121291
15	0.06600177	0.25272825	0.01247652	0.35059034	-33.9601690
16	0.06600177	0.25272825	0.01247652	0.35059034	-33.9601690
17	0.06449010	0.25142444	0.01730490	0.38513222	-38.0158220
18	0.06642718	0.25329163	0.01366534	0.35047236	-33.7889026
19	0.06642718	0.25329163	0.01366557	0.35047238	-33.7889026
20	0.06301772	0.24984035	0.01956135	0.41966544	-42.1566823

The following shows details of the best solution found:

Loadings and Uni. Vars. with ASE's and Wald CIs

```

-----
                                FAC_1                                FAC_2
X1  0.9129584  0.0464207                                0                0
     [ 0.821976, 1.003941] [                0,                0]

X2  0.8971286  0.0441942  0.4313792  0.0736614
     [ 0.810510, 0.983748] [ 0.287006, 0.575753]

X3  0.8652646  0.0459524  0.2271388  0.0476379
     [ 0.775200, 0.955330] [ 0.133770, 0.320507]

X4  0.9160986  0.0443564                                0                0
     [ 0.829162, 1.003035] [                0,                0]

X5          0          0 -0.0936520  0.0400532
     [          0,          0] [-0.172155,-0.015149]

X6          0          0          0          0
     [          0,          0] [          0,          0]

X7 -0.0888122  0.0486732                                0                0
     [-0.184210, 6.6e-003] [                0,                0]

X8          0          0  0.1844940  0.0609446
     [          0,          0] [ 0.065045, 0.303943]

                                FAC_3                                U_Var
X1  0.0465898  0.0328774  0.1220461  0.0171892

```

```

[-0.017849, 0.111028] [ 0.088356, 0.155736]
X2      0      0 1.00e-008 0.0590448
[      0,      0] [-0.115726, 0.115726]
X3      0      0 0.1951180 0.0244503
[      0,      0] [ 0.147196, 0.243040]
X4      0      0 0.1607634 0.0191277
[      0,      0] [ 0.123274, 0.198253]
X5 0.9482640 0.0444340 0.0895822 0.0275840
[ 0.861175, 1.035353] [ 0.035519, 0.143646]
X6 0.7985225 0.0488685 0.3623618 0.0352110
[ 0.702742, 0.894303] [ 0.293349, 0.431374]
X7 0.8079509 0.0577669 0.4106716 0.0393788
[ 0.694730, 0.921172] [ 0.333491, 0.487853]
X8 0.6940129 0.0513876 0.4878293 0.0474781
[ 0.593295, 0.794731] [ 0.394774, 0.580885]

```

Factor Correlations Phi

-----

```

          FAC_1          FAC_2
FAC_1  1.0000000      0      0
[ 1.000000, 1.000000] [      0,      0]
FAC_2      0      0 1.0000000      0
[      0,      0] [ 1.000000, 1.000000]
FAC_3  0.4971289 0.0491747      0      0
[ 0.400748, 0.593509] [      0,      0]

          FAC_3
FAC_1  0.4971289 0.0491747
[ 0.400748, 0.593509]
FAC_2      0      0
[      0,      0]
FAC_3  1.0000000      0

```

[ 1.000000, 1.000000]

		FAC_3
X1	0.0976812	0.0334375
	[ 0.032145,	0.163217]
X2	0	0
	[ 0,	0]
X3	0	0
	[ 0,	0]
X4	0.0616953	0.0356190
	[-8.1e-003,	0.131507]
X5	0.8557350	0.0436450
	[ 0.770192,	0.941278]
X6	0.7274247	0.0478260
	[ 0.633687,	0.821162]
X7	0.7128989	0.0492949
	[ 0.616283,	0.809515]
X8	0.5771781	0.0491124
	[ 0.480920,	0.673437]

(1) Standalone Fit Measures: -----  
Fit criterion . . . . . 0.0831  
Normal Th. Chi-square (df = 13) 25.2518 Prob>chi\*\*2 = 0.0214  
Normal Theory Reweighted LS Chi-square . . . . . 24.5913  
Probability of Close Fit . . . . . 0.3482  
Z-Test of Wilson & Hilferty (1931). . . . . 2.0254  
(2) Incremental Fit Measures: -----  
Null Model Chi-square (df = 28) . . . . . 2110.0336  
RMSEA Estimate . . . . . 0.0557 90%C.I.[ 0.0209, 0.0879]  
ECVI Estimate . . . . . 0.2390 90%C.I.[ 0.2043, 0.3000]  
McDonald's (1989) Centrality. . . . . 0.9801  
Tucker-Lewis Coefficient TLI. . . . . 0.9873  
Bentler & Bonett's (1980) NFI . . . . . 0.9880  
Bentler's Comparative Fit Index CFI . . . . . 0.9941  
Parsimonious NFI (James, Mulaik, & Brett,1982). . . . . 0.4587  
Bollen (1986) Normed Index Rho1 . . . . . 0.9742  
Bollen (1988) Non-normed Index Delta2 . . . . . 0.9942

```

(3) Information Criteria: -----
Akaike's Information Criterion. . . . . -0.7482
Bozdogan's (1987) CAIC. . . . . -62.1123
Schwarz's Bayesian Criterion. . . . . -49.1123
(4) Other Fit Measures: -----
Goodness of Fit Index (GFI) . . . . . 0.9802
Parsimonious GFI (Mulaik, 1989) . . . . . 0.4551
GFI Adjusted for Degrees of Freedom (AGFI). . . . . 0.9451
Root Mean Square Residual (RMR) . . . . . 0.0184
Hoelter's (1983) Critical N . . . . . 271

```

Standardized Factor Loadings

	FAC_1	FAC_2	FAC_3
X1	0.9129584	0	0.0465898
X2	0.9012260	0.4333494	0
X3	0.8672647	0.2276638	0
X4	0.9160986	0	0
X5	0	-0.0937666	0.9494242
X6	0	0	0.7985225
X7	-0.0888122	0	0.8079509
X8	0	0.1841701	0.6927942

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	X1	0.12204609	1.00000000	0.87795391
2	X2	1.000e-008	0.99092777	0.99999999
3	X3	0.19511797	0.99539281	0.80397892
4	X4	0.16076341	1.00000000	0.83923659
5	X5	0.08958223	0.99755756	0.91019844
6	X6	0.36236178	1.00000000	0.63763822
7	X7	0.41067164	1.00000000	0.58932836
8	X8	0.48782932	1.00352123	0.51388241

Total Determination of All Equations = 1.000  
Total Determination of Manifest Variables = 1.000

## 2.6 Data by McArdle (2011)

The following data is derived from raw data with  $N = 17351$  observations and  $n = 7$  variables:

sethi = [ 1. ,

```

.773 1. ,
.371 .359 1. ,
.189 .170 .227 1. ,
.283 .280 .255 .201 1. ,
.362 .345 .381 .221 .308 1. ,
.385 .352 .393 .185 .202 .403 1. ,
55.7 43.9 70.5 95.2 94.2 91.3 55.4 ,
18.5 22.3 34.1 21.1 14.3 16.7 21.2 ];
sethi = (tri2sym)sethi;
name = [" ID DR S7 BC NA DA VO "];
sethi = cname(sethi,name);

```

### 2.6.1 ML Analysis of the Covariance Matrix: nf=2

The following shows the input data when the covariance matrix is analyzed:

Name	Mean	Std Dev
ID	55.70000000	18.50000000
DR	43.90000000	22.30000000
S7	70.50000000	34.10000000
BC	95.20000000	21.10000000
NA	94.20000000	14.30000000
DA	91.30000000	16.70000000
VO	55.40000000	21.20000000

Variance Divisor = 17350

#### Covariance Matrix

	ID	DR	S7	BC	NA
ID	342.25000				
DR	318.90115	497.29000			
S7	234.04535	272.99437	1162.8100		
BC	73.776150	79.990100	163.32877	445.21000	
NA	74.867650	89.289200	124.34565	60.647730	204.49000
DA	111.83990	128.48145	216.96807	77.873770	73.553480
VO	150.99700	166.41152	284.10756	82.754200	61.238320

#### Covariance Matrix

	DA	VO
DA	278.89000	
VO	142.67812	449.44000

Determinant = 3.48541071e+017 (Ln = 4.0393e+001)

Due to the many observations  $N = 17351$  (which is a factor in the formula for the  $\chi^2$  value) we would have to start at an EFA model with  $p = 2.1e - 62$ . Our test computations showed that simple structures are relatively robust against reducing large numbers of observations for obtaining some "decent"  $\chi^2$  and therefore  $p$  value. Therefore, here, only for the optimization of the CFA zero pattern we divide the number of observations by 10. The starting  $\chi^2 = 31.06$  and  $p = 1.e - 4$  values then will be more suitable for the optimization.

```
*****  
Starting Model Improvement (ChiSq=310.763 p=2.10424e-062 DF=8)  
Maximizing the p Value for 1735 Observations  
*****
```

The 20 best solutions found:

```
*****  
Table of 20 Best Pattern Solutions  
*****
```

N	Npar	DF	Chisquared	P
1	17	11	32.2834799	6.868e-004
2	17	11	32.2834799	6.868e-004
3	17	11	32.2834799	6.868e-004
4	18	10	31.4142977	5.011e-004
5	18	10	31.4143019	5.011e-004
6	18	10	31.4143603	5.010e-004
7	18	10	31.4143605	5.010e-004
8	18	10	31.5589186	4.741e-004
9	16	12	35.4800636	3.926e-004
10	16	12	35.4800636	3.926e-004
11	16	12	35.4800639	3.926e-004
12	17	11	33.7839373	3.923e-004
13	18	10	32.2450398	3.645e-004
14	18	10	32.2450399	3.645e-004
15	18	10	32.2450604	3.645e-004
16	18	10	32.9358925	2.792e-004
17	19	9	31.1487163	2.791e-004
18	19	9	31.1487259	2.791e-004
19	19	9	31.1488122	2.791e-004
20	19	9	31.1488268	2.791e-004

N	RMSEA	ECVI	SRMR	PGFI	SBC
1	0.03340415	0.03831665	0.01830940	0.52103212	-49.7629098
2	0.03340415	0.03831665	0.01830940	0.52103212	-49.7629098
3	0.03340415	0.03831665	0.01830940	0.52103212	-49.7629098
4	0.03514207	0.03897414	0.01813111	0.47373431	-43.1733292
5	0.03514208	0.03897414	0.01813080	0.47373431	-43.1733250
6	0.03514212	0.03897418	0.01812838	0.47373448	-43.1732666
7	0.03514212	0.03897418	0.01812839	0.47373448	-43.1732665
8	0.03526054	0.03905754	0.01803283	0.47372611	-43.0287083
9	0.03359189	0.03900137	0.01998525	0.56812185	-54.0250887
10	0.03359189	0.03900137	0.01998525	0.56812185	-54.0250887
11	0.03359189	0.03900137	0.01998541	0.56812185	-54.0250884
12	0.03456157	0.03918196	0.01931998	0.52089873	-48.2624523
13	0.03581723	0.03945323	0.01829648	0.47366644	-42.3425871
14	0.03581723	0.03945323	0.01829648	0.47366644	-42.3425870
15	0.03581725	0.03945324	0.01829611	0.47366646	-42.3425665
16	0.03636916	0.03985164	0.01891900	0.47362444	-41.6517345
17	0.03767285	0.03997973	0.01800462	0.42638279	-35.9801479
18	0.03767286	0.03997973	0.01800395	0.42638273	-35.9801383
19	0.03767293	0.03997978	0.01800199	0.42638288	-35.9800521
20	0.03767294	0.03997979	0.01800233	0.42638274	-35.9800374

The following is the best solution found which has two still questionable small loadings for BC and DA of the first factor:

Loadings and Uni. Vars. with ASE's and Wald CIs

	FAC_1		FAC_2	
ID	16.701895	0.1315636	0	0
	[ 16.44404,	16.95976] [	0,	0]
DR	19.094064	0.1596966	0	0
	[ 18.78106,	19.40706] [	0,	0]
S7	0	0	20.625411	0.2761890
	[ 0,	0] [	20.08409,	21.16673]
BC	-1.6217927	0.2930368	8.5249130	0.3107876
	[-2.196134,	-1.047451] [	7.915781,	9.134045]
NA	0	0	6.2470714	0.1191234
	[ 0,	0] [	6.013594,	6.480549]

DA	-1.8444896	0.2650423	12.240044	0.2784739
	[-2.363963, -1.325016]		[ 11.69424, 12.78584]	
VO	0	0	12.707867	0.1717861
	[ 0,		0] [ 12.37117, 13.04456]	

		U_Var
ID	63.296695	2.5951222
	[ 58.21035, 68.38304]	
DR	132.70672	3.5700381
	[ 125.7096, 139.7039]	
S7	737.40242	9.9649026
	[ 717.8716, 756.9333]	
BC	389.23422	4.5227308
	[ 380.3698, 398.0986]	
NA	165.46410	1.9253255
	[ 161.6905, 169.2377]	
DA	157.23185	2.8118622
	[ 151.7207, 162.7430]	
VO	287.95011	3.8592242
	[ 280.3862, 295.5140]	

Factor Correlations Phi

	FAC_1	FAC_2
FAC_1	1.0000000	0.6990117
	[ 1.000000, 1.000000]	[ 0.684120, 0.713903]
FAC_2	0.6990117	1.0000000
	[ 0.684120, 0.713903]	[ 1.000000, 1.000000]

(1) Standalone Fit Measures: -----

Fit criterion . . . . .	0.0186
Normal Th. Chi-square (df = 11) . . . . .	323.0210 prob= 0.0000
Normal Theory Reweighted LS Chi-square . . . . .	323.6993
Probability of Close Fit . . . . .	1.0000

Z-Test of Wilson & Hilferty (1931) . . . . . 14.8126  
(2) Incremental Fit Measures: -----  
Null Model Chi-square (df = 21) . . . . . 32421.2870  
RMSEA Estimate . . . . . 0.0404 90%C.I.[ 0.0367, 0.0443]  
ECVI Estimate . . . . . 0.0206 90%C.I.[ 0.0174, 0.0242]  
McDonald's (1989) Centrality. . . . . 0.9910  
Tucker-Lewis Coefficient TLI. . . . . 0.9816  
Bentler & Bonett's (1980) NFI . . . . . 0.9900  
Bentler's Comparative Fit Index CFI . . . . . 0.9904  
Parsimonious NFI (James, Mulaik, & Brett,1982). . . . . 0.5186  
Bollen (1986) Normed Index Rho1 . . . . . 0.9810  
Bollen (1988) Non-normed Index Delta2 . . . . . 0.9904  
(3) Information Criteria: -----  
Akaike's Information Criterion. . . . . 301.0210  
Bozdogan's (1987) CAIC. . . . . 204.6455  
Schwarz's Bayesian Criterion. . . . . 215.6455  
(4) Other Fit Measures: -----  
Goodness of Fit Index (GFI) . . . . . 0.9947  
Parsimonious GFI (Mulaik, 1989) . . . . . 0.5210  
GFI Adjusted for Degrees of Freedom (AGFI). . . . . 0.9865  
Root Mean Square Residual (RMR) . . . . . 7.4215  
Hoelter's (1983) Critical N . . . . . 1058

Standardized Factor Loadings

	FAC_1	FAC_2
ID	0.9028051	0
DR	0.8562361	0
S7	0	0.6048508
BC	-0.0768622	0.4040243
NA	0	0.4368581
DA	-0.1104485	0.7329367
VO	0	0.5994277

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	ID	63.2966950	342.250000	0.81505714
2	DR	132.706722	497.290000	0.73314018
3	S7	737.402421	1162.81000	0.36584445
4	BC	389.234217	445.210000	0.12572894
5	NA	165.464099	204.490000	0.19084504
6	DA	157.231855	278.890000	0.43622269
7	VO	287.950109	449.440001	0.35931357

We now run an CFA with the two small loadings set to zero. The model would be:

```

l = [" L11  0., L21  0.,  0. L32, 0. L42,
      0.  L52, 0.  L62,  0. L72 "];
p = [" 1.  P21 , P21 1. "];
u = [" U1:U7 "];
list patt;
patt[1] = l; patt[2] = p; patt[3] = u;

```

First (for comparison of the  $p$  value) we use  $N = 1735$ :

Loadings and Uni. Vars. with ASE's and Wald CIs

	FAC_1		FAC_2	
ID	16.709322	0.4170521	0	0
	[ 15.89191,	17.52673]	[ 0,	0]
DR	19.085224	0.5061653	0	0
	[ 18.09316,	20.07729]	[ 0,	0]
S7	0	0	21.126660	0.8701078
	[ 0,	0]	[ 19.42128,	22.83204]
BC	0	0	7.2358149	0.5710882
	[ 0,	0]	[ 6.116503,	8.355127]
NA	0	0	6.2487945	0.3802425
	[ 0,	0]	[ 5.503533,	6.994056]
DA	0	0	10.606960	0.4247605
	[ 0,	0]	[ 9.774445,	11.43948]
VO	0	0	12.946389	0.5419721
	[ 0,	0]	[ 11.88414,	14.00864]

	U_Var	
ID	63.048560	8.2652702
	[ 46.84893,	79.24819]
DR	133.04423	11.352653
	[ 110.7934,	155.2950]

```

S7   716.47425  31.245459
     [ 655.2343, 777.7142]

BC   392.85298  13.989364
     [ 365.4343, 420.2716]

NA   165.44257  6.1230591
     [ 153.4416, 177.4435]

DA   166.38240  7.4393188
     [ 151.8016, 180.9632]

VO   281.83100  12.131495
     [ 258.0537, 305.6083]

```

Factor Correlations Phi  
-----

```

                FAC_1                FAC_2
FAC_1  1.0000000                0  0.6651695  0.0211041
      [ 1.000000, 1.000000] [ 0.623806, 0.706533]

FAC_2  0.6651695  0.0211041  1.0000000                0
      [ 0.623806, 0.706533] [ 1.000000, 1.000000]

```

Note, that this solution was not included in the best twenty listed above since its  $p$  is smaller:

```

(1) Standalone Fit Measures: -----
Fit criterion . . . . . 0.0230
Normal Th. Chi-square (df = 13) 39.9100 Prob>chi**2 = 0.0001
Normal Theory Reweighted LS Chi-square . . . . . 39.2820
Probability of Close Fit . . . . . 0.9804
Z-Test of Wilson & Hilferty (1931). . . . . 3.5984
(2) Incremental Fit Measures: -----
Null Model Chi-square (df = 21) . . . . . 3240.2600
RMSEA Estimate . . . . . 0.0346 90%C.I.[ 0.0227, 0.0470]
ECVI Estimate . . . . . 0.0404 90%C.I.[ 0.0315, 0.0537]
McDonald's (1989) Centrality. . . . . 0.9923
Tucker-Lewis Coefficient TLI. . . . . 0.9865
Bentler & Bonett's (1980) NFI . . . . . 0.9877
Bentler's Comparative Fit Index CFI . . . . . 0.9916
Parsimonious NFI (James, Mulaik, & Brett,1982). . . 0.6114

```

Bollen (1986) Normed Index Rho1 . . . . .	0.9801
Bollen (1988) Non-normed Index Delta2 . . . . .	0.9917
(3) Information Criteria: -----	
Akaike's Information Criterion. . . . .	13.9100
Bozdogan's (1987) CAIC. . . . .	-70.0539
Schwarz's Bayesian Criterion. . . . .	-57.0539
(4) Other Fit Measures: -----	
Goodness of Fit Index (GFI) . . . . .	0.9936
Parsimonious GFI (Mulaik, 1989) . . . . .	0.6151
GFI Adjusted for Degrees of Freedom (AGFI). . . . .	0.9861
Root Mean Square Residual (RMR) . . . . .	7.6173
Hoelter's (1983) Critical N . . . . .	973

Standardized Factor Loadings

	FAC_1	FAC_2
ID	0.9032066	0
DR	0.8558396	0
S7	0	0.6195501
BC	0	0.3429296
NA	0	0.4369786
DA	0	0.6351473
VO	0	0.6106787

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	ID	63.0485601	342.250000	0.81578215
2	DR	133.044230	497.290000	0.73246148
3	S7	716.474250	1162.81000	0.38384237
4	BC	392.852982	445.210000	0.11760072
5	NA	165.442567	204.490000	0.19095033
6	DA	166.382400	278.890000	0.40341210
7	VO	281.830999	449.440000	0.37292854

For  $N = 1735$  the  $p$  value is zero and there is not much difference of this sparse model with the optimal model found above:

Loadings and Uni. Vars. with ASE's and Wald CIs

		FAC_1		FAC_2
ID	16.709322	0.1318454	0	0
	[ 16.45091, 16.96773]	[	0,	0]

DR	19.085224	0.1600174	0	0
	[ 18.77160,	19.39885]	[ 0,	0]
S7	0	0	21.126660	0.2750729
	[ 0,	0]	[ 20.58753,	21.66579]
BC	0	0	7.2358149	0.1805419
	[ 0,	0]	[ 6.881959,	7.589671]
NA	0	0	6.2487945	0.1202086
	[ 0,	0]	[ 6.013190,	6.484399]
DA	0	0	10.606960	0.1342824
	[ 0,	0]	[ 10.34377,	10.87015]
VO	0	0	12.946389	0.1713372
	[ 0,	0]	[ 12.61057,	13.28220]

		U_Var
ID	63.048560	2.6129546
	[ 57.92726,	68.16986]
DR	133.04423	3.5889894
	[ 126.0099,	140.0785]
S7	716.47425	9.8778338
	[ 697.1141,	735.8344]
BC	392.85298	4.4225502
	[ 384.1849,	401.5210]
NA	165.44257	1.9357232
	[ 161.6486,	169.2365]
DA	166.38240	2.3518411
	[ 161.7729,	170.9919]
VO	281.83100	3.8352098
	[ 274.3141,	289.3479]

Factor Correlations Phi  
-----

	FAC_1		FAC_2	
FAC_1	1.0000000	0	0.6651695	0.0066718
	[ 1.000000, 1.000000]		[ 0.652093, 0.678246]	
FAC_2	0.6651695	0.0066718	1.0000000	0
	[ 0.652093, 0.678246]		[ 1.000000, 1.000000]	

(1) Standalone Fit Measures: -----  
Fit criterion . . . . . 0.0230  
Normal Th. Chi-square (df = 13) . . . . . 399.3305 prob= 0.0000  
Normal Theory Reweighted LS Chi-square . . . . . 393.0466  
Probability of Close Fit . . . . . 1.0000  
Z-Test of Wilson & Hilferty (1931). . . . . 16.4360

(2) Incremental Fit Measures: -----  
Null Model Chi-square (df = 21) . . . . . 32421.2870  
RMSEA Estimate . . . . . 0.0414 90%C.I.[ 0.0379, 0.0449]  
ECVI Estimate . . . . . 0.0247 90%C.I.[ 0.0212, 0.0287]  
McDonald's (1989) Centrality. . . . . 0.9889  
Tucker-Lewis Coefficient TLI. . . . . 0.9807  
Bentler & Bonett's (1980) NFI . . . . . 0.9877  
Bentler's Comparative Fit Index CFI . . . . . 0.9881  
Parsimonious NFI (James, Mulaik, & Brett,1982). . . . . 0.6114  
Bollen (1986) Normed Index Rho1 . . . . . 0.9801  
Bollen (1988) Non-normed Index Delta2 . . . . . 0.9881

(3) Information Criteria: -----  
Akaike's Information Criterion. . . . . 373.3305  
Bozdogan's (1987) CAIC. . . . . 259.4322  
Schwarz's Bayesian Criterion. . . . . 272.4322

(4) Other Fit Measures: -----  
Goodness of Fit Index (GFI) . . . . . 0.9936  
Parsimonious GFI (Mulaik, 1989) . . . . . 0.6151  
GFI Adjusted for Degrees of Freedom (AGFI). . . . . 0.9861  
Root Mean Square Residual (RMR) . . . . . 7.6173  
Hoelter's (1983) Critical N . . . . . 973

Standardized Factor Loadings

	FAC_1	FAC_2
ID	0.9032066	0
DR	0.8558396	0
S7	0	0.6195501
BC	0	0.3429296
NA	0	0.4369786
DA	0	0.6351473

V0 0 0.6106787

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	ID	63.0485601	342.250000	0.81578215
2	DR	133.044230	497.290000	0.73246148
3	S7	716.474250	1162.81000	0.38384237
4	BC	392.852982	445.210000	0.11760072
5	NA	165.442567	204.490000	0.19095033
6	DA	166.382400	278.890000	0.40341210
7	V0	281.830999	449.440000	0.37292854

2.6.2 ML Analysis of the Correlation Matrix: nf=2

We start with the EFA result, which again has a very small  $p$  value, and therefore the number of observations is reduced by dividing with 10, but only for the purpose of a meaningful optimization:

```
*****
Starting Model Improvement (ChiSq=310.762 p=2.10495e-062 DF=8)
Maximizing the p Value for 1735 Observations
*****
```

That changes the to some better values of  $\chi^2 = 31.06$  and  $p = 1.e - 4$ , the same as for analysing the covariance matrix. The following results are very similar to those of the analysis of the covariance matrix:

```
*****
Table of 20 Best Pattern Solutions
*****
```

N	Npar	DF	Chisquared	P
1	18	10	31.3322096	5.170e-004
2	17	11	33.8494784	3.828e-004
3	19	9	31.1487068	2.791e-004
4	19	9	31.1487069	2.791e-004
5	19	9	31.1487070	2.791e-004
6	19	9	31.2947461	2.635e-004
7	19	9	31.2947461	2.635e-004
8	18	10	33.8306223	1.973e-004
9	20	8	31.0583333	1.372e-004
10	20	8	31.0583335	1.372e-004

11	20	8	31.0583338	1.372e-004
12	20	8	31.0583338	1.372e-004
13	20	8	31.0583341	1.372e-004
14	20	8	31.0583343	1.372e-004
15	20	8	31.0583344	1.372e-004
16	20	8	31.0583346	1.372e-004
17	20	8	31.0583346	1.372e-004
18	18	10	96.6995912	2.489e-016
19	17	11	183.284372	2.150e-033
20	20	8	310.762474	2.105e-062

N	RMSEA	ECVI	SRMR	PGFI	SBC
1	0.03507465	0.03892680	0.01811613	0.47374445	-43.2554174
2	0.03461125	0.03921976	0.01908267	0.52090991	-48.1969113
3	0.03767284	0.03997972	0.01800534	0.42638272	-35.9801574
4	0.03767284	0.03997972	0.01800547	0.42638271	-35.9801573
5	0.03767284	0.03997972	0.01800548	0.42638271	-35.9801572
6	0.03779684	0.04006394	0.01812341	0.42637095	-35.8341182
7	0.03779684	0.04006394	0.01812344	0.42637095	-35.8341181
8	0.03707175	0.04036764	0.01907936	0.47355761	-40.7570046
9	0.04077035	0.04108635	0.01801949	0.37901031	-28.6117682
10	0.04077035	0.04108635	0.01801944	0.37901031	-28.6117680
11	0.04077035	0.04108635	0.01801934	0.37901031	-28.6117678
12	0.04077035	0.04108635	0.01801956	0.37901031	-28.6117677
13	0.04077035	0.04108635	0.01801969	0.37901030	-28.6117675
14	0.04077035	0.04108635	0.01801971	0.37901030	-28.6117672
15	0.04077035	0.04108635	0.01801930	0.37901031	-28.6117672
16	0.04077035	0.04108635	0.01801970	0.37901030	-28.6117670
17	0.04077035	0.04108635	0.01801933	0.37901031	-28.6117669
18	0.07071051	0.07662425	0.06008571	0.46882931	22.1119642
19	0.09503904	0.12539906	0.08023390	0.50895354	101.237983
20	0	0	0	0	0

The solution has three small loadings:

Loadings and Uni. Vars. with ASE's and Wald CIs

	FAC_1		FAC_2	
ID	0.9028836	0.0071108	0	0
	[ 0.888947, 0.916820] [		0,	0]
DR	0.8561902	0.0071607	0	0

			[ 0.842155, 0.870225]	[ 0, 0]
S7	0.0717923	0.0131783	0.5653448	0.0136323
			[ 0.045963, 0.097621]	[ 0.538626, 0.592064]
BC	0	0	0.3503089	0.0087428
	[ 0,	0]	[ 0.333173,	0.367444]
NA	0.0955753	0.0120580	0.3658937	0.0128627
			[ 0.071942, 0.119209]	[ 0.340683, 0.391104]
DA	0	0	0.6594657	0.0088863
	[ 0,	0]	[ 0.642049,	0.676883]
VO	0.0926820	0.0128616	0.5399763	0.0133247
			[ 0.067474, 0.117890]	[ 0.513860, 0.566092]

		U_Var
ID	0.1848012	0.0075812
	[ 0.169942,	0.199660]
DR	0.2669384	0.0071769
	[ 0.252872,	0.281005]
S7	0.6260197	0.0091663
	[ 0.608054,	0.643985]
BC	0.8772837	0.0099841
	[ 0.857715,	0.896852]
NA	0.8145862	0.0095345
	[ 0.795899,	0.833273]
DA	0.5651050	0.0097836
	[ 0.545930,	0.584280]
VO	0.6391557	0.0089937
	[ 0.621528,	0.656783]

Factor Correlations Phi

-----

FAC\_1

FAC\_2

```

FAC_1  1.000000      0  0.6062407  0.0100635
        [ 1.000000, 1.000000] [ 0.586517, 0.625965]

FAC_2  0.6062407  0.0100635  1.000000      0
        [ 0.586517, 0.625965] [ 1.000000, 1.000000]

```

```

(1) Standalone Fit Measures: -----
Fit criterion . . . . . 0.0181
Normal Th. Chi-square (df = 10)      313.5028  prob= 0.0000
Normal Theory Reweighted LS Chi-square . . . . . 313.5333
Probability of Close Fit . . . . . 0.9996
Z-Test of Wilson & Hilferty (1931). . . . . 14.5930
(2) Incremental Fit Measures: -----
Null Model Chi-square (df = 21) . . . . . 32421.2870
RMSEA Estimate . . . . . 0.0418  90%C.I.[ 0.0379, 0.0459]
ECVI Estimate . . . . . 0.0201  90%C.I.[ 0.0170, 0.0237]
McDonald's (1989) Centrality. . . . . 0.9913
Tucker-Lewis Coefficient TLI. . . . . 0.9803
Bentler & Bonett's (1980) NFI . . . . . 0.9903
Bentler's Comparative Fit Index CFI . . . . . 0.9906
Parsimonious NFI (James, Mulaik, & Brett,1982). . . 0.4716
Bollen (1986) Normed Index Rho1 . . . . . 0.9797
Bollen (1988) Non-normed Index Delta2 . . . . . 0.9906
(3) Information Criteria: -----
Akaike's Information Criterion. . . . . 293.5028
Bozdogan's (1987) CAIC. . . . . 205.8887
Schwarz's Bayesian Criterion. . . . . 215.8887
(4) Other Fit Measures: -----
Goodness of Fit Index (GFI) . . . . . 0.9949
Parsimonious GFI (Mulaik, 1989) . . . . . 0.4737
GFI Adjusted for Degrees of Freedom (AGFI). . . . . 0.9856
Root Mean Square Residual (RMR) . . . . . 0.0181
Hoelter's (1983) Critical N . . . . . 1015

```

Standardized Factor Loadings

	FAC_1	FAC_2
ID	0.9028836	0
DR	0.8561902	0
S7	0.0717923	0.5653448
BC	0	0.3503089
NA	0.0955753	0.3658937
DA	0	0.6594657
VO	0.0926820	0.5399763

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	ID	0.18480123	1.00000000	0.81519877
2	DR	0.26693836	1.00000000	0.73306164
3	S7	0.62601966	1.00000000	0.37398034
4	BC	0.87728368	1.00000000	0.12271633
5	NA	0.81458619	1.00000000	0.18541381
6	DA	0.56510499	1.00000000	0.43489501
7	VO	0.63915573	1.00000000	0.36084427

Now similar to the analysis of the covariance matrix, we run an CFA for  $N = 1735$  with a specified pattern where the small zero loadings are set to zero to compare the  $p$  value of that solution with that obtained by optimization:

Loadings and Uni. Vars. with ASE's and Wald CIs

-----

	FAC_1		FAC_2	
ID	0.9032066	0.0225434	0	0
	[ 0.859022,	0.947391]	[ 0,	0]
DR	0.8558396	0.0226980	0	0
	[ 0.811352,	0.900327]	[ 0,	0]
S7	0	0	0.6195501	0.0255164
	[ 0,	0]	[ 0.569539,	0.669561]
BC	0	0	0.3429296	0.0270658
	[ 0,	0]	[ 0.289882,	0.395978]
NA	0	0	0.4369786	0.0265904
	[ 0,	0]	[ 0.384862,	0.489095]
DA	0	0	0.6351473	0.0254348
	[ 0,	0]	[ 0.585296,	0.684999]
VO	0	0	0.6106787	0.0255647
	[ 0,	0]	[ 0.560573,	0.660785]

	U_Var	
ID	0.1842179	0.0241498
	[ 0.136885,	0.231551]

DR	0.2675385	0.0228290
	[ 0.222794,	0.312283]
S7	0.6161576	0.0268706
	[ 0.563492,	0.668823]
BC	0.8823993	0.0314219
	[ 0.820813,	0.943985]
NA	0.8090497	0.0299431
	[ 0.750362,	0.867737]
DA	0.5965879	0.0266747
	[ 0.544306,	0.648869]
VO	0.6270715	0.0269925
	[ 0.574167,	0.679976]

Factor Correlations Phi

-----

	FAC_1	FAC_2
FAC_1	1.0000000	0 0.6651695 0.0211041
	[ 1.000000,	1.000000] [ 0.623806, 0.706533]
FAC_2	0.6651695 0.0211041	1.0000000 0
	[ 0.623806,	0.706533] [ 1.000000, 1.000000]

(1) Standalone Fit Measures: -----

Fit criterion . . . . .	0.0230
Normal Th. Chi-square (df = 13)	39.9100 Prob>chi**2 = 0.0001
Normal Theory Reweighted LS Chi-square . . . . .	39.2820
Probability of Close Fit . . . . .	0.9804
Z-Test of Wilson & Hilferty (1931). . . . .	3.5984

(2) Incremental Fit Measures: -----

Null Model Chi-square (df = 21) . . . . .	3240.2600
RMSEA Estimate . . . . .	0.0346 90%C.I.[ 0.0227, 0.0470]
ECVI Estimate . . . . .	0.0404 90%C.I.[ 0.0315, 0.0537]
McDonald's (1989) Centrality. . . . .	0.9923
Tucker-Lewis Coefficient TLI. . . . .	0.9865
Bentler & Bonett's (1980) NFI . . . . .	0.9877

Bentler's Comparative Fit Index CFI . . . . .	0.9916
Parsimonious NFI (James, Mulaik, & Brett,1982). .	0.6114
Bollen (1986) Normed Index Rho1 . . . . .	0.9801
Bollen (1988) Non-normed Index Delta2 . . . . .	0.9917
(3) Information Criteria: -----	
Akaike's Information Criterion. . . . .	13.9100
Bozdogan's (1987) CAIC. . . . .	-70.0539
Schwarz's Bayesian Criterion. . . . .	-57.0539
(4) Other Fit Measures: -----	
Goodness of Fit Index (GFI) . . . . .	0.9936
Parsimonious GFI (Mulaik, 1989) . . . . .	0.6151
GFI Adjusted for Degrees of Freedom (AGFI). . . .	0.9861
Root Mean Square Residual (RMR) . . . . .	0.0214
Hoelter's (1983) Critical N . . . . .	973

Standardized Factor Loadings

	FAC_1	FAC_2
ID	0.9032066	0
DR	0.8558396	0
S7	0	0.6195501
BC	0	0.3429296
NA	0	0.4369786
DA	0	0.6351473
VO	0	0.6106787

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	ID	0.18421785	1.00000000	0.81578215
2	DR	0.26753852	1.00000000	0.73246148
3	S7	0.61615763	1.00000000	0.38384237
4	BC	0.88239928	1.00000000	0.11760072
5	NA	0.80904967	1.00000000	0.19095033
6	DA	0.59658790	1.00000000	0.40341210
7	VO	0.62707146	1.00000000	0.37292854

If we specify all  $N = 17351$  observations the  $p$  value becomes practically the same for both models, the optimized and the very sparse:

Loadings and Uni. Vars. with ASE's and Wald CIs

-----

	FAC_1	FAC_2
--	-------	-------

ID	0.9032066	0.0071268	0	0
	[ 0.889238,	0.917175]	[ 0,	0]
DR	0.8558396	0.0071757	0	0
	[ 0.841776,	0.869904]	[ 0,	0]
S7	0	0	0.6195501	0.0080667
	[ 0,	0]	[ 0.603740,	0.635360]
BC	0	0	0.3429296	0.0085565
	[ 0,	0]	[ 0.326159,	0.359700]
NA	0	0	0.4369786	0.0084062
	[ 0,	0]	[ 0.420503,	0.453454]
DA	0	0	0.6351473	0.0080409
	[ 0,	0]	[ 0.619388,	0.650907]
VO	0	0	0.6106787	0.0080819
	[ 0,	0]	[ 0.594838,	0.626519]

		U_Var
ID	0.1842179	0.0076346
	[ 0.169254,	0.199181]
DR	0.2675385	0.0072171
	[ 0.253393,	0.281684]
S7	0.6161576	0.0084948
	[ 0.599508,	0.632807]
BC	0.8823993	0.0099336
	[ 0.862930,	0.901869]
NA	0.8090497	0.0094661
	[ 0.790496,	0.827603]
DA	0.5965879	0.0084329
	[ 0.580060,	0.613116]
VO	0.6270715	0.0085333
	[ 0.610346,	0.643796]

Factor Correlations Phi

```

-----
                FAC_1                FAC_2
FAC_1  1.0000000          0  0.6651695  0.0066718
      [ 1.000000, 1.000000] [ 0.652093, 0.678246]

FAC_2  0.6651695  0.0066718  1.0000000          0
      [ 0.652093, 0.678246] [ 1.000000, 1.000000]

```

```

(1) Standalone Fit Measures: -----
Fit criterion . . . . . 0.0230
Normal Th. Chi-square (df = 13)      399.3305  prob= 0.0000
Normal Theory Reweighted LS Chi-square . . . . . 393.0466
Probability of Close Fit . . . . . 1.0000
Z-Test of Wilson & Hilferty (1931). . . . . 16.4360
(2) Incremental Fit Measures: -----
Null Model Chi-square (df = 21) . . . . . 32421.2870
RMSEA Estimate . . . . . 0.0414  90%C.I.[ 0.0379, 0.0449]
ECVI Estimate . . . . . 0.0247  90%C.I.[ 0.0212, 0.0287]
McDonald's (1989) Centrality. . . . . 0.9889
Tucker-Lewis Coefficient TLI. . . . . 0.9807
Bentler & Bonett's (1980) NFI . . . . . 0.9877
Bentler's Comparative Fit Index CFI . . . . . 0.9881
Parsimonious NFI (James, Mulaik, & Brett,1982). . . 0.6114
Bollen (1986) Normed Index Rho1 . . . . . 0.9801
Bollen (1988) Non-normed Index Delta2 . . . . . 0.9881
(3) Information Criteria: -----
Akaike's Information Criterion. . . . . 373.3305
Bozdogan's (1987) CAIC. . . . . 259.4322
Schwarz's Bayesian Criterion. . . . . 272.4322
(4) Other Fit Measures: -----
Goodness of Fit Index (GFI) . . . . . 0.9936
Parsimonious GFI (Mulaik, 1989) . . . . . 0.6151
GFI Adjusted for Degrees of Freedom (AGFI). . . . . 0.9861
Root Mean Square Residual (RMR) . . . . . 0.0214
Hoelter's (1983) Critical N . . . . . 973

```

Standardized Factor Loadings

	FAC_1	FAC_2
ID	0.9032066	0
DR	0.8558396	0
S7	0	0.6195501
BC	0	0.3429296

NA	0	0.4369786
DA	0	0.6351473
VO	0	0.6106787

#### Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	ID	0.18421785	1.00000000	0.81578215
2	DR	0.26753852	1.00000000	0.73246148
3	S7	0.61615763	1.00000000	0.38384237
4	BC	0.88239928	1.00000000	0.11760072
5	NA	0.80904967	1.00000000	0.19095033
6	DA	0.59658790	1.00000000	0.40341210
7	VO	0.62707146	1.00000000	0.37292854

### 2.6.3 EFA of ML with Oblique GEOMIN Rotation: nf=2

First we show the results of the analysis of the covariance matrix. Jennrichs iterative algorithm converges after 19 iterations:

#### Gradient Projection Algorithm (Bernaards & Jennrich)

##### Oblique Rotation: Geomin (Yates, 1984)

Iter	OptCrit	Termin	Alfa
1	425.046675	2.63051184	1.00000000
2	657.110929	3.71264402	0.00195313
3	240.936733	3.27096778	3.052e-005
4	68.0141593	2.94238086	6.104e-005
5	58.8152967	2.80519448	1.526e-005
6	55.3097263	2.28521252	1.526e-005
7	55.1668107	1.02729717	7.629e-006
8	55.1665080	0.63125714	3.815e-006
9	55.1664587	0.25091438	3.815e-006
10	55.1664500	-0.09719278	3.815e-006
11	55.1664482	-0.39158929	3.815e-006
12	55.1664475	-0.90761390	7.629e-006
13	55.1664474	-1.38209697	7.629e-006
14	55.1664474	-1.83779342	7.629e-006
15	55.1664474	-2.28654657	7.629e-006
16	55.1664474	-2.73293150	7.629e-006
17	55.1664474	-3.17853158	7.629e-006
18	55.1664474	-3.62387402	7.629e-006

19 55.1664474 -4.06913215 7.629e-006

Rotated Factor Loadings with Standard Errors

-----

		FAC_1		FAC_2
ID	15.530995	0.9537941	1.0518155	0.8702244
	[ 13.66159,	17.40040]	[-0.653793,	2.757424]
DR	19.697458	0.4837415	-0.0613617	0.0182379
	[ 18.74934,	20.64557]	[-0.097107,	-0.025616]
S7	0.0130254	0.0032414	20.870963	0.2914997
	[ 6.7e-003,	0.019378]	[ 20.29963,	21.44229]
BC	-1.2889542	0.3005380	8.2662530	0.3068495
	[-1.877998,	-0.699910]	[ 7.664839,	8.867667]
NA	0.7323044	0.1982060	5.6447326	0.2014838
	[ 0.343828,	1.120781]	[ 5.249832,	6.039634]
DA	-1.2280460	0.2957971	11.722542	0.2843065
	[-1.807798,	-0.648294]	[ 11.16531,	12.27977]
VO	0.3902479	0.3277066	12.501844	0.3160164
	[-0.252045,	1.032541]	[ 11.88246,	13.12122]

		U_Var
ID	78.338930	11.769961
	[ 55.27023,	101.4076]
DR	110.89552	18.835943
	[ 73.97775,	147.8133]
S7	726.85354	10.646377
	[ 705.9870,	747.7201]
BC	389.30167	4.5229628
	[ 380.4368,	398.1665]
NA	166.62672	1.9481911
	[ 162.8083,	170.4451]

DA 158.99286 2.7792911  
 [ 153.5456, 164.4402]

VO 286.54292 4.0563810  
 [ 278.5926, 294.4933]

Factor Correlations with Standard Errors

```

-----
                                FAC_1                FAC_2
FAC_1  1.0000000                0  0.6609174  0.0196315
      [ 1.000000, 1.000000] [ 0.622440, 0.699394]

FAC_2  0.6609174  0.0196315  1.0000000                0
      [ 0.622440, 0.699394] [ 1.000000, 1.000000]
  
```

(1) Standalone Fit Measures: -----  
 Fit criterion . . . . . 0.0179  
 Normal Th. Chi-square (df = 8) 310.7632 Prob>chi\*\*2 = 0.0000  
 Bartlett Corrected Chi-square (df = 8) 310.6826 p= 0.0000  
 Normal Theory Reweighted LS Chi-square . . . . . 311.1505  
 Probability of Close Fit . . . . . 0.8835  
 Z-Test of Wilson & Hilferty (1931). . . . . 14.4870  
 (2) Incremental Fit Measures: -----  
 Null Model Chi-square (df = 21) . . . . . 32421.2870  
 RMSEA Estimate . . . . . 0.0467 90%C.I.[ 0.0423, 0.0512]  
 ECVI Estimate . . . . . 0.0202 90%C.I.[ 0.0171, 0.0238]  
 McDonald's (1989) Centrality. . . . . 0.9913  
 Tucker-Lewis Coefficient TLI. . . . . 0.9755  
 Bentler & Bonett's (1980) NFI . . . . . 0.9904  
 Bentler's Comparative Fit Index CFI . . . . . 0.9907  
 Parsimonious NFI (James, Mulaik, & Brett,1982). . . . . 0.3773  
 Bollen (1986) Normed Index Rho1 . . . . . 0.9748  
 Bollen (1988) Non-normed Index Delta2 . . . . . 0.9907  
 (3) Information Criteria: -----  
 Akaike's Information Criterion. . . . . 294.7632  
 Bozdogan's (1987) CAIC. . . . . 224.6719  
 Schwarz's Bayesian Criterion. . . . . 232.6719  
 (4) Other Fit Measures: -----  
 Goodness of Fit Index (GFI) . . . . . 0.9949  
 Parsimonious GFI (Mulaik, 1989) . . . . . 0.3790  
 GFI Adjusted for Degrees of Freedom (AGFI). . . . . 0.9822  
 Root Mean Square Residual (RMR) . . . . . 6.8481  
 Hoelter's (1983) Critical N . . . . . 867

Standardized Factor Loadings

	FAC_1	FAC_2
ID	0.8395130	0.0568549
DR	0.8832928	-0.0027516
S7	3.82e-004	0.6120517
BC	-0.0610879	0.3917655
NA	0.0512101	0.3947365
DA	-0.0735357	0.7019486
VO	0.0184079	0.5897094

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	ID	78.3389298	342.250212	0.77110626
2	DR	110.895518	497.291463	0.77700096
3	S7	726.853535	1162.81014	0.37491641
4	BC	389.301666	445.210095	0.12557763
5	NA	166.626719	204.490014	0.18515963
6	DA	158.992862	278.890055	0.42990846
7	VO	286.542923	449.440306	0.36244498

Total Determination of All Equations = 0.947  
 Total Determination of Manifest Variables = 0.946

Now we show the results of the analysis of the correlation matrix. The GEOMIN algorithm converges after 17 iterations:

Gradient Projection Algorithm (Bernaards & Jennrich)

Oblique Rotation: Geomin (Yates, 1984)

Iter	OptCrit	Termin	Alfa
1	0.96691234	-0.06275166	1.00000000
2	0.90040283	-0.14032784	0.12500000
3	0.79895666	-0.03464587	0.25000000
4	0.73227476	-0.06755507	0.12500000
5	0.66581678	-0.07297478	0.12500000
6	0.56335375	0.16482321	0.25000000
7	0.51285785	-0.09648090	0.03125000
8	0.48638071	-0.36551510	0.06250000
9	0.47762486	-0.65724936	0.06250000

10	0.47545658	-1.03236771	0.06250000
11	0.47508596	-1.46133437	0.06250000
12	0.47503540	-1.91434928	0.06250000
13	0.47502916	-2.37652824	0.06250000
14	0.47502842	-2.84196567	0.06250000
15	0.47502833	-3.30852866	0.06250000
16	0.47502832	-3.77547702	0.06250000
17	0.47502832	-4.24255665	0.06250000

Rotated Factor Loadings with Standard Errors

-----

	FAC_1		FAC_2	
ID	0.8527124	0.0385122	0.0398485	0.0291566
	[ 0.777230,	0.928195]	[-0.017297,	0.096994]
DR	0.8917996	0.0319867	-0.0171253	0.0172974
	[ 0.829107,	0.954492]	[-0.051027,	0.016777]
S7	0.0384313	0.0099825	0.5871692	0.0114282
	[ 0.018866,	0.057997]	[ 0.564770,	0.609568]
BC	-0.0374927	0.0112693	0.3769952	0.0124535
	[-0.059580,	-0.015405]	[ 0.352587,	0.401404]
NA	0.0762644	0.0117185	0.3778781	0.0125682
	[ 0.053296,	0.099232]	[ 0.353245,	0.402511]
DA	-0.0307357	0.0070806	0.6747248	0.0100797
	[-0.044613,	-0.016858]	[ 0.654969,	0.694481]
VO	0.0551056	0.0106668	0.5655979	0.0117597
	[ 0.034199,	0.076012]	[ 0.542549,	0.588646]

	U_Var	
ID	0.2281843	0.0053683
	[ 0.217663,	0.238706]
DR	0.2237762	0.0053683
	[ 0.213255,	0.234298]
S7	0.6251264	0.0053683
	[ 0.614605,	0.635648]

BC	0.8744014	0.0053683
	[ 0.863880, 0.884923]	
NA	0.8148298	0.0053683
	[ 0.804308, 0.825351]	
DA	0.5701122	0.0053683
	[ 0.559591, 0.580634]	
VO	0.6375202	0.0053683
	[ 0.626999, 0.648042]	

Factor Correlations with Standard Errors

```

-----
                FAC_1                FAC_2
FAC_1  1.0000000                0  0.6343478  0.0089093
      [ 1.000000, 1.000000] [ 0.616886, 0.651810]
FAC_2  0.6343478  0.0089093  1.0000000                0
      [ 0.616886, 0.651810] [ 1.000000, 1.000000]

```

Standardized Factor Loadings

	FAC_1	FAC_2
ID	0.8527124	0.0398485
DR	0.8917996	-0.0171253
S7	0.0384313	0.5871692
BC	-0.0374927	0.3769952
NA	0.0762644	0.3778781
DA	-0.0307357	0.6747248
VO	0.0551056	0.5655979

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	ID	0.22818428	1.00000000	0.77181572
2	DR	0.22377615	1.00000001	0.77622385
3	S7	0.62512636	1.00000000	0.37487364
4	BC	0.87440136	1.00000000	0.12559864
5	NA	0.81482981	1.00000000	0.18517019
6	DA	0.57011220	1.00000000	0.42988780
7	VO	0.63752022	1.00000000	0.36247978

Total Determination of All Equations = 0.947  
 Total Determination of Manifest Variables = 0.946

As I was told, there are some differences between these results and those reported by McArdle (2011).

### 2.6.4 ML Analysis of the Covariance Matrix: nf=3

Now, we optimize w.r.t. three factors. For ML estimates with the EFA model we obtain.

```
*****
Starting Model Improvement (ChiSq=26.9934 p=5.90626e-006 DF=3)
Maximizing the p Value for 173 Observations
*****
```

Due to the large number of observations (17251) the *p* value is close to zero:

```
(1) Standalone Fit Measures: -----
Fit criterion . . . . . 0.0016
Normal Th. Chi-square (df = 3). 26.9934 Prob>chi**2 = 0.0000
Bartlett Corrected Chi-square (df = 3) 26.9853 prob= 0.0000
Normal Theory Reweighted LS Chi-square . . . . . 26.9637
Probability of Close Fit . . . . . 1.0000
Z-Test of Wilson & Hilferty (1931). . . . . 4.2400
(2) Incremental Fit Measures: -----
Null Model Chi-square (df = 21) . . . . . 32421.2870
RMSEA Estimate . . . . . 0.0215 90%C.I.[ 0.0145, 0.0292]
ECVI Estimate . . . . . 0.0044 90%C.I.[ 0.0037, 0.0056]
McDonald's (1989) Centrality. . . . . 0.9993
Tucker-Lewis Coefficient TLI. . . . . 0.9948
Bentler & Bonett's (1980) NFI . . . . . 0.9992
Bentler's Comparative Fit Index CFI . . . . . 0.9993
Parsimonious NFI (James, Mulaik, & Brett,1982). . . 0.1427
Bollen (1986) Normed Index Rho1 . . . . . 0.9942
Bollen (1988) Non-normed Index Delta2 . . . . . 0.9993
(3) Information Criteria: -----
Akaike's Information Criterion. . . . . 20.9934
Bozdogan's (1987) CAIC. . . . . -5.2909
Schwarz's Bayesian Criterion. . . . . -2.2909
(4) Other Fit Measures: -----
Goodness of Fit Index (GFI) . . . . . 0.9996
Parsimonious GFI (Mulaik, 1989) . . . . . 0.1428
GFI Adjusted for Degrees of Freedom (AGFI). . . . . 0.9959
```

Root Mean Square Residual (RMR) . . . . . 2.8867  
 Hoelter's (1983) Critical N . . . . . 5024

Rotated Factor Loadings with Standard Errors  
 -----

	FAC_1		FAC_2	
ID	13.832286	0.6006860	6.2238630	0.2906096
	[ 12.65496,	15.00961]	[ 5.654279,	6.793447]
DR	20.773764	0.9603427	5.0615724	0.3990759
	[ 18.89153,	22.65600]	[ 4.279398,	5.843747]
S7	8.6338812	0.2898870	18.367544	0.4274393
	[ 8.065713,	9.202049]	[ 17.52978,	19.20531]
BC	2.1987415	0.1733463	6.6514697	0.2423777
	[ 1.858989,	2.538494]	[ 6.176418,	7.126521]
NA	3.0198151	0.1302705	5.0364373	0.2173754
	[ 2.764490,	3.275141]	[ 4.610389,	5.462485]
DA	3.7812980	0.1460617	9.7032629	0.1809474
	[ 3.495022,	4.067574]	[ 9.348613,	10.05791]
VO	4.6803104	0.1775939	13.913024	0.4117647
	[ 4.332233,	5.028388]	[ 13.10598,	14.72007]

	FAC_3		U_Var	
ID	0.2196889	0.2547574	112.13365	13.241422
	[-0.279626,	0.719004]	[ 86.18094,	138.0864]
DR	0.2346563	0.1131624	40.072776	35.887195
	[ 0.012862,	0.456451]	[-30.26483,	110.4104]
S7	2.2157410	0.3931470	745.99176	10.725870
	[ 1.445187,	2.986295]	[ 724.9694,	767.0141]
BC	4.0870160	0.4108408	379.43008	4.9970858
	[ 3.281783,	4.892249]	[ 369.6360,	389.2242]
NA	5.0501879	0.4741234	144.50192	4.3968393
	[ 4.120923,	5.979453]	[ 135.8843,	153.1196]

DA 2.3019830 0.4085772 165.14046 2.6474320  
 [ 1.501186, 3.102780] [ 159.9516, 170.3293]

VO -4.6034008 0.8544240 212.77823 15.388701  
 [-6.278041,-2.928761] [ 182.6169, 242.9395]

The best solutions for simple structure are the following:

\*\*\*\*\*  
 Table of 20 Best Pattern Solutions  
 \*\*\*\*\*

N	Npar	DF	Chisquared	P
1	18	10	1.05681274	0.99977835
2	19	9	0.82973802	0.99973986
3	20	8	0.60803423	0.99972055
4	20	8	0.64736686	0.99964647
5	20	8	0.64737613	0.99964645
6	19	9	0.89844351	0.99963807
7	20	8	0.66900373	0.99960022
8	19	9	0.95283616	0.99953872
9	20	8	0.71358863	0.99949155
10	21	7	0.50743232	0.99941881
11	20	8	0.75414496	0.99937580
12	21	7	0.51960765	0.99937147
13	20	8	0.76623130	0.99933798
14	19	9	1.04468144	0.99932744
15	20	8	0.78024974	0.99929212
16	22	6	0.34907179	0.99922214
17	21	7	0.56105438	0.99919080
18	22	6	0.35450764	0.99918687
19	21	7	0.57594440	0.99911817
20	21	7	0.59480211	0.99902003

N	RMSEA	ECVI	SRMR	PGFI	SBC
1	0	0.22565646	0.01073546	0.47537045	-50.4761032
2	0	0.23653138	0.00871209	0.42798995	-45.5498863
3	0	0.24743752	0.00675025	0.38057055	-40.6182985
4	0	0.24766620	0.00736474	0.38054458	-40.5789659
5	0	0.24766625	0.00736282	0.38054460	-40.5789566
6	0	0.23693083	0.00961663	0.42793966	-45.4811808
7	0	0.24779200	0.00801976	0.38053456	-40.5573290

8	0	0.23724706	0.00984211	0.42789788	-45.4267882
9	0	0.24805121	0.00777729	0.38050760	-40.5127441
10	0	0.25904775	0.00621947	0.33305397	-35.5656088
11	0	0.24828700	0.00836732	0.38047915	-40.4721878
12	0	0.25911854	0.00651920	0.33304843	-35.5534335
13	0	0.24835727	0.00931404	0.38047243	-40.4601015
14	0	0.23778105	0.01058736	0.42783979	-45.3349429
15	0	0.24843877	0.00846600	0.38046371	-40.4460830
16	0	0.27032217	0.00575254	0.28554832	-30.5706778
17	0	0.25935951	0.00713220	0.33302723	-35.5119868
18	0	0.27035377	0.00576968	0.28554580	-30.5652419
19	0	0.25944607	0.00716497	0.33301646	-35.4970968
20	0	0.25955571	0.00805226	0.33300380	-35.4782391

Loadings and Uni. Vars. with ASE's and Wald CIs

		FAC_1		FAC_2
ID	16.719466	0.1316275	0	0
	[ 16.46148,	16.97745]	[ 0,	0]
DR	19.073644	0.1598069	0	0
	[ 18.76043,	19.38686]	[ 0,	0]
S7	0	0	20.759541	0.2732631
	[ 0,	0]	[ 20.22395,	21.29513]
BC	0	0	7.2883277	0.1923605
	[ 0,	0]	[ 6.911308,	7.665347]
NA	0	0	6.6592971	0.1266958
	[ 0,	0]	[ 6.410978,	6.907616]
DA	0	0	10.494758	0.1333456
	[ 0,	0]	[ 10.23341,	10.75611]
VO	0	0	13.614394	0.1764118
	[ 0,	0]	[ 13.26863,	13.96015]
		FAC_3		U_Var
ID	0	0	62.709453	2.6032825
	[ 0,	0]	[ 57.60711,	67.81179]
DR	0	0	133.48609	3.5727845

	[	0,	0]	[	126.4836,	140.4886]
S7	0	0	731.85147	9.8153664		
	[	0,	0]	[	712.6137,	751.0892]
BC	2.6039975	0.3235364	385.30948	4.7548129		
	[	1.969878,	3.238117]	[	375.9902,	394.6287]
NA	4.6515370	0.6099560	138.50697	5.7785682		
	[	3.456045,	5.847029]	[	127.1812,	149.8328]
DA	0	0	168.75005	2.3308522		
	[	0,	0]	[	164.1817,	173.3184]
VO	-6.3256458	0.9098426	224.07448	12.727777		
	[	-8.108905,	-4.542387]	[	199.1285,	249.0205]

Factor Correlations Phi

-----

		FAC_1		FAC_2		
FAC_1	1.0000000	0	0.6571400	0.0066259		
	[	1.000000,	1.000000]	[	0.644153,	0.670127]
FAC_2	0.6571400	0.0066259	1.0000000	0		
	[	0.644153,	0.670127]	[	1.000000,	1.000000]
FAC_3	0	0	0	0		
	[	0,	0]	[	0,	0]
			FAC_3			
FAC_1	0	0				
	[	0,	0]			
FAC_2	0	0				
	[	0,	0]			
FAC_3	1.0000000	0				
	[	1.000000,	1.000000]			

(1) Standalone Fit Measures: -----  
Fit criterion . . . . . 0.0061  
Normal Th. Chi-square (df = 10) . . . . . 106.6029 prob= 0.0000  
Normal Theory Reweighted LS Chi-square . . . . . 104.7526

Probability of Close Fit . . . . .	1.0000
Z-Test of Wilson & Hilferty (1931). . . . .	8.2046
(2) Incremental Fit Measures: -----	
Null Model Chi-square (df = 21) . . . . .	32421.2870
RMSEA Estimate . . . . .	0.0236 90%C.I.[ 0.0197, 0.0277]
ECVI Estimate . . . . .	0.0082 90%C.I.[ 0.0065, 0.0104]
McDonald's (1989) Centrality. . . . .	0.9972
Tucker-Lewis Coefficient TLI. . . . .	0.9937
Bentler & Bonett's (1980) NFI . . . . .	0.9967
Bentler's Comparative Fit Index CFI . . . . .	0.9970
Parsimonious NFI (James, Mulaik, & Brett,1982). . . . .	0.4746
Bollen (1986) Normed Index Rho1 . . . . .	0.9931
Bollen (1988) Non-normed Index Delta2 . . . . .	0.9970
(3) Information Criteria: -----	
Akaike's Information Criterion. . . . .	86.6029
Bozdogan's (1987) CAIC. . . . .	-1.0111
Schwarz's Bayesian Criterion. . . . .	8.9889
(4) Other Fit Measures: -----	
Goodness of Fit Index (GFI) . . . . .	0.9983
Parsimonious GFI (Mulaik, 1989) . . . . .	0.4754
GFI Adjusted for Degrees of Freedom (AGFI). . . . .	0.9952
Root Mean Square Residual (RMR) . . . . .	5.3527
Hoelter's (1983) Critical N . . . . .	2981

Standardized Factor Loadings

	FAC_1	FAC_2	FAC_3
ID	0.9037553	0	0
DR	0.8553200	0	0
S7	0	0.6087781	0
BC	0	0.3454182	0.1234089
NA	0	0.4656885	0.3252822
DA	0	0.6284330	0
VO	0	0.6421882	-0.2983861

Squared Multiple Correlations

N	Variable	ErrVariance	TotVariance	R-squared
1	ID	62.7091975	342.250000	0.81677371
2	DR	133.486425	497.290000	0.73157227
3	S7	731.860053	1162.81000	0.37061080
4	BC	385.309895	445.210002	0.13454349
5	NA	138.506337	204.490003	0.32267429
6	DA	168.748525	278.890000	0.39492802

There are other local maxima with different zero patterns in  $\mathbf{P}$  when optimizing for  $N = 173$ .

However, this nice simple structure which maximizes the  $p$  value for  $N = 173$  would not maximize the  $p$  value for  $N = 17531$ . When using  $N = 17531$  this zero pattern gives  $p = 2.6e - 18$  which is even worse than the  $p = 5.9e - 006$  for EFA.

Maybe one of the loadings L43, L53, or L73 could be set to zero? If we set only L73 to zero we obtain  $p = 2.9e - 63$  and if we set both L53 and L73 to zero we obtain  $p = 5.2e - 78$ . Our statement, that the  $p$  optimal CFA structure is very robust when the number of observations is reduced is contradicted by this example.

On the other side, when optimizing the  $p$  value for a  $\chi^2$  value with  $N = 17531$  we would be able to obtain a slightly better  $p$  value than that of the EFA solution, but only by generating one additional zero loading. That means the  $p$  optimal pattern for the large  $N = 17531$  is almost as dense as EFA. What that means is, that for such large  $N$  an optimization for maximum  $p$  may not get very sparse factor loadings patterns. And the reason is probably, that for the large  $\chi^2$  the small increase of the degrees of freedom by introducing a few zero loadings does not contribute to much increase of the  $p$  value.

### 3 The Bibliography

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