# Automatic Model Improvement using the `cfa(.)` Function in CMAT

Wolfgang M. Hartmann: cmat.wolfgang@gmail.com

May 2011

## Contents

This is a first preliminary version of a paper describing the automatic CFA model improvement algorithm implemented in the `cfa()` function.

# 1 General Remarks

The syntax of the function

< gof,est,resi,cov,mod1,mod2,boci> = cfa(data,optn<,patt<,scal<,wgt<,xini<,targ<,wtrg> . >)

is explained in detail in the *Reference Manual* of CMAT. (Note, that some of the input and output arguments are lists of matrices.)
The factor model

$$\mathbf{C} = \mathbf{LPL}^T + diag(\mathbf{U})$$

where $\mathbf{C}$ is a $n \times n$ covariance or correlation matrix, $\mathbf{L}$ is a $n \times f$ factor loading matrix, $\mathbf{P}$ the symmetric $f \times f$ matrix of factor correlations (with unit diagonal), and $\mathbf{U} = diag$ is a $n \times n$ diagonal matrix of unique variances. Currently, the model is restricted to $diag(\mathbf{U})$ but an extension is intended for specifying CFA patterns for general symmetric $\mathbf{U}$.
The `cfa()` function implements ULS, ML, GLS, WLS, and DWLS estimation for confirmatory (CFA) and exploratory (EFA) factor analysis. From the estimation methods, WLS and DWLS estimation require either raw data input or the specification of a weight matrix with the fifth input argument. The input data may be a $N \times n$ matrix, $N$ observations, $n$ variables, of raw data or a symmetric $n \times n$ matrix of correlations or covariances. Analyzed is

- the covariance matrix,
- the correlation matrix like a covariance matrix,
- the correlation matrix without fitting the unit diagonal.

The `cfa()` function is able to analyze multiple $s \geq 1$ data samples with a simultaneous model. For multiple sample analysis the raw data may contain an ID variable specifying the sample number. But the user may also specify lists of input covariance or correlation matrices. Different parameter patterns and even different factor numbers can be specified for the different samples.
If the pattern input argument `patt` is specified as nonmissing, the number $f$ of factors of the CFA is defined, otherwise it must be specified with the `nfact` option.
One of the larger new developments for the `cfa()` function is an algorithm which tries to find a zero pattern in loading matrix $\mathbf{F}$ and factor correlations $\mathbf{P}$ which corresponds to a large $p$ value of the $\chi^2$ fit criterion. Since the maximization of $p$ is numerically very difficult we cannot guarantee that the function will always find a global maximum, but in many applications it found CFA models with surprisingly large $p$ values.
The CFA model improvement algorithm works within a known number $f$ of factors. There are two versions:

- if the `patt` input argument is specified as missing: the algorithm start with an EFA and computes a CFA pattern which corresponds to a larger $p$ value

- if the `patt` input argument is specified as nonmissing: the algorithm start with that CFA specification and tries to improve the CFA pattern for obtaining a larger $p$ value.

Especially the second version of starting point permits the testing of optimiality (w.r.t the $p$ value) of published CFA specifications. For many well know published model small modifications exist which show larger $p$ values.
The `cfa()` function is able to analyze ordinal data by computing polychoric correlations. The `scal` input argument should be a binary vector specifying which of the variables of the input raw data are scaled ordinary (set to 1) and which is metric (continuous,interval) (set to 0). This input argument may be a missing value indicating the default, that all variables are continuous. If there are variables of different scaling type (ordinal and continuous) the continuous variables are binned to ordinal (by default into 16 bins, but maybe specified differently using the `"nbin"` option). Note, that nominal variables with $k$ levels may always be rescaled to $k-1$ binary dummy variables.
Except for GLS estimation the `cfa` function is able to perform *genuine* correlation analysis. The difference between the classical covariance analysis (even of correlation matrices) and genuine correlation analysis are:

**covariance analysis** For EFA both, the loadings $\mathbf{L}$ and unique variances $\mathbf{U}$ are estimated by fitting the model to the covariance or correlation matrix inluding its diagonal.

**correlation analysis** The diagonal of the correlation matrix is considered fixed to unity and the model is not fitting the diagonal of the matrix. That means, for EFA only the loadings $\mathbf{L}$ are estimated, afterward the unique variances $\mathbf{U}$ are simply set to the difference between the unit diagonal and the communalities $h_i^2 = \sum_{j=1}^{nvar} l_{ij}^2 \quad i = 1, \ldots, n$.

Genuine ML correlation analysis is computationally expensive and may become very messy for Heywood cases, which occurs when the communalities $h_i^2$ become larger than 1. And because the degrees of freedom are reduced by $n$ it is harder to fit applications for small $n$. Especially for ordinal data when the polychoric correlation matrix is fitted, the asymptotic covariance matrix becomes indefinite and the ML fit criterion may become negative. For polycghoric correlation matrices with negative eigenvalues (indefinite) the `svd()` function will not permit the fit of ML or GLS estimates, therefore ULS, WLS or DWLS estimation maybe used instead.
*Note also, that the ML fit criterion for genuine correlation analysis is changed and does not include the constant term* $log(det(\mathbf{S}))$*, since the polychoric correlation matrix may have negative eigenvalues and therefore a negative determinant.*
Note, that WLS estimation requires many data observations so that the large

$\binom{n}{2} \times \binom{n}{2}$ weight matrix $\mathbf{W}$ (based on fourth order moments) becomes nonsingular.
Except for ULS estimation `cfa()` is able to compute standard normal and S-B robust asymptotic standard errors and confidence intervals.
The degrees of freedom are defined as:

- analyzing covariance matrix or correlation as covariance matrix: $df = s\binom{n}{2} - q$
- genuine correlation analysis: $df = s(\binom{n}{2} - n) - q$

using $s$ for the number of samples and $q$ for the number of model parameters.
For exploratory analysis it also implements a number of popular orthogonal and oblique rotation methods. In addition it incorporates robust (nonnormal, Satorra-Bentler) goodness of fit indices, robust ASEs, and nonnormal (robust) standardized residuals for ULS, ML, and GLS estimates.
The following remarks on rotation apply only to EFA, since CFA solutions are not invariant to rotation:

1. The same rotation methods as in the `frotate` or the older `factor` function are also available in the `cfa()` function.
2. Except for ULS, asymptotic standard errors and confidence intervals are computed for rotated factor loadings, the unique variances, and the factor correlations (when oblique rotation is applied).
3. Also for oblique rotation, the *factor structure* is computed with asymptotic standard errors and confidence intervals.

The asymptotic standard errors and Wald confidence intervals of rotated factor loadings depend on:

1. the kind of rotation method appplied and especially the fact whether the rotation is orthogonal or oblique;
2. if the (unstandardized) covariance or the (standardized) correlation matrix is analysed. (Note, you may read in a correlation matrix, but if you specify the analysis option `"anal"` as `"cov"`, the results correspond to those of a covariance analysis.)
3. if Kaiser normalization of the loadings is applied for the specific rotation method.

Comparison of `factor()`, `sem`, and `cfa` functions in CMAT:

| Feature | factor | cfa | sem |
|---|---|---|---|
| automatic model improvement | - | + | - |
| multiple sample analysis | - | + | + |
| standard normal ASEs and CIs | + | + | + |
| robust CIs and fit measures | - | + | + |
| profile likelihood CIs | + | + | + |
| bootstrapping CIs and GOF | + | + | + |
| Bollen-Stine bootstrap for $p$ | + | + | + |
| pattern specification (*1) | - | + | + |
| modification indices (*2) | - | + | + |
| different factor numbers (*3) | - | + | + |
| ordinal data (polychoric corrs) (*4) | - | + | - |
| genuine correlation analysis (*5) | - | + | +/- |
| ULS | + | + | + |
| GLS (*6) | - | +/- | + |
| ML (*7) | + | + | + |
| WLS (*8) | - | + | + |
| DWLS | - | + | + |
| standardized factor loadings | - | + | + |
| determining number of factors (*9) | + | - | - |
| rotation to simple structure (*10) | + | +/- | + |

Of course, the `sem` function is able to analyze models with more general form than that of the exploratory (EFA) or confirmatory (CFA) factor analysis.

**(*1)** there is a default parameter pattern for EFA

**(*2)** for EFA the asymptotic standard errors and the confidence intervals replace the modification indices

**(*3)** pattern definition may specify different number of factors for different samples

**(*4)** for the analysis of ordinal data the polychoric correlation matrix (or $s$ matrices) may also be computed using the `polychor()` function and provided as input into the `cfa`, `sem`, or `factor` functions

**(*5)** correlation data can be analyzed with all functions, `sem`, `factor` and `cfa`, however the `cfa` function is able to model correlation data with a specific model which results in a model matrix with fixed unit diagonal; the `sem` function permits specifying constraints for models similar to the `cfa` correlation structure model

**(*6)** GLS estimation is not available for polychoric correlations and correlation analysis (polychoric correlation matrices may not be pd)

**(*7)** ML estimation for correlation type analysis is still computational inefficient

**(*8)** note that WLS estimation requires a minimum set of $\binom{n}{2}$ independent observations (cases) for a nonsingular weight matrix (based on the fourth order moments of the data)

**(*9)** for CFA, usually the pattern definition defines the number of factors; for EFA the `factor` function offers a number of test criteria for the number of factors

**(*10)** only EFA solutions can be rotated, for CFA the specified pattern and goodness of fit is affected by rotation

Flow of computations in `factor()` and `cfa()`:

**FACTOR**

1. input data
2. compute criteria for number of factors
3. obtain suitable starting values for default pattern
4. estimate optimal parameters and standard normal ASEs
5. compute goodness-of-fit measures
6. orthogonal or oblique rotation to simple structure with recomputation of standard normal ASEs
7. optional: compute profile likelihood CIs
8. optional: bootstrap for GOFs and confidence intervals

**CFA**

1. input data (for multiple samples)
2. for CFA: input parameter pattern (models for all samples) (unnecessary for EFA)
3. obtain suitable starting values for default pattern
4. estimate optimal parameters and standard normal ASEs
5. compute goodness-of-fit measures
6. optional: compute robust goodness-of-fit measures and robust ASEs
7. compute modification indices
8. optional: compute profile likelihood CIs
9. optional: perform automatic model improvement starting with EFA or CFA depending on step 2; at the end repeat steps 5, 6, 7 for best solution
10. optional: bootstrap for GOFs and confidence intervals

Depending on our experience with this new algorithm we may extend it to solve some more complicate models, like

1. optimization w.r.t. other fit criteria, like GFI, RMSEA etc.

2. permit offdiagonal parameter estimates in a symmetric matrix $\mathbf{U}$

$$\mathbf{C} = \mathbf{LPL}^T + \mathbf{U}$$

3. hierarchical factor analysis models,
4. linear and nonlinear constraints on the parameters.

Some of our findings:

- Most CFA models with a large $p$ value are not very sparse. Usually they are close to an EFA, but with moderate increase in the $\chi^2$ value and an substantial increase in the degrees of freedom.
- Not surprisingly, for larger $f$ values it becomes easier to obtain $p > 0.01$ values. In all applications we were able to find CFA models with better fit than EFA for the same number $f$ of factors.
- It should be quite disturbing to researchers in the field that there are normally many very different CFA models structures among those with $p > 0.01$ values, depending on the value of $f$ (`cfa` stores the best 20 models found). And it may be hard to explain to researchers who want a foundation of their theory.

## 2 Some CFA Applications Revisited

The following tables show the $\chi^2$, degrees of reedom, and the $p$ value for a number of well known CFA examples. The values are reported for:

**EFA** the exploratory factor analysis ($f$ factors)

**EFA(imp)** the CFA result of the automatic model improvement algorithm starting at the EFA result

**CFA** the most common CFA model reported in the literature

**CFA(imp)** the CFA result of the automatic model improvement algorithm starting at the (formerly published) CFA result

Our algorithm tries to find a CFA model with corresponding $p$ value as large as possible. In practice, this must not be the model which is most appropriate for the underlying theory. However, in our experience researchers in Behavioral Sciences are many times hard pressed for developing a CFA model with a $p$ value large enough to be accepted by others. And this is especially so when the data have more than $n = 20$ variables. As the example with the *Import/Export Car Data* shows, our algorithm is not always able to find a model with $p > 0.01$ or even $p > 0.05$, especially when the specified number $f$ of factors is relatively small.
The following examples are in `tcfa11.inp, tcfa12.inp, tcfa13.inp`:

| Lot Data (Maydeu-Olivares) N=389, n=8, f=2, ML P.-M. CORR like COV | | | |
|---|---|---|---|
| | $\chi^2$ | $df$ | $p$ |
| EFA | 17.51 | 13 | 0.177 |
| EFA(imp) | 17.55 | 15 | 0.287 |
| CFA | 28.66 | 19 | 0.072 |
| CFA(imp) | 17.55 | 15 | 0.287 |
| Lot Data (Maydeu-Olivares) N=389, n=8, f=2, ML Poly gen. CORR | | | |
| | $\chi^2$ | $df$ | $p$ |
| EFA | 21.8634 | 13 | 0.0575 |
| EFA(imp) | 23.4828 | 16 | 0.1014 |
| CFA | 35.5829 | 19 | 0.0119 |
| CFA(imp) | 21.9038 | 15 | 0.1104 |
| Lot Data (Maydeu-Olivares) N=389, n=8, f=2, WLS Poly gen. CORR | | | |
| | $\chi^2$ | $df$ | $p$ |
| EFA | 16.0461 | 13 | 0.2467 |
| EFA(imp) | 17.4932 | 17 | 0.4215 |
| CFA | 25.2397 | 19 | 0.1528 |
| CFA(imp) | 17.1661 | 17 | 0.4432 |
| Lot Data (Maydeu-Olivares) N=389, n=8, f=2, DWLS Poly gen. CORR | | | |
| | $\chi^2$ | $df$ | $p$ |
| EFA | 5.0867 | 13 | 0.9732 |
| EFA(imp) | 5.0928 | 14 | 0.9845 |
| CFA | 12.7332 | 19 | 0.8519 |
| CFA(imp) | 5.1180 | 15 | 0.9911 |
| Import/Export Car Data (Sarle) SEM48: N=82, n=10, f=2, ML CORR | | | |
| | $\chi^2$ | $df$ | $p$ |
| EFA | 68.69 | 26 | 1.027e-5 |
| EFA(imp) | 69.78 | 31 | 8.159e-5 |
| 24 Psychological Tests(Harman,1976), SEM28: N=145, n=13, f=2, ML CORR | | | |
| | $\chi^2$ | $df$ | $p$ |
| EFA | 30.22 | 23 | 0.143 |
| EFA(imp) | 32.92 | 37 | 0.6608 |
| EFA(imp) | 36.65 | 43 | 0.7402 |

The following examples are in `tcfa14.inp`:

| Lawley-Maxwell Data, SEM06: N=100, n=8, f=3, ML CORR | | | |
|---|---|---|---|
| | $\chi^2$ | *df* | *p* |
| EFA | 1.78 | 7 | 0.9708 |
| EFA(imp) | 2.29 | 12 | 0.9988 |
| CFA | 2.45 | 10 | 0.9916 |
| CFA(imp) | 2.29 | 12 | 0.9988 |
| Simulated CFA (Bentler, 1985) SEM12: N=100, n=8, f=2, ML CORR | | | |
| | $\chi^2$ | *df* | *p* |
| EFA | 9.731 | 4 | 0.0452 |
| EFA(imp) | 10.88 | 7 | 0.1439 |
| CFA | 16.69 | 8 | 0.0335 |
| CFA(imp) | 10.88 | 7 | 0.1439 |
| Ability and Aspiration, LISREL VI, SEM13: N=556, n=6, f=2, ML CORR | | | |
| | $\chi^2$ | *df* | *p* |
| EFA | 4.592 | 4 | 0.3317 |
| EFA(imp) | 5.366 | 7 | 0.6154 |
| CFA | 9.256 | 8 | 0.3212 |
| CFA(imp) | 5.366 | 7 | 0.6154 |
| Nine Psychological Variables, LISREL VI, SEM16: N=145, n=9, f=3, ML CORR | | | |
| | $\chi^2$ | *df* | *p* |
| EFA | 9.9482 | 12 | 0.6205 |
| EFA(imp) | 11.251 | 20 | 0.9394 |
| CFA | 29.008 | 23 | 0.1801 |
| CFA(imp) | 11.251 | 20 | 0.9394 |

The following examples are in `tcfa15.inp, tcfa16.inp`:

| Thurstone (1931) Data, SEM09: N=213, n=9, f=3, ML CORR | | | |
|---|---|---|---|
| | $\chi^2$ | *df* | *p* |
| EFA | 2.9032 | 12 | 0.9962 |
| EFA(imp) | 3.1213 | 16 | 0.9998 |
| CFA | 38.196 | 24 | 0.0331 |
| CFA(imp) | 3.1065 | 16 | 0.9998 |
| Speed factor Data by Lord (1956), SEM33: N=649, n=18, f=6, ML COV | | | |
| | $\chi^2$ | *df* | *p* |
| EFA | 77.248 | 60 | 0.0662 |
| EFA(imp) | 83.5408 | 90 | 0.6713 |
| CFA | 174.503 | 117 | 0.0005 |
| CFA(imp) | 81.1435 | 94 | 0.8251 |
| Miller-Lutz Data, SEM34: N=51, n=8, f=4, ML COV | | | |
| | $\chi^2$ | *df* | *p* |
| EFA | 0.7566 | 2 | 0.6850 |
| EFA(imp) | 0.7819 | 6 | 0.9925 |
| CFA (2) | 265.34 | 28 | 0.0000 |
| CFA (4) | 220.93 | 18 | 0.0000 |
| Data from LAWLEY & MAXWELL (1971,p.96), SEM43: N=73, n=9, f=3, ML COV | | | |
| | $\chi^2$ | *df* | *p* |
| EFA | 5.5922 | 12 | 0.9352 |
| EFA(imp) | 6.1829 | 17 | 0.9919 |
| CFA | 5.5922 | 12 | 0.9352 |
| CFA(imp) | 6.0717 | 18 | 0.9959 |
| JAMES, MULAIK & BRETT (1982), SEM45: N=800, n=12, f=3, ML COV | | | |
| | $\chi^2$ | *df* | *p* |
| EFA | 22.3663 | 33 | 0.9191 |
| EFA(imp) | 23.4282 | 43 | 0.9935 |
| CFA | 42.3613 | 51 | 0.8001 |
| CFA(imp) | 24.7974 | 46 | 0.9955 |

The following examples are in `tcfa16.inp`:

| JOERESKOG (Enslein, Ralston & Wilf,1977), SEM46: N=286, n=9, f=3, ML CORR | | | |
|---|---|---|---|
| | $\chi^2$ | *df* | $p$ |
| EFA | 33.5157 | 12 | 0.0008 |
| EFA(imp) | 34.4539 | 16 | 0.0047 |
| CFA (2) | 138.4970 | 19 | 0.0000 |
| CFA(2,imp) | 138.1978 | 22 | 0.0000 |
| CFA (3) | 33.5157 | 12 | 0.0008 |
| CFA(3,imp) | 35.3875 | 16 | 0.0035 |
| Attitudes of Morality and Equality, SEM61: N=200, n=8, f=2, ML CORR | | | |
| | $\chi^2$ | *df* | $p$ |
| EFA | 21.3073 | 13 | 0.0671 |
| EFA(imp) | 22.3793 | 15 | 0.0982 |
| CFA | 41.5842 | 19 | 0.0020 |
| CFA(imp) | 22.3793 | 15 | 0.0982 |
| Sympathy and Anger, BOLLEN (1989), SEM72: N=138, n=6, f=2, ML COV | | | |
| | $\chi^2$ | *df* | $p$ |
| EFA | 5.7654 | 4 | 0.2174 |
| EFA(imp) | 6.0878 | 6 | 0.4134 |
| CFA | 126.57 | 8 | 0.0000 |
| CFA(imp) | 6.0878 | 6 | 0.4134 |
| Eight Physical Variables (Harman, 1967), SEM80: N=305, n=8, f=2, ML CORR | | | |
| | $\chi^2$ | *df* | $p$ |
| EFA | 76.9612 | 13 | 4.09e-11 |
| EFA(imp) | 78.6508 | 16 | 2.91e-10 |
| CFA (1) | 169.66 | 20 | 1.01e-25 |
| CFA(1,imp) | 78.6508 | 16 | 2.91e-10 |
| CFA (2) | 102.25 | 19 | 2.09e-13 |
| CFA(2,imp) | 78.6508 | 16 | 2.91e-10 |
| Eight Physical Variables (Harman, 1967), SEM80: N=305, n=8, f=3, ML CORR | | | |
| | $\chi^2$ | *df* | $p$ |
| EFA | 23.0148 | 7 | 0.0017 |
| EFA(imp) | 25.2517 | 13 | 0.0214 |

We note, that we were always able to find CFA models with $p$ values larger than those obtained from the EFA. Of course we cannot say the same thing for the $\chi^2$ value, since any constraint will increase the $\chi^2$ value of the EFA solution. The increase of the $p$ value thus is based on the increase of the degrees of freedom associated with a moderate increase of the $\chi^2$ value.

Also, the $p$ values of the models found with our algorithm (EFA(imp)) are always larger than those reported for models in the literature (CFA). That means, even when the models found with our algorithm must not be the best, from a theoretical point of view it may be a good starting guess for a model that better describes the theory.

## 2.1 Lot Data (Maydeu-Olivares, 2009)

We analyzed the data in two ways by

1. traditional approach: treating the ordinal data as metric and fitting product-moment correlations including the diagonal of the correlation matrix,
2. new approach: computing polychoric correlations for the ordinal data only fitting the offdiagonal entries of the correlation matrix.

### 2.1.1 Lot Data: ML for Product Moment Correlations

Here we want to show the most common approach, fitting product moment correlations including the diagonal (i.e. equivalent to covariance analysis). Some univariate moments when treating data as metric:

```
Name             Mean     Std Dev    Skewness    Kurtosis
---------------------------------------------------------
i1        2.244215938  1.00744584 -0.12460927 -0.64820145
i4        2.439588689  0.99201754 -0.34919434 -0.35531466
i5        2.596401028  0.98916158 -0.56683881 -0.11488129
i11       2.336760925  0.98557158 -0.56994653 -0.09588667
i3        1.845758355  1.05364549  0.24526977 -0.72410856
i8        1.393316195  1.03149204  0.62842416 -0.14311245
i9        1.321336761  1.00107276  0.68497038  0.00779773
i12       1.398457584  1.06881652  0.70560377 -0.23352418

                 Variance Divisor = 388


Mardia's Multivariate Kurtosis . . . . . . . .       18.39
Relative Multivariate Kurtosis . . . . . . . .        1.23
Normalized Multivariate Kurtosis . . . . . . .       14.34
Mardia Based Kappa (Browne, 1982). . . . . . .      0.2298
Mean Scaled Univariate Kurtosis  . . . . . . .    -0.09613
Adjusted Mean Scaled Univariate Kurtosis . . .    -0.08896

Observation numbers with largest contribution to kurtosis


           294        94        376        381        335
```

```
1760.120  904.9803  841.3284  743.3313  528.4092
```

The matrix of product moment correslations:

```
                         Correlation Matrix

                 i1          i4          i5         i11          i3
    i1    1.0000000
    i4    0.5112302  1.0000000
    i5    0.4405557  0.5332253  1.0000000
    i11   0.2518052  0.3437831  0.2243751  1.0000000
    i3   -0.1562358 -0.2234606 -0.2552427 -0.1086935  1.0000000
    i8   -0.2786839 -0.3759395 -0.3315373 -0.1940056  0.5041608
    i9   -0.2390110 -0.2905373 -0.3007501 -0.2222896  0.5089277
    i12  -0.2150692 -0.3455020 -0.2960516 -0.2549389  0.4369115


                         Correlation Matrix

                             i8          i9         i12
               i8    1.0000000
               i9    0.7009527  1.0000000
               i12   0.5377681  0.5183549  1.0000000
        Determinant =  7.69133189e-002 (Ln = -2.5651e+000)
```

This output shows the EFA model fit:

```
*****************************************************************
Starting Model Improvement (ChiSq=17.5081 Pvalue=0.177104 DF=13)
*****************************************************************
```

The following table presents a summary of the best 13 solutions. Note, that some models have been found with the same $\chi^2$ values and degrees of freedom but with different parametrizations in **L** and **P**. In this example we obtain equivalent fit for models with uncorrelated **P**=**I** and correlated factors $p[1,2] \neq 0$.

```
**********************************
Table of 20 Best Pattern Solutions
**********************************

N  Npar    DF  Chisquared           P

1    21    15  17.5451923  0.28732998
2    21    15  17.5451923  0.28732998
```

| | | | | |
|---|---|---|---|---|
| 3 | 21 | 15 | 17.6276394 | 0.28274721 |
| 4 | 20 | 16 | 19.0657624 | 0.26526770 |
| 5 | 20 | 16 | 19.0657624 | 0.26526770 |
| 6 | 19 | 17 | 20.9052521 | 0.23055711 |
| 7 | 19 | 17 | 20.9052521 | 0.23055711 |
| 8 | 22 | 14 | 17.5231748 | 0.22936779 |
| 9 | 22 | 14 | 17.5231748 | 0.22936779 |
| 10 | 22 | 14 | 17.5231748 | 0.22936779 |
| 11 | 22 | 14 | 17.5231749 | 0.22936779 |
| 12 | 22 | 14 | 17.5231749 | 0.22936779 |
| 13 | 22 | 14 | 17.5231749 | 0.22936779 |
| 14 | 20 | 16 | 19.8038723 | 0.22918436 |
| 15 | 22 | 14 | 17.5289063 | 0.22908589 |
| 16 | 22 | 14 | 17.5289063 | 0.22908589 |
| 17 | 22 | 14 | 17.5289063 | 0.22908589 |
| 18 | 20 | 16 | 19.8263737 | 0.22814273 |
| 19 | 20 | 16 | 19.8263737 | 0.22814273 |
| 20 | 21 | 15 | 18.9084259 | 0.21791361 |

| N | RMSEA | ECVI | SRMR | PGFI | SBC |
|---|---|---|---|---|---|
| 1 | 0.02091216 | 0.15603751 | 0.02059178 | 0.52983547 | -71.9084978 |
| 2 | 0.02091216 | 0.15603751 | 0.02059178 | 0.52983547 | -71.9084978 |
| 3 | 0.02124816 | 0.15625000 | 0.02071255 | 0.52980257 | -71.8260507 |
| 4 | 0.02222252 | 0.15467946 | 0.02452283 | 0.56460867 | -76.3515071 |
| 5 | 0.02222252 | 0.15467946 | 0.02452283 | 0.56460867 | -76.3515071 |
| 6 | 0.02433236 | 0.15414337 | 0.02692954 | 0.59928327 | -80.4755967 |
| 7 | 0.02433236 | 0.15414337 | 0.02692954 | 0.59928327 | -80.4755967 |
| 8 | 0.02546755 | 0.16125781 | 0.02045578 | 0.49451849 | -65.9669360 |
| 9 | 0.02546755 | 0.16125781 | 0.02045576 | 0.49451849 | -65.9669360 |
| 10 | 0.02546755 | 0.16125781 | 0.02045568 | 0.49451849 | -65.9669360 |
| 11 | 0.02546755 | 0.16125781 | 0.02045591 | 0.49451849 | -65.9669359 |
| 12 | 0.02546755 | 0.16125781 | 0.02045593 | 0.49451849 | -65.9669359 |
| 13 | 0.02546755 | 0.16125781 | 0.02045593 | 0.49451849 | -65.9669359 |
| 14 | 0.02475353 | 0.15658181 | 0.02553610 | 0.56440874 | -75.6133972 |
| 15 | 0.02548826 | 0.16127258 | 0.02061294 | 0.49451876 | -65.9612045 |
| 16 | 0.02548826 | 0.16127258 | 0.02061293 | 0.49451876 | -65.9612045 |
| 17 | 0.02548826 | 0.16127258 | 0.02061291 | 0.49451876 | -65.9612045 |
| 18 | 0.02482663 | 0.15663980 | 0.02460313 | 0.56435741 | -75.5908958 |
| 19 | 0.02482663 | 0.15663980 | 0.02460315 | 0.56435741 | -75.5908958 |
| 20 | 0.02591430 | 0.15955100 | 0.02415190 | 0.52937518 | -70.5452643 |

The following shows details of the best solution found:

```
Loadings and Uni. Vars. with ASE's and Wald CIs
```

```
-----------------------------------------------

                     FAC_1                 FAC_2
i1     0.6297975  0.0519471          0          0
     [ 0.527983, 0.731612] [        0,         0]

i4     0.8255722  0.0517764          0          0
     [ 0.724092, 0.927052] [        0,         0]

i5     0.6110768  0.0617574 -0.0887525  0.0576947
     [ 0.490034, 0.732119] [-0.201832, 0.024327]

i11    0.3656166  0.0643032 -0.0765299  0.0617991
     [ 0.239585, 0.491649] [-0.197654, 0.044594]

i3             0          0  0.6179993  0.0492443
     [        0,         0] [ 0.521482, 0.714516]

i8    -0.0972515  0.0510824  0.7888890  0.0528204
     [-0.197371, 2.9e-003] [ 0.685363, 0.892415]

i9             0          0  0.8350369  0.0458291
     [        0,         0] [ 0.745214, 0.924860]

i12   -0.1573878  0.0565717  0.5679202  0.0554314
     [-0.268266,-0.046509] [ 0.459277, 0.676564]


                      U_Var
i1     0.6033552  0.0536629
     [ 0.498178, 0.708532]

i4     0.3184306  0.0565624
     [ 0.207570, 0.429291]

i5     0.5712696  0.0520766
     [ 0.469201, 0.673338]

i11    0.8359933  0.0628656
     [ 0.712779, 0.959208]

i3     0.6180768  0.0498835
     [ 0.520307, 0.715847]

i8     0.3010894  0.0378062
     [ 0.226991, 0.375188]
```

```
i9    0.3027134  0.0405970
     [ 0.223145, 0.382282]

i12   0.5745127  0.0462982
     [ 0.483770, 0.665256]
```

```
   Factor Correlations Phi
   -----------------------

                      FAC_1                FAC_2
FAC_1   1.0000000         0 -0.4373454  0.0606098
      [ 1.000000, 1.000000] [-0.556138,-0.318552]

FAC_2  -0.4373454  0.0606098  1.0000000         0
      [-0.556138,-0.318552] [ 1.000000, 1.000000]
```

```
(1) Standalone Fit Measures: ------------------------------
Fit criterion . . . . . . . . . . . . . . . . . .    0.0452
Normal Th. Chi-square (df = 15) 17.5452 Prob>chi**2 = 0.2873
Elliptic cor. Chi-square. . . . 14.2661 Prob>chi**2 = 0.5054
Normal Theory Reweighted LS Chi-square  . . . . .   17.2203
Probability of Close Fit  . . . . . . . . . . . .    0.9158
Z-Test of Wilson & Hilferty (1931). . . . . . . .    0.5623
(2) Incremental Fit Measures: -----------------------------
Null Model Chi-square (df = 28) . . . . . . . .    995.2496
RMSEA Estimate  . . . . . . .  0.0209  90%C.I.[ . ,  0.0544]
ECVI Estimate . . . . . . . .  0.1560  90%C.I.[ . ,  0.1943]
McDonald's (1989) Centrality. . . . . . . . . . .    0.9967
Tucker-Lewis Coefficient TLI. . . . . . . . . . .    0.9951
Bentler & Bonett's (1980) NFI . . . . . . . . . .    0.9824
Bentler's Comparative Fit Index CFI . . . . . . .    0.9974
Parsimonious NFI (James, Mulaik, & Brett,1982). .    0.5263
Bollen (1986) Normed Index Rho1 . . . . . . . . .    0.9671
Bollen (1988) Non-normed Index Delta2 . . . . . .    0.9974
(3) Information Criteria: ---------------------------------
Akaike's Information Criterion. . . . . . . . . .  -12.4548
Bozdogan's (1987) CAIC. . . . . . . . . . . . . .  -86.9085
Schwarz's Bayesian Criterion. . . . . . . . . . .  -71.9085
(4) Other Fit Measures: -----------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .    0.9890
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .    0.5298
GFI Adjusted for Degrees of Freedom (AGFI). . . .    0.9737
Root Mean Square Residual (RMR) . . . . . . . . .    0.0206
```

```
Hoelter's (1983) Critical N . . . . . . . . . . .        554
```

Standardized Factor Loadings

| | FAC_1 | FAC_2 |
|---|---|---|
| i1 | 0.6297975 | 0 |
| i4 | 0.8255722 | 0 |
| i5 | 0.6110768 | -0.0887525 |
| i11 | 0.3656166 | -0.0765299 |
| i3 | 0 | 0.6179993 |
| i8 | -0.0972515 | 0.7888890 |
| i9 | 0 | 0.8350369 |
| i12 | -0.1573878 | 0.5679202 |

Squared Multiple Correlations

| N | Variable | ErrVariance | TotVariance | R-squared |
|---|---|---|---|---|
| 1 | i1 | 0.60335517 | 1.00000000 | 0.39664483 |
| 2 | i4 | 0.31843058 | 1.00000000 | 0.68156942 |
| 3 | i5 | 0.57126962 | 1.00000000 | 0.42873038 |
| 4 | i11 | 0.83599328 | 1.00000000 | 0.16400672 |
| 5 | i3 | 0.61807681 | 1.00000000 | 0.38192319 |
| 6 | i8 | 0.30108944 | 1.00000000 | 0.69891056 |
| 7 | i9 | 0.30271343 | 1.00000000 | 0.69728657 |
| 8 | i12 | 0.57451268 | 1.00000000 | 0.42548732 |

Maydeu-Olivares found the CFA model with zero loadings in $L[1,2], L[2,2], L[3,2], L[4,2], L[5,1], L[6,1], L[7,1],$ is better suitable for his purpose. The following shows some of the results for this model:

```
         Loadings and Uni. Vars. with ASE's and Wald CIs
         -----------------------------------------------

                           FAC_1                  FAC_2
        i1    0.6358694  0.0516265          0          0
             [ 0.534683, 0.737055] [        0,         0]

        i4    0.7990097  0.0501158          0          0
             [ 0.700785, 0.897235] [        0,         0]

        i5    0.6728991  0.0512115          0          0
             [ 0.572526, 0.773272] [        0,         0]

        i11   0.4062557  0.0547096          0          0
```

```
     [ 0.299027, 0.513485] [        0,        0]

i3           0          0  0.6130510  0.0490786
     [        0,        0] [ 0.516859, 0.709243]

i8           0          0  0.8473233  0.0445086
     [        0,        0] [ 0.760088, 0.934559]

i9           0          0  0.8158311  0.0451235
     [        0,        0] [ 0.727391, 0.904271]

i12          0          0  0.6521216  0.0483176
     [        0,        0] [ 0.557421, 0.746822]


                              U_Var
           i1    0.5956700  0.0530467
                [ 0.491700, 0.699640]

           i4    0.3615835  0.0510587
                [ 0.261510, 0.461657]

           i5    0.5472068  0.0517912
                [ 0.445698, 0.648716]

           i11   0.8349563  0.0633126
                [ 0.710866, 0.959047]

           i3    0.6241685  0.0498275
                [ 0.526508, 0.721829]

           i8    0.2820432  0.0368046
                [ 0.209908, 0.354179]

           i9    0.3344197  0.0376715
                [ 0.260585, 0.408255]

           i12   0.5747374  0.0471383
                [ 0.482348, 0.667127]


             Factor Correlations Phi
             -----------------------

                      FAC_1                  FAC_2
FAC_1   1.0000000          0 -0.5365535  0.0484893
```

```
            [ 1.000000, 1.000000] [-0.631591,-0.441516]

    FAC_2  -0.5365535  0.0484893  1.0000000          0
           [-0.631591,-0.441516] [ 1.000000, 1.000000]
```

Note, that the $p$ value for this model is still sufficiently large ($p > 0.05$):

```
(1) Standalone Fit Measures: -------------------------------
Fit criterion . . . . . . . . . . . . . . . . . . .    0.0739
Normal Th. Chi-square (df = 19) 28.6588 Prob>chi**2 = 0.0715
Elliptic cor. Chi-square. . . . 23.3027 Prob>chi**2 = 0.2243
Normal Theory Reweighted LS Chi-square  . . . . .   27.8669
Probability of Close Fit  . . . . . . . . . . . .    0.7905
Z-Test of Wilson & Hilferty (1931). . . . . . . .    1.4659
(2) Incremental Fit Measures: ------------------------------
Null Model Chi-square (df = 28) . . . . . . . . .  995.2496
RMSEA Estimate  . . . . . . .  0.0362  90%C.I.[ . ,  0.0618]
ECVI Estimate . . . . . . . .  0.1636  90%C.I.[ . ,  0.2118]
McDonald's (1989) Centrality. . . . . . . . . . .    0.9877
Tucker-Lewis Coefficient TLI. . . . . . . . . . .    0.9853
Bentler & Bonett's (1980) NFI . . . . . . . . . .    0.9712
Bentler's Comparative Fit Index CFI . . . . . . .    0.9900
Parsimonious NFI (James, Mulaik, & Brett,1982). .    0.6590
Bollen (1986) Normed Index Rho1 . . . . . . . . .    0.9576
Bollen (1988) Non-normed Index Delta2 . . . . . .    0.9901
(3) Information Criteria: ----------------------------------
Akaike's Information Criterion. . . . . . . . . .   -9.3412
Bozdogan's (1987) CAIC. . . . . . . . . . . . . . -103.6492
Schwarz's Bayesian Criterion. . . . . . . . . . .  -84.6492
(4) Other Fit Measures: ------------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .    0.9824
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .    0.6666
GFI Adjusted for Degrees of Freedom (AGFI). . . .    0.9666
Root Mean Square Residual (RMR) . . . . . . . . .    0.0334
Hoelter's (1983) Critical N . . . . . . . . . . .       410


                Standardized Factor Loadings

                       FAC_1      FAC_2
                i1    0.6358694          0
                i4    0.7990097          0
                i5    0.6728991          0
                i11   0.4062557          0
                i3            0  0.6130510
```

```
i8            0  0.8473233
i9            0  0.8158311
i12           0  0.6521216
```

```
         Squared Multiple Correlations

N Variable ErrVariance TotVariance   R-squared
1 i1        0.59567005  1.00000000  0.40432995
2 i4        0.36158355  1.00000000  0.63841645
3 i5        0.54720684  1.00000000  0.45279316
4 i11       0.83495631  1.00000000  0.16504369
5 i3        0.62416848  1.00000000  0.37583152
6 i8        0.28204318  1.00000000  0.71795682
7 i9        0.33441968  1.00000000  0.66558032
8 i12       0.57473739  1.00000000  0.42526261
```

### 2.1.2 Lot Data: ML Fit of Polychoric Correlations

The following shows some of the results when fitting only the offdiagonal entries of the polychoric correlation matrix:

```
     ************************************************
     Sums of Weights and Frequencies for Class Levels
     ************************************************

Variable       Value    Nobs  Proportion      Freq     Weight
                   0      13  0.03341902        13 13.0000000
                   1      84  0.21593830        84 84.0000000
                   2     125  0.32133676       125 125.000000
                   3     129  0.33161954       129 129.000000
                   4      38  0.09768638        38 38.0000000

                   0      12  0.03084833        12 12.0000000
                   1      56  0.14395887        56 56.0000000
                   2     121  0.31105398       121 121.000000
                   3     149  0.38303342       149 149.000000
                   4      51  0.13110540        51 51.0000000
            ..........................................
                   0      69  0.17737789        69 69.0000000
                   1     186  0.47814910       186 186.000000
                   2      60  0.15424165        60 60.0000000
                   3      58  0.14910026        58 58.0000000
                   4      16  0.04113111        16 16.0000000
```

```
                   **********
                   Thresholds
                   **********

 1  -1.83276143 -0.67651357  0.17814153  1.29484771
 2  -1.86846944 -0.93533747 -0.03544818  1.12118112
 3  -1.90673221 -1.05143443 -0.27068619  0.97603151
 4  -1.60738861 -0.88654975 -0.00322189  1.42470796
 5  -1.44271677 -0.17159847  0.55195841  1.54068570
 6  -0.91556101  0.31102177  0.99699679  1.79924808
 7  -0.86761683  0.40028580  1.07410343  1.90673221
 8  -0.92540400  0.40028580  0.87704399  1.73770831


           Polychoric Correlation(s)

  1.0000000
  0.5577219  1.0000000
  0.4859851  0.5877967  1.0000000
  0.2803616  0.3723243  0.2652058  1.0000000
 -0.1636465 -0.2297360 -0.2780625 -0.1216389  1.0000000
 -0.3080950 -0.3925849 -0.3718812 -0.1979306  0.5536118
 -0.2654568 -0.3005806 -0.3364019 -0.2294538  0.5587520
 -0.2585184 -0.3656555 -0.3312995 -0.3016573  0.4891194


           Polychoric Correlation(s)

           1.0000000
           0.7644719  1.0000000
           0.5980033  0.5932275  1.0000000
    Matrix is Positive Definite: Determinant=0.0414436


S-B Mean-Corr. MVN Test: Chi^2=1070.92 df=420 pval= 0.0000
              (See A. Maydeu-Olivares, 2003)


              ******************************
              Correlation ML Factor Analysis
              (Exploratory Factor Analysis)
              ******************************

Input Data. . . . . . . . . . . . . . . . . . . . Raw Data
Analysis of . . . . . . . . . . . . . . . Polychoric W/O Diag
Number of Items . . . . . . . . . . . . . . . . . .      8
Rotation Method . . . . . . . . . . . . . . . . . .      2
```

```
Number of Subjects. . . . . . . . . . . . . . . . .       389
Number of Parameters. . . . . . . . . . . . . . . .        15
Version . . . . . . . . . . . . . . . . SEM-Type Matrix Model
Orthogonal Rotation Method. . . . . .Varimax (Kaiser, 1958)
Unnormed Rotation: Parameter. . . . . . . . . . . 1.0000000
ASE for Rotated Solution. . . . . . . . . . .Wald (Analyt.)
```

This output shows the EFA model fit:

```
*****************************************************************
Starting Model Improvement (ChiSq=21.8634 Pvalue=0.0575151 DF=13)
*****************************************************************


                *********************************
                Table of 20 Best Pattern Solutions
                *********************************

                 N  Npar    DF  Chisquared           P

                 1    13    15  21.9038083  0.11035108
                 2    12    16  23.4828056  0.10142491
                 3    14    14  21.9014747  0.08066159
                 4    14    14  21.9014747  0.08066159
                 5    13    15  23.4803746  0.07445955
                 6    11    17  26.2344729  0.07029331
                 7    13    15  24.0281274  0.06461737
                 8    15    13  21.8633824  0.05751505
                 9    15    13  21.8633824  0.05751505
                10    12    16  26.2339567  0.05082391
                11    10    18  30.3830713  0.03388456
                12     9    19  35.5829340  0.01187108
                13    10    18  34.5596562  0.01073329
                14    11    17  33.8939594  0.00866469
                15    13    15  43.2016837  1.465e-004
                16    13    15  43.2016837  1.465e-004
                17    13    15  43.3605522  1.384e-004
                18    12    16  46.1921741  9.092e-005
                19    12    16  46.1921741  9.092e-005
                20    11    17  52.6930884  1.598e-005

    N       RMSEA        ECVI        SRMR        PGFI         SBC

    1  0.03444156  0.12505470  0.03870009  0.53177191 -67.5498819
    2  0.03471813  0.12384723  0.04194483  0.56699868 -71.9344639
```

```
 3  0.03813943  0.13032573  0.03865676  0.49631866 -61.5886361
 4  0.03813943  0.13032573  0.03865681  0.49631866 -61.5886361
 5  0.03817210  0.12911801  0.04189964  0.53155910 -65.9733156
 6  0.03741672  0.12566211  0.04518281  0.60188640 -75.1463759
 7  0.03938559  0.13052975  0.04151720  0.53140157 -65.4255628
 8           0           0           0           0           0
 9  0.04191912  0.13550460  0.03848420  0.46086941 -55.6631490
10  0.04060186  0.13093783  0.04516349  0.56648006 -69.1833128
11  0.04210775  0.13107733  0.04974364  0.63646480 -76.9613569
12  0.04742835  0.13920200  0.05649097  0.67059668 -77.7250735
13  0.04869379  0.14184173  0.05589841  0.63549228 -72.7847720
14  0.05060873  0.14540306  0.05465688  0.60025733 -67.4868895
15  0.06961070  0.17994613  0.08948907  0.52921459 -46.2520065
16  0.06961070  0.17994613  0.08948917  0.52921460 -46.2520065
17  0.06980650  0.18035558  0.08932140  0.52943257 -46.0931379
18  0.06973830  0.18237653  0.09455658  0.56441320 -49.2250954
19  0.06973830  0.18237653  0.09455668  0.56441321 -49.2250954
20  0.07356170  0.19385442  0.10245884  0.59905854 -48.6877604


      Loadings and Uni. Vars. with ASE's and Wald CIs
      -----------------------------------------------

                          FAC_1                 FAC_2
     i1    0.6477315  0.0484128          0          0
          [ 0.552844, 0.742619] [        0,         0]

     i4    0.8284240  0.0480574          0          0
          [ 0.734233, 0.922615] [        0,         0]

     i5    0.6294115  0.0566221 -0.1003851  0.0525502
          [ 0.518434, 0.740389] [-0.203382, 2.6e-003]

     i11   0.3718424  0.0594592 -0.0712707  0.0564553
          [ 0.255304, 0.488380] [-0.181921, 0.039380]

     i3            0          0  0.6208688  0.0437439
          [        0,         0] [ 0.535132, 0.706605]

     i8   -0.1024494  0.0473546  0.7837677  0.0453847
          [-0.195263,-9.6e-003] [ 0.694815, 0.872720]

     i9            0          0  0.8453638  0.0396660
          [        0,         0] [ 0.767620, 0.923108]

     i12  -0.1597340  0.0525165  0.5814684  0.0488632
```

```
[-0.262664,-0.056804] [ 0.485698, 0.677239]


                   U_Var
i1    0.5804440          0
     [ 0.580444, 0.580444]

i4    0.3137136          0
     [ 0.313714, 0.313714]

i5    0.5427439          0
     [ 0.542744, 0.542744]

i11   0.8352540          0
     [ 0.835254, 0.835254]

i3    0.6145219          0
     [ 0.614522, 0.614522]

i8    0.3103736          0
     [ 0.310374, 0.310374]

i9    0.2853600          0
     [ 0.285360, 0.285360]

i12   0.5613796          0
     [ 0.561380, 0.561380]


      Factor Correlations Phi
      -----------------------

                  FAC_1                 FAC_2
FAC_1   1.0000000          0 -0.4037454  0.0568514
       [ 1.000000, 1.000000] [-0.515172,-0.292319]

FAC_2  -0.4037454  0.0568514  1.0000000          0
       [-0.515172,-0.292319] [ 1.000000, 1.000000]


(1) Standalone Fit Measures: ------------------------------
Fit criterion . . . . . . . . . . . . . . . . . .     0.0565
Normal Th. Chi-square (df = 15) 21.9038 Prob>chi**2 = 0.1104
Normal Theory Reweighted LS Chi-square  . . . . .    10.9519
Probability of Close Fit  . . . . . . . . . . . .     0.7844
Z-Test of Wilson & Hilferty (1931). . . . . . . .     1.2269
```

```
(2) Incremental Fit Measures: ------------------------------
Null Model Chi-square (df = 28) . . . . . . . .   1235.1674
RMSEA Estimate  . . . . . . .  0.0344  90%C.I.[ . ,  0.0636]
ECVI Estimate . . . . . . . .  0.1251  90%C.I.[ . ,  0.1895]
McDonald's (1989) Centrality. . . . . . . . . . .    0.9912
Tucker-Lewis Coefficient TLI. . . . . . . . . . .    0.9893
Bentler & Bonett's (1980) NFI . . . . . . . . . .    0.9823
Bentler's Comparative Fit Index CFI . . . . . . .    0.9943
Parsimonious NFI (James, Mulaik, & Brett,1982). .    0.5262
Bollen (1986) Normed Index Rho1 . . . . . . . . .    0.9669
Bollen (1988) Non-normed Index Delta2 . . . . . .    0.9943
(3) Information Criteria: ---------------------------------
Akaike's Information Criterion. . . . . . . . . .   -8.0962
Bozdogan's (1987) CAIC. . . . . . . . . . . . . .  -82.5499
Schwarz's Bayesian Criterion. . . . . . . . . . .  -67.5499
(4) Other Fit Measures: -----------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .    0.9934
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .    0.5322
GFI Adjusted for Degrees of Freedom (AGFI). . . .    0.9841
Root Mean Square Residual (RMR) . . . . . . . . .    0.0387
Hoelter's (1983) Critical N . . . . . . . . . . .       444


                Standardized Factor Loadings

                           FAC_1      FAC_2
                  i1    0.6477315          0
                  i4    0.8284240          0
                  i5    0.6294115 -0.1003851
                  i11   0.3718424 -0.0712707
                  i3            0  0.6208688
                  i8   -0.1024494  0.7837677
                  i9            0  0.8453638
                  i12  -0.1597340  0.5814684


               Squared Multiple Correlations

        N Variable ErrVariance TotVariance   R-squared
        1 i1        0.58044397  1.00000000  0.41955603
        2 i4        0.31371361  1.00000000  0.68628639
        3 i5        0.54274387  1.00000000  0.45725613
        4 i11       0.83525399  1.00000000  0.16474601
        5 i3        0.61452187  1.00000000  0.38547813
        6 i8        0.31037364  1.00000000  0.68962636
        7 i9        0.28536004  1.00000000  0.71463996
```

```
8 i12        0.56137964  1.00000000  0.43862036
```

Again, we are fitting only the offdiagonal entries of the polychoric correlation matrix however now with EFA.

```
                 ******************************
                 Correlation ML Factor Analysis
                 (Exploratory Factor Analysis)
                 ******************************

   Input Data. . . . . . . . . . . . . . . . . . . . . . Raw Data
   Analysis of . . . . . . . . . . . . . . . Polychoric W/O Diag
   Number of Items . . . . . . . . . . . . . . . . . . .        8
   Rotation Method . . . . . . . . . . . . . . . . . . .        2
   Number of Subjects. . . . . . . . . . . . . . . . . .      389
   Number of Parameters. . . . . . . . . . . . . . . . .       15
   Version . . . . . . . . . . . . . . . . SEM-Type Matrix Model
   Orthogonal Rotation Method. . . . . .Varimax (Kaiser, 1958)
   Unnormed Rotation: Parameter. . . . . . . . . . . 1.0000000
   ASE for Rotated Solution. . . . . . . . . . .Wald (Analyt.)


   (1) Standalone Fit Measures: ------------------------------
   Fit criterion . . . . . . . . . . . . . . . . . .    0.0563
   Normal Th. Chi-square (df = 13) 21.8634 Prob>chi**2 = 0.0575
   Bartlett Corrected Chi-square (df = 13)    21.5910 p= 0.0575
   Normal Theory Reweighted LS Chi-square  . . . . .   10.9317
   Probability of Close Fit  . . . . . . . . . . . .    0.6344
   Z-Test of Wilson & Hilferty (1931). . . . . . . .    1.5779
   (2) Incremental Fit Measures: -----------------------------
   Null Model Chi-square (df = 28) . . . . . . . . .  -388.0000
   RMSEA Estimate  . . . . . . .  0.0419  90%C.I.[ . ,  0.0716]
   ECVI Estimate . . . . . . . .  0.1355  90%C.I.[ . ,  0.2009]
   McDonald's (1989) Centrality. . . . . . . . . . .    0.9887
   Tucker-Lewis Coefficient TLI. . . . . . . . . . .    1.0459
   Bentler & Bonett's (1980) NFI . . . . . . . . . .    1.0563
   Bentler's Comparative Fit Index CFI . . . . . . .    0.0000
   Parsimonious NFI (James, Mulaik, & Brett,1982). .    0.4904
   Bollen (1986) Normed Index Rho1 . . . . . . . . .    1.1214
   Bollen (1988) Non-normed Index Delta2 . . . . . .    1.0221
   (3) Information Criteria: ---------------------------------
   Akaike's Information Criterion. . . . . . . . . .   -4.1366
   Bozdogan's (1987) CAIC. . . . . . . . . . . . . .  -68.6631
   Schwarz's Bayesian Criterion. . . . . . . . . . .  -55.6631
   (4) Other Fit Measures: -----------------------------------
```

```
Goodness of Fit Index (GFI) . . . . . . . . . . .    0.9934
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .    0.4612
GFI Adjusted for Degrees of Freedom (AGFI). . . .    0.9817
Root Mean Square Residual (RMR) . . . . . . . . .    0.0385
Hoelter's (1983) Critical N . . . . . . . . . . .       398
```

```
        Rotated Factor Loadings with Standard Errors
        --------------------------------------------

                     FAC_1                     FAC_2
i1    0.6235542  0.0422227 -0.1644182  0.0428581
     [ 0.540799, 0.706309] [-0.248419,-0.080418]

i4    0.8077756  0.0399853 -0.1989762  0.0347872
     [ 0.729406, 0.886145] [-0.267158,-0.130795]

i5    0.6259097  0.0413365 -0.2527542  0.0420524
     [ 0.544892, 0.706928] [-0.335175,-0.170333]

i11   0.3725920  0.0511884 -0.1611601  0.0509114
     [ 0.272265, 0.472919] [-0.260945,-0.061376]

i3   -0.1064721  0.0457239  0.6115087  0.0374094
     [-0.196089,-0.016855] [ 0.538188, 0.684830]

i8   -0.2315572  0.0348744  0.7975712  0.0274638
     [-0.299910,-0.163205] [ 0.743743, 0.851399]

i9   -0.1414585  0.0325585  0.8342607  0.0266517
     [-0.205272,-0.077645] [ 0.782024, 0.886497]

i12  -0.2527785  0.0443307  0.6122940  0.0363948
     [-0.339665,-0.165892] [ 0.540962, 0.683627]
```

```
                    U_Var
i1    0.5841468          0
     [ 0.584147, 0.584147]

i4    0.3079070          0
     [ 0.307907, 0.307907]

i5    0.5443523          0
     [ 0.544352, 0.544352]
```

```
i11   0.8352026          0
     [ 0.835203, 0.835203]

i3    0.6147208          0
     [ 0.614721, 0.614721]

i8    0.3102614          0
     [ 0.310261, 0.310261]

i9    0.2839986          0
     [ 0.283999, 0.283999]

i12   0.5611990          0
     [ 0.561199, 0.561199]
```

Standardized Factor Loadings

| | FAC_1 | FAC_2 |
|---|---|---|
| i1 | 0.6235542 | -0.1644182 |
| i4 | 0.8077756 | -0.1989762 |
| i5 | 0.6259097 | -0.2527542 |
| i11 | 0.3725920 | -0.1611601 |
| i3 | -0.1064721 | 0.6115087 |
| i8 | -0.2315572 | 0.7975712 |
| i9 | -0.1414585 | 0.8342607 |
| i12 | -0.2527785 | 0.6122940 |

Squared Multiple Correlations

| N | Variable | ErrVariance | TotVariance | R-squared |
|---|---|---|---|---|
| 1 | i1 | 0.58414683 | 1.00000000 | 0.41585317 |
| 2 | i4 | 0.30790700 | 1.00000000 | 0.69209300 |
| 3 | i5 | 0.54435232 | 1.00000000 | 0.45564768 |
| 4 | i11 | 0.83520259 | 1.00000000 | 0.16479741 |
| 5 | i3 | 0.61472076 | 1.00000000 | 0.38527924 |
| 6 | i8 | 0.31026142 | 1.00000000 | 0.68973858 |
| 7 | i9 | 0.28399858 | 1.00000000 | 0.71600142 |
| 8 | i12 | 0.56119902 | 1.00000000 | 0.43880098 |

Total Determination of All Equations = 0.998
Total Determination of Manifest Variables = 0.940

### 2.1.3 Lot Data: WLS Fit of Polychoric Correlations

Again, only the off-digonal entries of the polychoric correlaton matrix are fitted.

```
***********************************************************
Starting Model Improvement (ChiSq=16.0461 p=0.246651 DF=13)
***********************************************************


                *********************************
                Table of 16 Best Pattern Solutions
                *********************************

                N  Npar    DF  Chisquared          P

                1    11    17  17.4932261  0.42146361
                2    12    16  16.7221159  0.40379538
                3    13    15  16.0839664  0.37649325
                4    12    16  17.1562985  0.37555349
                5    10    18  19.9382364  0.33630766
                6    14    14  16.0747095  0.30883317
                7    11    17  19.3922544  0.30648710
                8    11    17  19.5929426  0.29557704
                9    12    16  18.9872319  0.26932586
               10    10    18  22.6323496  0.20511575
               11     9    19  25.2396995  0.15280258
               12     9    19  25.2396995  0.15280258
               13     9    19  25.2396995  0.15280258
               14    10    18  24.3006715  0.14542201
               15     8    20  110.269653  1.754e-014
               16     8    20  110.269653  1.754e-014


 N       RMSEA        ECVI        SRMR        PGFI         SBC

 1  0.00864734  0.10178667  0.04035342  0.59710684 -83.8876227
 2  0.01078518  0.10495391  0.03710830  0.56239927 -78.6951536
 3  0.01364729  0.10846383  0.03419656  0.52757236 -73.3697237
 4  0.01364769  0.10607293  0.03886014  0.56216483 -78.2609710
 5  0.01665909  0.10293360  0.04400652  0.63074553 -87.4061918
 6  0.01954334  0.11359461  0.03407470  0.49240524 -67.4154013
 7  0.01904423  0.10668107  0.04121360  0.59601735 -81.9885944
 8  0.01982696  0.10719831  0.04323028  0.59590221 -81.7879063
 9  0.02193606  0.11079183  0.04113782  0.56117620 -76.4300376
10  0.02575423  0.10987719  0.04705782  0.62910897 -84.7120786
11           0           0           0           0           0
12  0.02909304  0.11144252  0.04960816  0.66238763 -88.0683081
```

```
13  0.02909304  0.11144252  0.04960712  0.66238763 -88.0683081
14  0.03003595  0.11417699  0.04760267  0.62809554 -83.0437567
15  0.10785494  0.32543725  0.19892470  0.63985901 -9.00193377
16  0.10785494  0.32543725  0.19892470  0.63985901 -9.00193377


      Loadings and Uni. Vars. with ASE's and Wald CIs
      -----------------------------------------------

                          FAC_1                  FAC_2
     i1    0.6757371  0.0442865          0          0
          [ 0.588937, 0.762537] [        0,         0]

     i4    0.8518933  0.0351081          0          0
          [ 0.783083, 0.920704] [        0,         0]

     i5    0.6460490  0.0606375 -0.1093036  0.0663641
          [ 0.527202, 0.764896] [-0.239375, 0.020768]

     i11   0.4604004  0.0558383          0          0
          [ 0.350959, 0.569842] [        0,         0]

     i3            0          0  0.6724844  0.0459350
          [        0,         0] [ 0.582453, 0.762515]

     i8            0          0  0.9178680  0.0257476
          [        0,         0] [ 0.867404, 0.968332]

     i9            0          0  0.8988828  0.0242453
          [        0,         0] [ 0.851363, 0.946403]

     i12  -0.1536135  0.0640301  0.6577492  0.0572753
          [-0.279110,-0.028117] [ 0.545492, 0.770007]


                                     U_Var
               i1    0.5433794          0
                    [ 0.543379, 0.543379]

               i4    0.2742779          0
                    [ 0.274278, 0.274278]

               i5    0.5011459          0
                    [ 0.501146, 0.501146]

               i11   0.7880315          0
```

```
     [ 0.788031, 0.788031]

i3    0.5477648          0
     [ 0.547765, 0.547765]

i8    0.1575183          0
     [ 0.157518, 0.157518]

i9    0.1920096          0
     [ 0.192010, 0.192010]

i12   0.4442864          0
     [ 0.444286, 0.444286]
```

```
                 Factor Correlations Phi
                 -----------------------

                        FAC_1                 FAC_2
FAC_1   1.0000000           0 -0.4922966  0.0557069
      [ 1.000000, 1.000000] [-0.601480,-0.383113]

FAC_2  -0.4922966  0.0557069  1.0000000           0
      [-0.601480,-0.383113] [ 1.000000, 1.000000]
```

```
(1) Standalone Fit Measures: ------------------------------
Fit criterion . . . . . . . . . . . . . . . . . .    0.0451
Normal Th. Chi-square (df = 17) 17.4932 Prob>chi**2 = 0.4215
Probability of Close Fit  . . . . . . . . . . . .    0.9656
Z-Test of Wilson & Hilferty (1931). . . . . . . .    0.1981
(2) Incremental Fit Measures: -----------------------------
Null Model Chi-square (df = 28) . . . . . . . . . 11415.9663
RMSEA Estimate  . . . . . . .  0.0086  90%C.I.[ . ,  0.0472]
ECVI Estimate . . . . . . . .  0.1018  90%C.I.[ . ,  0.1590]
McDonald's (1989) Centrality. . . . . . . . . . .    0.9994
Tucker-Lewis Coefficient TLI. . . . . . . . . . .    0.9999
Bentler & Bonett's (1980) NFI . . . . . . . . . .    0.9985
Bentler's Comparative Fit Index CFI . . . . . . .    1.0000
Parsimonious NFI (James, Mulaik, & Brett,1982). .    0.6062
Bollen (1986) Normed Index Rho1 . . . . . . . . .    0.9975
Bollen (1988) Non-normed Index Delta2 . . . . . .    1.0000
(3) Information Criteria: ---------------------------------
Akaike's Information Criterion. . . . . . . . . .  -16.5068
Bozdogan's (1987) CAIC. . . . . . . . . . . . . . -100.8876
Schwarz's Bayesian Criterion. . . . . . . . . . .  -83.8876
```

```
(4) Other Fit Measures: ------------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .    0.9835
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .    0.5971
GFI Adjusted for Degrees of Freedom (AGFI). . . .    0.9650
Root Mean Square Residual (RMR) . . . . . . . . .    0.0404
Hoelter's (1983) Critical N . . . . . . . . . . .       613

                 Standardized Factor Loadings

                          FAC_1      FAC_2
                 i1    0.6757371          0
                 i4    0.8518933          0
                 i5    0.6460490 -0.1093036
                 i11   0.4604004          0
                 i3            0  0.6724844
                 i8            0  0.9178680
                 i9            0  0.8988828
                 i12  -0.1536135  0.6577492


               Squared Multiple Correlations

        N Variable ErrVariance TotVariance   R-squared
        1 i1        0.54337942  1.00000000  0.45662058
        2 i4        0.27427788  1.00000000  0.72572212
        3 i5        0.50114592  1.00000000  0.49885408
        4 i11       0.78803144  1.00000000  0.21196856
        5 i3        0.54776477  1.00000000  0.45223523
        6 i8        0.15751825  1.00000000  0.84248175
        7 i9        0.19200963  1.00000000  0.80799037
        8 i12       0.44428644  1.00000000  0.55571356


    Total Determination of All Equations = 0.983
 Total Determination of Manifest Variables = 1.000
```

The following shows the EFA fit of the polychoric correlation matrix:

```
(1) Standalone Fit Measures: -------------------------------
Fit criterion . . . . . . . . . . . . . . . . . .    0.0414
Normal Th. Chi-square (df = 13) 16.0461 Prob>chi**2 = 0.2467
Probability of Close Fit  . . . . . . . . . . . .    0.8725
Z-Test of Wilson & Hilferty (1931). . . . . . . .    0.6867
(2) Incremental Fit Measures: ------------------------------
Null Model Chi-square (df = 28) . . . . . . . .  11415.9663
```

```
RMSEA Estimate  . . . . . . .  0.0246  90%C.I.[ . ,  0.0588]
ECVI Estimate . . . . . . . .  0.1187  90%C.I.[ . ,  0.1764]
McDonald's (1989) Centrality. . . . . . . . . . .     0.9961
Tucker-Lewis Coefficient TLI. . . . . . . . . . .     0.9994
Bentler & Bonett's (1980) NFI . . . . . . . . . .     0.9986
Bentler's Comparative Fit Index CFI . . . . . . .     0.9997
Parsimonious NFI (James, Mulaik, & Brett,1982). .     0.4636
Bollen (1986) Normed Index Rho1 . . . . . . . . .     0.9970
Bollen (1988) Non-normed Index Delta2 . . . . . .     0.9997
(3) Information Criteria: ---------------------------------
Akaike's Information Criterion. . . . . . . . . .    -9.9539
Bozdogan's (1987) CAIC. . . . . . . . . . . . . .   -74.4805
Schwarz's Bayesian Criterion. . . . . . . . . . .   -61.4805
(4) Other Fit Measures: -----------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .     0.9848
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .     0.4572
GFI Adjusted for Degrees of Freedom (AGFI). . . .     0.9580
Root Mean Square Residual (RMR) . . . . . . . . .     0.0342
Hoelter's (1983) Critical N . . . . . . . . . . .        542


        Rotated Factor Loadings with Standard Errors
        --------------------------------------------

                         FAC_1                FAC_2
     i1    0.6562847  0.0382584 -0.1974561  0.0400465
          [ 0.581300, 0.731270] [-0.275946,-0.118966]

     i4    0.8261019  0.0351926 -0.2338132  0.0333772
          [ 0.757126, 0.895078] [-0.299231,-0.168395]

     i5    0.6406256  0.0378389 -0.2873865  0.0394141
          [ 0.566463, 0.714789] [-0.364637,-0.210136]

     i11   0.4077281  0.0484299 -0.1766353  0.0480273
          [ 0.312807, 0.502649] [-0.270767,-0.082504]

     i3   -0.1239008  0.0421997  0.6635098  0.0312584
          [-0.206611,-0.041191] [ 0.602245, 0.724775]

     i8   -0.2252224  0.0288431  0.8715148  0.0177197
          [-0.281754,-0.168691] [ 0.836785, 0.906245]

     i9   -0.1720307  0.0284790  0.8860567  0.0173419
          [-0.227849,-0.116213] [ 0.852067, 0.920046]
```

```
i12  -0.2582669  0.0387080  0.6991847  0.0278767
     [-0.334133,-0.182401] [ 0.644547, 0.753822]


            i1    0.5303015          0
                  [ 0.530302, 0.530302]

            i4    0.2628870          0
                  [ 0.262887, 0.262887]

            i5    0.5070078          0
                  [ 0.507008, 0.507008]

            i11   0.8025578          0
                  [ 0.802558, 0.802558]

            i3    0.5444033          0
                  [ 0.544403, 0.544403]

            i8    0.1897369          0
                  [ 0.189737, 0.189737]

            i9    0.1853089          0
                  [ 0.185309, 0.185309]

            i12   0.4444390          0
                  [ 0.444439, 0.444439]
```

The following shows the CFA fit of the polychoric correlation matrix using the model by Maydeu-Olivares:

```
 Loadings and Uni. Vars. with ASE's and Wald CIs
 -----------------------------------------------

                      FAC_1                 FAC_2
i1    0.6818860  0.0432233          0           0
     [ 0.597170, 0.766602] [        0,          0]

i4    0.8388110  0.0341808          0           0
     [ 0.771818, 0.905804] [        0,          0]

i5    0.7215944  0.0396250          0           0
     [ 0.643931, 0.799258] [        0,          0]

i11   0.4302503  0.0554110          0           0
```

```
     [ 0.321647, 0.538854] [        0,        0]

 i3           0          0  0.6736862  0.0464487
     [        0,         0] [ 0.582648, 0.764724]

 i8           0          0  0.9153590  0.0252889
     [        0,         0] [ 0.865794, 0.964924]

 i9           0          0  0.8965837  0.0240553
     [        0,         0] [ 0.849436, 0.943731]

 i12          0          0  0.7567778  0.0423568
     [        0,         0] [ 0.673760, 0.839796]


                               U_Var
           i1    0.5350315          0
                [ 0.535032, 0.535032]

           i4    0.2963962          0
                [ 0.296396, 0.296396]

           i5    0.4793015          0
                [ 0.479302, 0.479302]

           i11   0.8148847          0
                [ 0.814885, 0.814885]

           i3    0.5461469          0
                [ 0.546147, 0.546147]

           i8    0.1621178          0
                [ 0.162118, 0.162118]

           i9    0.1961377          0
                [ 0.196138, 0.196138]

           i12   0.4272873          0
                [ 0.427287, 0.427287]


             Factor Correlations Phi
             -----------------------

                     FAC_1                  FAC_2
FAC_1   1.0000000          0 -0.5618570  0.0488902
```

```
          [ 1.000000, 1.000000] [-0.657680,-0.466034]

    FAC_2  -0.5618570  0.0488902  1.0000000         0
          [-0.657680,-0.466034] [ 1.000000, 1.000000]


(1) Standalone Fit Measures: ------------------------------
Fit criterion . . . . . . . . . . . . . . . . . .    0.0651
Normal Th. Chi-square (df = 19) 25.2397 Prob>chi**2 = 0.1528
Probability of Close Fit  . . . . . . . . . . . .    0.8850
Z-Test of Wilson & Hilferty (1931). . . . . . . .    1.0262
(2) Incremental Fit Measures: -----------------------------
Null Model Chi-square (df = 28) . . . . . . . .  11415.9663
RMSEA Estimate  . . . . . . .  0.0291  90%C.I.[ . ,  0.0564]
ECVI Estimate . . . . . . . .  0.1114  90%C.I.[ . ,  0.1764]
McDonald's (1989) Centrality. . . . . . . . . . .    0.9920
Tucker-Lewis Coefficient TLI. . . . . . . . . . .    0.9992
Bentler & Bonett's (1980) NFI . . . . . . . . . .    0.9978
Bentler's Comparative Fit Index CFI . . . . . . .    0.9995
Parsimonious NFI (James, Mulaik, & Brett,1982). .    0.6771
Bollen (1986) Normed Index Rho1 . . . . . . . . .    0.9967
Bollen (1988) Non-normed Index Delta2 . . . . . .    0.9995
(3) Information Criteria: ---------------------------------
Akaike's Information Criterion. . . . . . . . . .  -12.7603
Bozdogan's (1987) CAIC. . . . . . . . . . . . . . -107.0683
Schwarz's Bayesian Criterion. . . . . . . . . . .  -88.0683
(4) Other Fit Measures: -----------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .    0.9762
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .    0.6624
GFI Adjusted for Degrees of Freedom (AGFI). . . .    0.9548
Root Mean Square Residual (RMR) . . . . . . . . .    0.0496
Hoelter's (1983) Critical N . . . . . . . . . . .       465
```

### 2.1.4 Lot Data: DWLS Fit of Polychoric Correlations

Again, only the off-digonal entries of the polychoric correlaton matrix are fitted.

```
**********************************
Table of 15 Best Pattern Solutions
**********************************

N  Npar    DF  Chisquared           P

1    13    15  5.11806027  0.99107875
2    14    14  5.09283788  0.98447823
```

| | | | | |
|---|---|---|---|---|
| 3 | 12 | 16 | 6.35525440 | 0.98378366 |
| 4 | 11 | 17 | 8.14378935 | 0.96337469 |
| 5 | 10 | 18 | 9.14447449 | 0.95630138 |
| 6 | 11 | 17 | 10.3320077 | 0.88911341 |
| 7 | 10 | 18 | 11.3720935 | 0.87790496 |
| 8 | 10 | 18 | 11.5809997 | 0.86809803 |
| 9 | 12 | 16 | 10.2630614 | 0.85253419 |
| 10 | 9 | 19 | 12.7331886 | 0.85194528 |
| 11 | 9 | 19 | 12.7331886 | 0.85194528 |
| 12 | 9 | 19 | 12.7331886 | 0.85194528 |
| 13 | 11 | 17 | 11.2644477 | 0.84249104 |
| 14 | 8 | 20 | 374.505125 | 3.896e-067 |
| 15 | 8 | 20 | 374.505125 | 3.896e-067 |

| N | RMSEA | ECVI | SRMR | PGFI | SBC |
|---|---|---|---|---|---|
| 1 | 0 | 0.08020119 | 0.02200114 | 0.53410883 | -84.3356299 |
| 2 | 0 | 0.08529082 | 0.02191276 | 0.49850896 | -78.3972729 |
| 3 | 0 | 0.07823519 | 0.02461146 | 0.56930212 | -89.0620151 |
| 4 | 0 | 0.07769018 | 0.02791070 | 0.60424766 | -93.2370595 |
| 5 | 0 | 0.07511462 | 0.02965252 | 0.63941496 | -98.1999537 |
| 6 | 0 | 0.08332992 | 0.03151421 | 0.60346973 | -91.0488411 |
| 7 | 0 | 0.08085591 | 0.03291566 | 0.63857643 | -95.9723346 |
| 8 | 0 | 0.08139433 | 0.03320311 | 0.63849780 | -95.7634285 |
| 9 | 0 | 0.08830686 | 0.03148998 | 0.56799458 | -85.1542081 |
| 10 | 0 | 0.07920925 | 0.03478164 | 0.67351209 | -100.574819 |
| 11 | 0 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0.07920925 | 0.03478150 | 0.67351209 | -100.574819 |
| 13 | 0 | 0.08573311 | 0.03270651 | 0.60313824 | -90.1164012 |
| 14 | 0.21373734 | 1.00645651 | 0.19215615 | 0.55765013 | 255.233538 |
| 15 | 0.21373734 | 1.00645651 | 0.19215615 | 0.55765013 | 255.233538 |

```
        Loadings and Uni. Vars. with ASE's and Wald CIs
        ----------------------------------------------

                           FAC_1                    FAC_2
       i1    0.6703500  0.0397637         0           0
            [ 0.592415, 0.748285] [        0,         0]

       i4    0.8447774  0.0488002         0           0
            [ 0.749131, 0.940424] [        0,         0]

       i5    0.6526541  0.0678055 -0.1030619  0.0685362
            [ 0.519758, 0.785550] [-0.237390, 0.031267]
```

```
i11   0.3873697  0.0621310 -0.0804273  0.0615312
     [ 0.265595, 0.509144] [-0.201026, 0.040172]

i3           0          0  0.6529447  0.0387571
     [        0,        0] [ 0.576982, 0.728907]

i8   -0.0947586  0.0813314  0.8198499  0.0717772
     [-0.254165, 0.064648] [ 0.679169, 0.960531]

i9           0          0  0.8736692  0.0460049
     [        0,        0] [ 0.783501, 0.963837]

i12  -0.1640964  0.0737977  0.6259643  0.0623590
     [-0.308737,-0.019456] [ 0.503743, 0.748186]


                                 U_Var
           i1    0.5506309          0
                [ 0.550631, 0.550631]

           i4    0.2863512          0
                [ 0.286351, 0.286351]

           i5    0.5060607          0
                [ 0.506061, 0.506061]

           i11   0.8169081          0
                [ 0.816908, 0.816908]

           i3    0.5736632          0
                [ 0.573663, 0.573663]

           i8    0.2526176          0
                [ 0.252618, 0.252618]

           i9    0.2367022          0
                [ 0.236702, 0.236702]

           i12   0.4936464          0
                [ 0.493646, 0.493646]


             Factor Correlations Phi
             -----------------------
```

```
                    FAC_1                   FAC_2
FAC_1   1.0000000          0 -0.4263825  0.0545373
       [ 1.000000, 1.000000] [-0.533274,-0.319491]

FAC_2  -0.4263825  0.0545373  1.0000000          0
       [-0.533274,-0.319491] [ 1.000000, 1.000000]
```

```
(1) Standalone Fit Measures: ------------------------------
Fit criterion . . . . . . . . . . . . . . . . . .    0.0132
Normal Th. Chi-square (df = 15)  5.1181 Prob>chi**2 = 0.9911
Probability of Close Fit  . . . . . . . . . . . .    1.0000
Z-Test of Wilson & Hilferty (1931). . . . . . . .   -2.3531
(2) Incremental Fit Measures: -----------------------------
Null Model Chi-square (df = 28) . . . . . . . . .  4648.2583
RMSEA Estimate  . . . . . . . . . . . . . . . . . .  0.0000
ECVI Estimate . . . . . . . . . . . . . . . . . . .  0.0802
McDonald's (1989) Centrality. . . . . . . . . . .    1.0128
Tucker-Lewis Coefficient TLI. . . . . . . . . . .    1.0040
Bentler & Bonett's (1980) NFI . . . . . . . . . .    0.9989
Bentler's Comparative Fit Index CFI . . . . . . .    1.0000
Parsimonious NFI (James, Mulaik, & Brett,1982). .    0.5351
Bollen (1986) Normed Index Rho1 . . . . . . . . .    0.9979
Bollen (1988) Non-normed Index Delta2 . . . . . .    1.0021
(3) Information Criteria: ---------------------------------
Akaike's Information Criterion. . . . . . . . . .  -24.8819
Bozdogan's (1987) CAIC. . . . . . . . . . . . . .  -99.3356
Schwarz's Bayesian Criterion. . . . . . . . . . .  -84.3356
(4) Other Fit Measures: -----------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .    0.9970
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .    0.5341
GFI Adjusted for Degrees of Freedom (AGFI). . . .    0.9928
Root Mean Square Residual (RMR) . . . . . . . . .    0.0220
Hoelter's (1983) Critical N . . . . . . . . . . .      1896
```

Standardized Factor Loadings

| | FAC_1 | FAC_2 |
|---|---|---|
| i1 | 0.6703500 | 0 |
| i4 | 0.8447774 | 0 |
| i5 | 0.6526541 | -0.1030619 |
| i11 | 0.3873697 | -0.0804273 |
| i3 | 0 | 0.6529447 |
| i8 | -0.0947586 | 0.8198499 |
| i9 | 0 | 0.8736692 |

```
i12  -0.1640964  0.6259643

N Variable ErrVariance TotVariance   R-squared
1 i1        0.55063085  1.00000000  0.44936915
2 i4        0.28635120  1.00000000  0.71364880
3 i5        0.50606066  1.00000000  0.49393934
4 i11       0.81690815  1.00000000  0.18309185
5 i3        0.57366321  1.00000000  0.42633679
6 i8        0.25261760  1.00000000  0.74738240
7 i9        0.23670217  1.00000000  0.76329783
8 i12       0.49364638  1.00000000  0.50635362


    Total Determination of All Equations = 0.975
 Total Determination of Manifest Variables = 0.973
```

The following shows the EFA fit of the polychoric correlation matrix:

```
(1) Standalone Fit Measures: ------------------------------
Fit criterion . . . . . . . . . . . . . . . . . .     0.0131
Normal Th. Chi-square (df = 13)  5.0867 Prob>chi**2 = 0.9732
Probability of Close Fit  . . . . . . . . . . . .     0.9997
Z-Test of Wilson & Hilferty (1931). . . . . . . .    -1.9235
(2) Incremental Fit Measures: -----------------------------
Null Model Chi-square (df = 28) . . . . . . . . .  4587.5178
RMSEA Estimate  . . . . . . . . . . . . . . . . . .   0.0000
ECVI Estimate . . . . . . . . . . . . . . . . . . .   0.0904
McDonald's (1989) Centrality. . . . . . . . . . .     1.0102
Tucker-Lewis Coefficient TLI. . . . . . . . . . .     1.0037
Bentler & Bonett's (1980) NFI . . . . . . . . . .     0.9989
Bentler's Comparative Fit Index CFI . . . . . . .     1.0000
Parsimonious NFI (James, Mulaik, & Brett,1982). .     0.4638
Bollen (1986) Normed Index Rho1 . . . . . . . . .     0.9976
Bollen (1988) Non-normed Index Delta2 . . . . . .     1.0017
(3) Information Criteria: ---------------------------------
Akaike's Information Criterion. . . . . . . . . .   -20.9133
Bozdogan's (1987) CAIC. . . . . . . . . . . . . .   -85.4398
Schwarz's Bayesian Criterion. . . . . . . . . . .   -72.4398
(4) Other Fit Measures: -----------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .     0.9970
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .     0.4629
GFI Adjusted for Degrees of Freedom (AGFI). . . .     0.9918
Root Mean Square Residual (RMR) . . . . . . . . .     0.0219
Hoelter's (1983) Critical N . . . . . . . . . . .       1707
```

```
   Rotated Factor Loadings with Standard Errors
   --------------------------------------------

                     FAC_1                 FAC_2
i1    0.6503989  0.0394191 -0.1719797  0.0410431
     [ 0.573139, 0.727659] [-0.252423,-0.091537]

i4    0.8120786  0.0363689 -0.2225821  0.0341954
     [ 0.740797, 0.883360] [-0.289604,-0.155560]

i5    0.6484993  0.0385200 -0.2715475  0.0401367
     [ 0.573001, 0.723997] [-0.350214,-0.192881]

i11   0.3885002  0.0496874 -0.1799930  0.0493350
     [ 0.291115, 0.485886] [-0.276688,-0.083298]

i3   -0.1077119  0.0435602  0.6473818  0.0338529
     [-0.193088,-0.022335] [ 0.581031, 0.713732]

i8   -0.2355034  0.0320577  0.8325272  0.0226060
     [-0.298335,-0.172671] [ 0.788220, 0.876834]

i9   -0.1592354  0.0309847  0.8554850  0.0220635
     [-0.219964,-0.098506] [ 0.812241, 0.898729]

i12  -0.2679428  0.0412792  0.6595682  0.0319459
     [-0.348849,-0.187037] [ 0.596955, 0.722181]


                                 U_Var
           i1    0.5474043           0
                [ 0.547404, 0.547404]

           i4    0.2909855           0
                [ 0.290985, 0.290985]

           i5    0.5057106           0
                [ 0.505711, 0.505711]

           i11   0.8166701           0
                [ 0.816670, 0.816670]

           i3    0.5692950           0
                [ 0.569295, 0.569295]

           i8    0.2514367           0
```

```
                [ 0.251437, 0.251437]

          i9     0.2427895         0
                [ 0.242789, 0.242789]

          i12    0.4931764         0
                [ 0.493176, 0.493176]
```

The following shows the CFA fit of the polychoric correlation matrix using the model by Maydeu-Olivares:

```
 Loadings and Uni. Vars. with ASE's and Wald CIs
 ----------------------------------------------

                     FAC_1                FAC_2
i1     0.6554355  0.0376231          0         0
     [ 0.581696, 0.729175] [         0,         0]

i4     0.8123717  0.0423662          0         0
     [ 0.729335, 0.895408] [         0,         0]

i5     0.7339822  0.0405931          0         0
     [ 0.654421, 0.813543] [         0,         0]

i11    0.4450190  0.0379459          0         0
     [ 0.370646, 0.519392] [         0,         0]

i3             0          0  0.6361935  0.0365814
     [         0,         0] [ 0.564495, 0.707892]

i8             0          0  0.8796857  0.0402388
     [         0,         0] [ 0.800819, 0.958552]

i9             0          0  0.8398419  0.0384565
     [         0,         0] [ 0.764468, 0.915215]

i12            0          0  0.7362067  0.0382401
     [         0,         0] [ 0.661257, 0.811156]


                          U_Var
          i1     0.5704043         0
                [ 0.570404, 0.570404]

          i4     0.3400523         0
```

```
         [ 0.340052, 0.340052]

   i5     0.4612701          0
         [ 0.461270, 0.461270]

   i11    0.8019581          0
         [ 0.801958, 0.801958]

   i3     0.5952578          0
         [ 0.595258, 0.595258]

   i8     0.2261530          0
         [ 0.226153, 0.226153]

   i9     0.2946657          0
         [ 0.294666, 0.294666]

   i12    0.4579997          0
         [ 0.458000, 0.458000]


           Factor Correlations Phi
           -----------------------

                     FAC_1                 FAC_2
 FAC_1   1.0000000          0 -0.5408324  0.0322304
        [ 1.000000, 1.000000] [-0.604003,-0.477662]

 FAC_2  -0.5408324  0.0322304  1.0000000          0
        [-0.604003,-0.477662] [ 1.000000, 1.000000]


(1) Standalone Fit Measures: -------------------------------
Fit criterion . . . . . . . . . . . . . . . . . . .    0.0328
Normal Th. Chi-square (df = 19) 12.7332 Prob>chi**2 = 0.8519
Probability of Close Fit  . . . . . . . . . . . .     0.9987
Z-Test of Wilson & Hilferty (1931). . . . . . . .    -1.0467
(2) Incremental Fit Measures: ------------------------------
Null Model Chi-square (df = 28) . . . . . . . . .   4174.0836
RMSEA Estimate  . . . . . . .  0.0000  90%C.I.[ . ,  0.0251]
ECVI Estimate . . . . . . . .  0.0792  90%C.I.[ . ,  0.1279]
McDonald's (1989) Centrality. . . . . . . . . . .     1.0081
Tucker-Lewis Coefficient TLI. . . . . . . . . . .     1.0022
Bentler & Bonett's (1980) NFI . . . . . . . . . .     0.9969
Bentler's Comparative Fit Index CFI . . . . . . .     1.0000
Parsimonious NFI (James, Mulaik, & Brett,1982). .     0.6765
```

```
Bollen (1986) Normed Index Rho1 . . . . . . . . . .     0.9955
Bollen (1988) Non-normed Index Delta2 . . . . . .     1.0015
(3) Information Criteria: --------------------------------
Akaike's Information Criterion. . . . . . . . . . .   -25.2668
Bozdogan's (1987) CAIC. . . . . . . . . . . . . . . -119.5748
Schwarz's Bayesian Criterion. . . . . . . . . . . . -100.5748
(4) Other Fit Measures: ----------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .     0.9925
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .     0.6735
GFI Adjusted for Degrees of Freedom (AGFI). . . .     0.9859
Root Mean Square Residual (RMR) . . . . . . . . .     0.0348
Hoelter's (1983) Critical N . . . . . . . . . . .        920
```

## 2.2 Nine Psychological Variables (H & S, 1937): SEM16

```
print "SEM16: Nine Psychological Variables, LISREL VI, p. III.106";
print "Confirmatory Factor Analysis, Data of Holzinger \& Swineford (1937)";
print "EFA: p= 0.6205, chi= 9.95, df=12";


c16 = [ 1. ,
         .318   1. ,
         .436   .419   1. ,
         .335   .234   .323   1. ,
         .304   .157   .283   .722   1. ,
         .326   .195   .350   .714   .685   1. ,
         .116   .057   .056   .203   .246   .170   1. ,
         .314   .145   .229   .095   .181   .113   .585   1. ,
         .489   .239   .361   .309   .345   .280   .408   .512   1. ];
```

This output shows the EFA model fit:

```
****************************************************************
Starting Model Improvement (ChiSq=9.94823 Pvalue=0.620503 DF=12)
****************************************************************
```

The following table presents a summary of the best 15 solutions.

```
**********************************
Table of 20 Best Pattern Solutions
**********************************

N  Npar    DF  Chisquared          P
```

| | | | | |
|---|---|---|---|---|
| 1 | 25 | 20 | 11.2513458 | 0.93944202 |
| 2 | 26 | 19 | 10.7247316 | 0.93275083 |
| 3 | 26 | 19 | 10.7263835 | 0.93269941 |
| 4 | 25 | 20 | 11.5091596 | 0.93193552 |
| 5 | 24 | 21 | 12.3712894 | 0.92904775 |
| 6 | 26 | 19 | 10.9716580 | 0.92478811 |
| 7 | 26 | 19 | 10.9826233 | 0.92442157 |
| 8 | 25 | 20 | 11.8806178 | 0.92012372 |
| 9 | 27 | 18 | 10.3471365 | 0.91998450 |
| 10 | 27 | 18 | 10.3785326 | 0.91884778 |
| 11 | 26 | 19 | 11.1772709 | 0.91773130 |
| 12 | 27 | 18 | 10.4235965 | 0.91719925 |
| 13 | 27 | 18 | 10.5682819 | 0.91177126 |
| 14 | 27 | 18 | 10.5805499 | 0.91130155 |
| 15 | 24 | 21 | 12.9248614 | 0.91123297 |
| 16 | 25 | 20 | 12.1851493 | 0.90955583 |
| 17 | 27 | 18 | 10.7111720 | 0.90620882 |
| 18 | 25 | 20 | 12.4203547 | 0.90084840 |
| 19 | 27 | 18 | 10.8867485 | 0.89910059 |
| 20 | 26 | 19 | 11.7020850 | 0.89796157 |

| N | RMSEA | ECVI | SRMR | PGFI | SBC |
|---|---|---|---|---|---|
| 1 | 0 | 0.45126867 | 0.02525887 | 0.54589698 | -88.2833290 |
| 2 | 0 | 0.46253700 | 0.02167876 | 0.51911584 | -83.8332095 |
| 3 | 0 | 0.46254848 | 0.02210502 | 0.51902253 | -83.8315576 |
| 4 | 0 | 0.45305905 | 0.02650475 | 0.54573851 | -88.0255153 |
| 5 | 0 | 0.44412069 | 0.02785032 | 0.57213963 | -92.1401191 |
| 6 | 0 | 0.46425177 | 0.02411269 | 0.51881167 | -83.5862831 |
| 7 | 0 | 0.46432792 | 0.02432238 | 0.51885331 | -83.5753178 |
| 8 | 0 | 0.45563862 | 0.03146928 | 0.54548695 | -87.6540570 |
| 9 | 0 | 0.47484019 | 0.02137929 | 0.49195998 | -79.2340709 |
| 10 | 0 | 0.47505822 | 0.02066109 | 0.49198665 | -79.2026748 |
| 11 | 0 | 0.46567964 | 0.02510962 | 0.51866077 | -83.3806702 |
| 12 | 0 | 0.47537116 | 0.02127910 | 0.49195562 | -79.1576109 |
| 13 | 0 | 0.47637592 | 0.02154743 | 0.49178819 | -79.0129255 |
| 14 | 0 | 0.47646112 | 0.02341660 | 0.49182332 | -79.0006575 |
| 15 | 0 | 0.44796494 | 0.03152857 | 0.57172483 | -91.5865472 |
| 16 | 0 | 0.45775342 | 0.02736283 | 0.54508076 | -87.3495256 |
| 17 | 0 | 0.47736821 | 0.02289020 | 0.49166657 | -78.8700354 |
| 18 | 0 | 0.45938679 | 0.02754853 | 0.54484200 | -87.1143201 |
| 19 | 0 | 0.47858749 | 0.02522464 | 0.49153255 | -78.6944589 |
| 20 | 0 | 0.46932418 | 0.03001944 | 0.51834954 | -82.8558561 |

The following shows details of the best solution found:

```
Loadings and Uni. Vars. with ASE's and Wald CIs
-----------------------------------------------

                    FAC_1                   FAC_2
V1   0.6987112  0.0857264  0.0937473  0.0842428
    [ 0.530690, 0.866732] [-0.071366, 0.258860]

V2   0.4881935  0.0903112          0          0
    [ 0.311187, 0.665200] [        0,        0]

V3   0.6718046  0.0869520          0          0
    [ 0.501382, 0.842227] [        0,        0]

V4           0          0          0          0
    [        0,        0] [        0,        0]

V5           0          0  0.1143820  0.0630808
    [        0,        0] [-9.3e-003, 0.238018]

V6           0          0          0          0
    [        0,        0] [        0,        0]

V7           0          0  0.7712926  0.1010557
    [        0,        0] [ 0.573227, 0.969358]

V8   0.4197159  0.1058744  0.7332916  0.0970976
    [ 0.212206, 0.627226] [ 0.542984, 0.923599]

V9   0.5893710  0.0858883  0.4441484  0.0825194
    [ 0.421033, 0.757709] [ 0.282413, 0.605883]


                    FAC_3                   U_Var
V1           0          0  0.5046143  0.0853445
    [        0,        0] [ 0.337342, 0.671886]

V2           0          0  0.7616671  0.1000075
    [        0,        0] [ 0.565656, 0.957678]

V3           0          0  0.5486786  0.0901259
    [        0,        0] [ 0.372035, 0.725322]

V4   0.8738382  0.0698783  0.2364069  0.0507738
    [ 0.736879, 1.010797] [ 0.136892, 0.335922]

V5   0.8219073  0.0714973  0.3088296  0.0522597
```

```
    [ 0.681775, 0.962039] [ 0.206402, 0.411257]

V6   0.8242760  0.0716267  0.3205690  0.0539320
    [ 0.683890, 0.964662] [ 0.214864, 0.426274]

V7   0.1991920  0.0852976  0.3612541  0.1222980
    [ 0.032012, 0.366372] [ 0.121554, 0.600954]

V8  -0.1246921  0.0967887  0.3383583  0.0955084
    [-0.314394, 0.065010] [ 0.151165, 0.525551]

V9           0          0  0.4617680  0.0719642
    [        0,         0] [ 0.320721, 0.602815]


             Factor Correlations Phi
             -----------------------

                       FAC_1                 FAC_2
FAC_1   1.0000000          0          0          0
      [ 1.000000, 1.000000] [        0,         0]

FAC_2           0          0  1.0000000          0
      [        0,         0] [ 1.000000, 1.000000]

FAC_3   0.5520284  0.0801574          0          0
      [ 0.394923, 0.709134] [        0,         0]

                                  FAC_3
           FAC_1   0.5520284  0.0801574
                 [ 0.394923, 0.709134]

           FAC_2           0          0
                 [        0,         0]

           FAC_3   1.0000000          0
                 [ 1.000000, 1.000000]


(1) Standalone Fit Measures: -------------------------------
Fit criterion . . . . . . . . . . . . . . . . . .     0.0781
Normal Th. Chi-square (df = 20) 11.2513 Prob>chi**2 = 0.9394
Normal Theory Reweighted LS Chi-square  . . . . .    11.4651
Probability of Close Fit  . . . . . . . . . . . .     0.9916
Z-Test of Wilson & Hilferty (1931). . . . . . . .    -1.5499
(2) Incremental Fit Measures: ------------------------------
```

```
Null Model Chi-square (df = 36) . . . . . . . .     496.6694
RMSEA Estimate  . . . . . . .  0.0000  90%C.I.[ . ,  0.0159]
ECVI Estimate . . . . . . . . . . . . . . . . . . .  0.4513
McDonald's (1989) Centrality. . . . . . . . . . .    1.0306
Tucker-Lewis Coefficient TLI. . . . . . . . . . .    1.0342
Bentler & Bonett's (1980) NFI . . . . . . . . . .    0.9773
Bentler's Comparative Fit Index CFI . . . . . . .    1.0000
Parsimonious NFI (James, Mulaik, & Brett,1982). .    0.5430
Bollen (1986) Normed Index Rho1 . . . . . . . . .    0.9592
Bollen (1988) Non-normed Index Delta2 . . . . . .    1.0184
(3) Information Criteria: --------------------------------
Akaike's Information Criterion. . . . . . . . . .  -28.7487
Bozdogan's (1987) CAIC. . . . . . . . . . . . . . -108.2833
Schwarz's Bayesian Criterion. . . . . . . . . . .  -88.2833
(4) Other Fit Measures: ----------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .    0.9826
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .    0.5459
GFI Adjusted for Degrees of Freedom (AGFI). . . .    0.9609
Root Mean Square Residual (RMR) . . . . . . . . .    0.0253
Hoelter's (1983) Critical N . . . . . . . . . . .       404
```

```
                Standardized Factor Loadings

                FAC_1      FAC_2      FAC_3
           V1   0.6981528  0.0936724          0
           V2   0.4881935          0          0
           V3   0.6718046          0          0
           V4           0          0  0.8738382
           V5           0  0.1145285  0.8229595
           V6           0          0  0.8242760
           V7           0  0.7729082  0.1996092
           V8   0.4176322  0.7296512 -0.1240731
           V9   0.5874958  0.4427352          0
```

```
              Squared Multiple Correlations

        N Variable ErrVariance TotVariance   R-squared
        1 V1        0.50461428  1.00160011  0.49619186
        2 V2        0.76166711  1.00000000  0.23833289
        3 V3        0.54867857  1.00000000  0.45132143
        4 V4        0.23640687  1.00000000  0.76359313
        5 V5        0.30882955  0.99744437  0.69037917
        6 V6        0.32056901  1.00000000  0.67943099
        7 V7        0.36125406  0.99582374  0.63723093
```

```
        8 V8        0.33835828  1.01000332  0.66499291
        9 V9        0.46176796  1.00639398  0.54116582
     Total Determination of All Equations = 0.992
  Total Determination of Manifest Variables = 0.991
```

The CFA model is reported in the LISREL VI Manual and also shown in `sem16.inp`:

```
l = [  .  0. 0.,  .  0. 0.,  .  0. 0.,
      0.   . 0., 0.   . 0., 0.   . 0.,
      0.  0.  ., 0.  0.  .,  .  0.  .  ];
p = [ 1. , . 1., .  .  1. ];
u = [ 9 # . ];
list patt;
patt[1] = l; patt[2] = (tri2sym)p; patt[3] = u;
```

```
         Loadings and Uni. Vars. with ASE's and Wald CIs
         -----------------------------------------------

                              FAC_1                   FAC_2
         V1   0.7081902  0.0868147          0          0
             [ 0.538036, 0.878344] [        0,         0]

         V2   0.4833306  0.0907553          0          0
             [ 0.305453, 0.661208] [        0,         0]

         V3   0.6498856  0.0874669          0          0
             [ 0.478454, 0.821318] [        0,         0]

         V4           0          0  0.8679084  0.0701579
             [        0,         0] [ 0.730402, 1.005415]

         V5           0          0  0.8298926  0.0714842
             [        0,         0] [ 0.689786, 0.969999]

         V6           0          0  0.8250112  0.0716510
             [        0,         0] [ 0.684578, 0.965445]

         V7           0          0          0          0
             [        0,         0] [        0,         0]

         V8           0          0          0          0
             [        0,         0] [        0,         0]
```

```
 V9   0.4566634  0.0886175          0          0
     [ 0.282976, 0.630350] [        0,        0]


                    FAC_3                  U_Var
 V1           0          0  0.4984667  0.0901823
     [        0,         0] [ 0.321713, 0.675221]

 V2           0          0  0.7663915  0.1006133
     [        0,         0] [ 0.569193, 0.963590]

 V3           0          0  0.5776487  0.0911299
     [        0,         0] [ 0.399037, 0.756260]

 V4           0          0  0.2467350  0.0513112
     [        0,         0] [ 0.146167, 0.347303]

 V5           0          0  0.3112783  0.0536661
     [        0,         0] [ 0.206095, 0.416462]

 V6           0          0  0.3193565  0.0540504
     [        0,         0] [ 0.213420, 0.425293]

 V7   0.6808743  0.0886616  0.5364102  0.0931701
     [ 0.507101, 0.854648] [ 0.353800, 0.719020]

 V8   0.8591342  0.0916707  0.2618885  0.1132573
     [ 0.679463, 1.038805] [ 0.039908, 0.483869]

 V9   0.4188227  0.0884722  0.4669924  0.0726962
     [ 0.245420, 0.592225] [ 0.324511, 0.609474]


             Factor Correlations Phi
             -----------------------

                      FAC_1                  FAC_2
FAC_1   1.0000000          0  0.5574397  0.0811712
       [ 1.000000, 1.000000] [ 0.398347, 0.716532]

FAC_2   0.5574397  0.0811712  1.0000000          0
       [ 0.398347, 0.716532] [ 1.000000, 1.000000]

FAC_3   0.3896605  0.1045577  0.2229733  0.0960244
       [ 0.184731, 0.594590] [ 0.034769, 0.411178]
```

```
                       FAC_3
FAC_1   0.3896605  0.1045577
       [ 0.184731, 0.594590]

FAC_2   0.2229733  0.0960244
       [ 0.034769, 0.411178]

FAC_3   1.0000000          0
       [ 1.000000, 1.000000]


(1) Standalone Fit Measures: -------------------------------
Fit criterion . . . . . . . . . . . . . . . . . .    0.2014
Normal Th. Chi-square (df = 23) 29.0077 Prob>chi**2 = 0.1801
Normal Theory Reweighted LS Chi-square  . . . . .   28.7441
Probability of Close Fit  . . . . . . . . . . . .    0.5674
Z-Test of Wilson & Hilferty (1931). . . . . . . .    0.9165
(2) Incremental Fit Measures: ------------------------------
Null Model Chi-square (df = 36) . . . . . . . .    496.6694
RMSEA Estimate  . . . . . . .  0.0426  90%C.I.[ . ,  0.0851]
ECVI Estimate . . . . . . . .  0.5298  90%C.I.[ . ,  0.6580]
McDonald's (1989) Centrality. . . . . . . . . . .    0.9795
Tucker-Lewis Coefficient TLI. . . . . . . . . . .    0.9796
Bentler & Bonett's (1980) NFI . . . . . . . . . .    0.9416
Bentler's Comparative Fit Index CFI . . . . . . .    0.9870
Parsimonious NFI (James, Mulaik, & Brett,1982). .    0.6016
Bollen (1986) Normed Index Rho1 . . . . . . . . .    0.9086
Bollen (1988) Non-normed Index Delta2 . . . . . .    0.9873
(3) Information Criteria: ----------------------------------
Akaike's Information Criterion. . . . . . . . . .  -16.9923
Bozdogan's (1987) CAIC. . . . . . . . . . . . . . -108.4571
Schwarz's Bayesian Criterion. . . . . . . . . . .  -85.4571
(4) Other Fit Measures: ------------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .    0.9575
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .    0.6118
GFI Adjusted for Degrees of Freedom (AGFI). . . .    0.9169
Root Mean Square Residual (RMR) . . . . . . . . .    0.0451
Hoelter's (1983) Critical N . . . . . . . . . . .       176


                Standardized Factor Loadings

            FAC_1       FAC_2       FAC_3
   V1   0.7081902           0           0
   V2   0.4833306           0           0
```

```
V3   0.6498856          0          0
V4           0  0.8679084          0
V5           0  0.8298926          0
V6           0  0.8250112          0
V7           0          0  0.6808743
V8           0          0  0.8591342
V9   0.4566634          0  0.4188227
```

```
              Squared Multiple Correlations

      N Variable ErrVariance TotVariance   R-squared
      1 V1        0.49846668  1.00000000  0.50153332
      2 V2        0.76639154  1.00000000  0.23360846
      3 V3        0.57764865  1.00000000  0.42235135
      4 V4        0.24673495  1.00000000  0.75326505
      5 V5        0.31127826  1.00000000  0.68872174
      6 V6        0.31935650  1.00000000  0.68064350
      7 V7        0.53641025  1.00000000  0.46358975
      8 V8        0.26188849  1.00000000  0.73811151
      9 V9        0.46699238  1.00000000  0.53300762


  Total Determination of All Equations = 0.991
Total Determination of Manifest Variables = 0.989
```

## 2.3 Data by Thurstone (1931): SEM09

```
thur = [ 1. ,
         .828  1. ,
         .776   .779  1. ,
         .439   .493   .460  1. ,
         .432   .464   .425   .674  1. ,
         .447   .489   .443   .590   .541  1. ,
         .447   .432   .401   .381   .402   .288  1. ,
         .541   .537   .534   .350   .367   .320   .555  1. ,
         .380   .358   .359   .424   .446   .325   .598   .452  1. ];
```

This output shows the EFA model fit:

```
**************************************************************
Starting Model Improvement (ChiSq=2.9032 Pvalue=0.99619 DF=12)
**************************************************************
```

*********************************
Table of 20 Best Pattern Solutions
*********************************

| N | Npar | DF | Chisquared | P |
|---|---|---|---|---|
| 1 | 29 | 16 | 3.10652944 | 0.99978601 |
| 2 | 28 | 17 | 3.65139986 | 0.99972255 |
| 3 | 29 | 16 | 3.23210757 | 0.99972182 |
| 4 | 30 | 15 | 2.97690527 | 0.99961664 |
| 5 | 30 | 15 | 2.98291727 | 0.99961181 |
| 6 | 30 | 15 | 2.98343774 | 0.99961139 |
| 7 | 27 | 18 | 4.30652327 | 0.99959675 |
| 8 | 29 | 16 | 3.48509077 | 0.99954465 |
| 9 | 29 | 16 | 3.48509077 | 0.99954465 |
| 10 | 30 | 15 | 3.06858017 | 0.99953741 |
| 11 | 30 | 15 | 3.07668316 | 0.99952981 |
| 12 | 29 | 16 | 3.50299596 | 0.99952927 |
| 13 | 30 | 15 | 3.09050625 | 0.99951663 |
| 14 | 30 | 15 | 3.09050627 | 0.99951663 |
| 15 | 29 | 16 | 3.53120039 | 0.99950420 |
| 16 | 30 | 15 | 3.11376352 | 0.99949378 |
| 17 | 28 | 17 | 4.02829078 | 0.99945776 |
| 18 | 29 | 16 | 3.59905747 | 0.99943942 |
| 19 | 29 | 16 | 3.59905748 | 0.99943942 |
| 20 | 30 | 15 | 3.20987961 | 0.99938997 |

| N | RMSEA | ECVI | SRMR | PGFI | SBC |
|---|---|---|---|---|---|
| 1 | 0 | 0.30178215 | 0.00624752 | 0.44301711 | -82.6741452 |
| 2 | 0 | 0.29445131 | 0.00831949 | 0.47042501 | -87.4905670 |
| 3 | 0 | 0.30237450 | 0.00662794 | 0.44295734 | -82.5485671 |
| 4 | 0 | 0.31107171 | 0.00573816 | 0.41538856 | -77.4424772 |
| 5 | 0 | 0.31110007 | 0.00579723 | 0.41538446 | -77.4364652 |
| 6 | 0 | 0.31110252 | 0.00581218 | 0.41538478 | -77.4359447 |
| 7 | 0 | 0.28764052 | 0.00986447 | 0.49774912 | -92.1967357 |
| 8 | 0 | 0.30356782 | 0.00716500 | 0.44284805 | -82.2955839 |
| 9 | 0 | 0.30356782 | 0.00716498 | 0.44284805 | -82.2955839 |
| 10 | 0 | 0.31150414 | 0.00614370 | 0.41534420 | -77.3508023 |
| 11 | 0 | 0.31154236 | 0.00571654 | 0.41534617 | -77.3426993 |
| 12 | 0 | 0.30365228 | 0.00914108 | 0.44283839 | -82.2776787 |
| 13 | 0 | 0.31160756 | 0.00627418 | 0.41533361 | -77.3288762 |
| 14 | 0 | 0.31160756 | 0.00627426 | 0.41533362 | -77.3288762 |
| 15 | 0 | 0.30378532 | 0.00790744 | 0.44281245 | -82.2494743 |
| 16 | 0 | 0.31171727 | 0.00620186 | 0.41533461 | -77.3056190 |

```
17          0  0.29622909  0.00992466  0.47025539 -87.1136760
18          0  0.30410540  0.00812188  0.44277769 -82.1816172
19          0  0.30410540  0.00812215  0.44277769 -82.1816172
20          0  0.31217064  0.00668120  0.41528353 -77.2095029
```

The following shows details of the best solution found:

```
          Loadings and Uni. Vars. with ASE's and Wald CIs
          -----------------------------------------------

                           FAC_1                    FAC_2
         OBS1   0.9471675  0.0617547          0          0
               [ 0.826131, 1.068204] [        0,         0]

         OBS2   0.9130025  0.0538786          0          0
               [ 0.807402, 1.018603] [        0,         0]

         OBS3   0.8560433  0.0559666          0          0
               [ 0.746351, 0.965736] [        0,         0]

         OBS4           0          0 -0.4002501  0.0935147
               [        0,         0] [-0.583536,-0.216965]

         OBS5           0          0 -0.4411033  0.0888934
               [        0,         0] [-0.615331,-0.266876]

         OBS6   0.1814241  0.0824835 -0.2078905  0.0880482
               [ 0.019759, 0.343089] [-0.380462,-0.035319]

         OBS7   0.1135365  0.1196842 -0.7938235  0.1086546
               [-0.121040, 0.348113] [-1.006783,-0.580864]

         OBS8   0.4334076  0.0940989 -0.4145607  0.0758386
               [ 0.248977, 0.617838] [-0.563202,-0.265920]

         OBS9           0          0 -0.6962829  0.0770788
               [        0,         0] [-0.847354,-0.545211]


         OBS1  -0.0868570  0.0521968  0.1730363  0.0289007
               [-0.189161, 0.015447] [ 0.116392, 0.229681]

         OBS2           0          0  0.1669560  0.0270921
               [        0,         0] [ 0.113856, 0.220056]
```

```
 OBS3          0          0  0.2676555  0.0329201
      [        0,        0] [ 0.203133, 0.332178]

 OBS4   0.7571450  0.0701888  0.2674337  0.0583942
      [ 0.619577, 0.894713] [ 0.152983, 0.381884]

 OBS5   0.6587992  0.0737304  0.3722769  0.0532866
      [ 0.514290, 0.803308] [ 0.267837, 0.476717]

 OBS6   0.5417772  0.0829298  0.5043147  0.0596782
      [ 0.379238, 0.704317] [ 0.387348, 0.621282]

 OBS7          0          0  0.2751764  0.0987070
      [        0,        0] [ 0.081714, 0.468639]

 OBS8  -0.0569989  0.0775350  0.4972177  0.0581030
      [-0.208965, 0.094967] [ 0.383338, 0.611098]

 OBS9   0.1983391  0.0878593  0.4762630  0.0714034
      [ 0.026138, 0.370540] [ 0.336315, 0.616211]

           Factor Correlations Phi
            -----------------------

                     FAC_1                 FAC_2
FAC_1   1.0000000          0 -0.4544176  0.1157565
      [ 1.000000, 1.000000] [-0.681296,-0.227539]

FAC_2  -0.4544176  0.1157565  1.0000000          0
      [-0.681296,-0.227539] [ 1.000000, 1.000000]

FAC_3   0.4694901  0.1007843          0          0
      [ 0.271957, 0.667024] [        0,        0]

                                FAC_3
           FAC_1   0.4694901  0.1007843
                 [ 0.271957, 0.667024]

           FAC_2          0          0
                 [        0,        0]

           FAC_3   1.0000000          0
                 [ 1.000000, 1.000000]

(1) Standalone Fit Measures: -------------------------------
```

```
Fit criterion . . . . . . . . . . . . . . . . . . .     0.0147
Normal Th. Chi-square (df = 16)  3.1065 Prob>chi**2 = 0.9998
Normal Theory Reweighted LS Chi-square  . . . . .     3.0736
Probability of Close Fit  . . . . . . . . . . . .     1.0000
Z-Test of Wilson & Hilferty (1931). . . . . . . .    -3.4540
(2) Incremental Fit Measures: -----------------------------
Null Model Chi-square (df = 36) . . . . . . . . .  1101.8924
RMSEA Estimate  . . . . . . . . . . . . . . . . . .   0.0000
ECVI Estimate . . . . . . . . . . . . . . . . . . .   0.3018
McDonald's (1989) Centrality. . . . . . . . . . .     1.0307
Tucker-Lewis Coefficient TLI. . . . . . . . . . .     1.0272
Bentler & Bonett's (1980) NFI . . . . . . . . . .     0.9972
Bentler's Comparative Fit Index CFI . . . . . . .     1.0000
Parsimonious NFI (James, Mulaik, & Brett,1982). .     0.4432
Bollen (1986) Normed Index Rho1 . . . . . . . . .     0.9937
Bollen (1988) Non-normed Index Delta2 . . . . . .     1.0119
(3) Information Criteria: ---------------------------------
Akaike's Information Criterion. . . . . . . . . .   -28.8935
Bozdogan's (1987) CAIC. . . . . . . . . . . . . .   -98.6741
Schwarz's Bayesian Criterion. . . . . . . . . . .   -82.6741
(4) Other Fit Measures: -----------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .     0.9968
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .     0.4430
GFI Adjusted for Degrees of Freedom (AGFI). . . .     0.9910
Root Mean Square Residual (RMR) . . . . . . . . .     0.0062
Hoelter's (1983) Critical N . . . . . . . . . . .       1796
```

Standardized Factor Loadings

| | FAC_1 | FAC_2 | FAC_3 |
|---|---|---|---|
| OBS1 | 0.9469504 | 0 | -0.0868371 |
| OBS2 | 0.9127608 | 0 | 0 |
| OBS3 | 0.8558441 | 0 | 0 |
| OBS4 | 0 | -0.4000696 | 0.7568036 |
| OBS5 | 0 | -0.4409126 | 0.6585143 |
| OBS6 | 0.1813749 | -0.2078342 | 0.5416304 |
| OBS7 | 0.1135289 | -0.7937702 | 0 |
| OBS8 | 0.4333498 | -0.4145054 | -0.0569913 |
| OBS9 | 0 | -0.6961397 | 0.1982984 |

Squared Multiple Correlations

| N | Variable | ErrVariance | TotVariance | R-squared |
|---|---|---|---|---|
| 1 | OBS1 | 0.17303634 | 1.00045863 | 0.82704299 |

```
    2 OBS2      0.16695601  1.00052958  0.83313236
    3 OBS3      0.26765547  1.00046555  0.73246907
    4 OBS4      0.26743374  1.00090241  0.73280738
    5 OBS5      0.37227686  1.00086534  0.62804501
    6 OBS6      0.50431474  1.00054204  0.49595848
    7 OBS7      0.27517642  1.00013413  0.72486049
    8 OBS8      0.49721772  1.00026677  0.50291489
    9 OBS9      0.47626301  1.00041126  0.52393278
  Total Determination of All Equations = 0.995
Total Determination of Manifest Variables = 0.995
```

McDONALD (1985, p.57) is using the following CFA Model where all entries of **L** are zeroed except $L[1,1], L[2,1], L[3,1], L[4,2], L[5,2], L[6,2], L[7,3], L[8,3], L[9,3]$ and all offdiagonal entries of **P** are parameters. This model is also specified in `sem09.inp`

```
      Loadings and Uni. Vars. with ASE's and Wald CIs
      -----------------------------------------------

                        FAC_1                 FAC_2
     OBS1   0.9047110  0.0542156          0          0
          [ 0.798450, 1.010972] [        0,         0]

     OBS2   0.9138216  0.0538657          0          0
          [ 0.808247, 1.019396] [        0,         0]

     OBS3   0.8560762  0.0560231          0          0
          [ 0.746273, 0.965879] [        0,         0]

     OBS4           0          0  0.8357615  0.0607813
          [        0,         0] [ 0.716632, 0.954891]

     OBS5           0          0  0.7971823  0.0617654
          [        0,         0] [ 0.676124, 0.918240]

     OBS6           0          0  0.7025557  0.0642509
          [        0,         0] [ 0.576626, 0.828485]

     OBS7           0          0          0          0
          [        0,         0] [        0,         0]

     OBS8           0          0          0          0
          [        0,         0] [        0,         0]

     OBS9           0          0          0          0
```

```
      [        0,        0] [        0,        0]

                     FAC_3               U_Var
 OBS1          0         0  0.1814980  0.0284775
      [        0,        0] [ 0.125683, 0.237313]

 OBS2          0         0  0.1649300  0.0277694
      [        0,        0] [ 0.110503, 0.219357]

 OBS3          0         0  0.2671335  0.0333634
      [        0,        0] [ 0.201742, 0.332525]

 OBS4          0         0  0.3015026  0.0510219
      [        0,        0] [ 0.201502, 0.401504]

 OBS5          0         0  0.3645004  0.0526354
      [        0,        0] [ 0.261337, 0.467664]

 OBS6          0         0  0.5064156  0.0596261
      [        0,        0] [ 0.389551, 0.623281]

 OBS7   0.7808184  0.0642916  0.3903225  0.0593363
      [ 0.654809, 0.906828] [ 0.274026, 0.506620]

 OBS8   0.7201523  0.0656347  0.4813806  0.0622489
      [ 0.591511, 0.848794] [ 0.359375, 0.603386]

 OBS9   0.7034975  0.0660291  0.5050912  0.0633281
      [ 0.574083, 0.832912] [ 0.380970, 0.629212]

              Factor Correlations Phi
              -----------------------

                     FAC_1               FAC_2
FAC_1   1.0000000         0  0.6426983  0.0505065
      [ 1.000000, 1.000000] [ 0.543707, 0.741689]

FAC_2   0.6426983  0.0505065  1.0000000         0
      [ 0.543707, 0.741689] [ 1.000000, 1.000000]

FAC_3   0.6699934  0.0510583  0.6371772  0.0585959
      [ 0.569921, 0.770066] [ 0.522331, 0.752023]

                                 FAC_3
```

```
FAC_1   0.6699934  0.0510583
       [ 0.569921, 0.770066]

FAC_2   0.6371772  0.0585959
       [ 0.522331, 0.752023]

FAC_3   1.0000000          0
       [ 1.000000, 1.000000]


(1) Standalone Fit Measures: -------------------------------
Fit criterion . . . . . . . . . . . . . . . . . .     0.1802
Normal Th. Chi-square (df = 24) 38.1963 Prob>chi**2 = 0.0331
Normal Theory Reweighted LS Chi-square  . . . . .    40.1947
Probability of Close Fit  . . . . . . . . . . . .     0.4088
Z-Test of Wilson & Hilferty (1931). . . . . . . .     1.8373
(2) Incremental Fit Measures: ------------------------------
Null Model Chi-square (df = 36) . . . . . . . . .  1101.8924
RMSEA Estimate  . . . . .  0.0528  90%C.I.[ 0.0153,  0.0831]
ECVI Estimate . . . . . . . .  0.3881  90%C.I.[ . ,  0.4888]
McDonald's (1989) Centrality. . . . . . . . . . .     0.9672
Tucker-Lewis Coefficient TLI. . . . . . . . . . .     0.9800
Bentler & Bonett's (1980) NFI . . . . . . . . . .     0.9653
Bentler's Comparative Fit Index CFI . . . . . . .     0.9867
Parsimonious NFI (James, Mulaik, & Brett,1982). .     0.6436
Bollen (1986) Normed Index Rho1 . . . . . . . . .     0.9480
Bollen (1988) Non-normed Index Delta2 . . . . . .     0.9868
(3) Information Criteria: ----------------------------------
Akaike's Information Criterion. . . . . . . . . .    -9.8037
Bozdogan's (1987) CAIC. . . . . . . . . . . . . .  -114.4747
Schwarz's Bayesian Criterion. . . . . . . . . . .   -90.4747
(4) Other Fit Measures: ------------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .     0.9596
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .     0.6397
GFI Adjusted for Degrees of Freedom (AGFI). . . .     0.9242
Root Mean Square Residual (RMR) . . . . . . . . .     0.0436
Hoelter's (1983) Critical N . . . . . . . . . . .        204


                Standardized Factor Loadings

                  FAC_1       FAC_2       FAC_3
           OBS1   0.9047110           0           0
           OBS2   0.9138216           0           0
           OBS3   0.8560762           0           0
           OBS4           0   0.8357615           0
```

```
OBS5           0  0.7971823          0
OBS6           0  0.7025557          0
OBS7           0          0  0.7808184
OBS8           0          0  0.7201523
OBS9           0          0  0.7034975
```

```
        Squared Multiple Correlations

N Variable ErrVariance TotVariance   R-squared
1 OBS1      0.18149802  1.00000000  0.81850198
2 OBS2      0.16493005  1.00000000  0.83506995
3 OBS3      0.26713349  1.00000000  0.73286651
4 OBS4      0.30150265  1.00000000  0.69849735
5 OBS5      0.36450044  1.00000000  0.63549956
6 OBS6      0.50641556  1.00000000  0.49358444
7 OBS7      0.39032255  1.00000000  0.60967745
8 OBS8      0.48138061  1.00000000  0.51861939
9 OBS9      0.50509124  1.00000000  0.49490876


  Total Determination of All Equations = 0.994
Total Determination of Manifest Variables = 0.992
```

## 2.4 Speed factor Data by Lord(1956): SEM33

The automatic model improvement algorithm starting at the $f = 6$ factor EFA obtains:

```
*****************************************************************
Starting Model Improvement (ChiSq=77.2481 Pvalue=0.0662436 DF=60)
*****************************************************************


                *********************************
                Table of 20 Best Pattern Solutions
                *********************************

                N  Npar    DF  Chisquared           P

                1    81    90  83.5408048  0.67127810
                2    80    91  84.5127706  0.67113528
                3    82    89  82.5913392  0.67078703
                4    79    92  85.6235961  0.66710531
                5    82    89  82.7220320  0.66705714
                6    80    91  84.6690951  0.66672585
```

| | | | | |
|---|---|---|---|---|
| 7 | 83 | 88 | 81.8036686 | 0.66564614 |
| 8 | 78 | 93 | 86.8054509 | 0.66111865 |
| 9 | 81 | 90 | 83.9222530 | 0.66042589 |
| 10 | 83 | 88 | 82.0392250 | 0.65884309 |
| 11 | 83 | 88 | 82.0392426 | 0.65884258 |
| 12 | 84 | 87 | 81.1057439 | 0.65781014 |
| 13 | 80 | 91 | 85.0045103 | 0.65720598 |
| 14 | 82 | 89 | 83.2192018 | 0.65275594 |
| 15 | 80 | 91 | 85.2134775 | 0.65123653 |
| 16 | 82 | 89 | 83.3072553 | 0.65020554 |
| 17 | 82 | 89 | 83.3196657 | 0.64984568 |
| 18 | 83 | 88 | 82.3715996 | 0.64917838 |
| 19 | 81 | 90 | 84.3317432 | 0.64866368 |
| 20 | 81 | 90 | 84.3727857 | 0.64747878 |

| N | RMSEA | ECVI | SRMR | PGFI | SBC |
|---|---|---|---|---|---|
| 1 | 0 | 0.38647266 | 0.00966930 | 0.57988505 | -499.248140 |
| 2 | 0 | 0.38479296 | 0.01033847 | 0.58623987 | -504.751607 |
| 3 | 0 | 0.38818709 | 0.00951834 | 0.57350654 | -493.722173 |
| 4 | 0 | 0.38332755 | 0.00976527 | 0.59250184 | -510.116214 |
| 5 | 0 | 0.38838878 | 0.00972279 | 0.57352962 | -493.591480 |
| 6 | 0 | 0.38503420 | 0.00926660 | 0.58612022 | -504.595282 |
| 7 | 0 | 0.39015120 | 0.00942374 | 0.56714585 | -488.034411 |
| 8 | 0 | 0.38197175 | 0.01060256 | 0.59883830 | -515.409792 |
| 9 | 0 | 0.38706132 | 0.01055991 | 0.57982539 | -498.866692 |
| 10 | 0 | 0.39051471 | 0.00956565 | 0.56714384 | -487.798854 |
| 11 | 0 | 0.39051474 | 0.00956506 | 0.56714371 | -487.798836 |
| 12 | 0 | 0.39225381 | 0.00939487 | 0.56075812 | -482.256902 |
| 13 | 0 | 0.38555182 | 0.00993579 | 0.58613672 | -504.259867 |
| 14 | 0 | 0.38915601 | 0.00918856 | 0.57344398 | -493.094310 |
| 15 | 0 | 0.38587430 | 0.00954506 | 0.58610865 | -504.050900 |
| 16 | 0 | 0.38929190 | 0.00994625 | 0.57344592 | -493.006256 |
| 17 | 0 | 0.38931105 | 0.00995461 | 0.57343891 | -492.993846 |
| 18 | 0 | 0.39102764 | 0.00893986 | 0.56708063 | -487.466480 |
| 19 | 0 | 0.38769325 | 0.00993271 | 0.57978047 | -498.457201 |
| 20 | 0 | 0.38775659 | 0.00936871 | 0.57975575 | -498.416159 |

The following shows details of the best solution found:

```
Loadings and Uni. Vars. with ASE's and Wald CIs
-----------------------------------------------

                    FAC_1                FAC_2
```

```
V1    6.0719262  0.3200259          0          0
     [ 5.444687, 6.699166] [        0,         0]

V2    6.2437766  0.3100025  2.1423596  0.9009982
     [ 5.636183, 6.851370] [ 0.376436, 3.908284]

V3    6.8310094  0.3172840          0          0
     [ 6.209144, 7.452875] [        0,         0]

V4    6.9658948  0.3072605          0          0
     [ 6.363675, 7.568114] [        0,         0]

V5    7.6929515  0.2810391 -1.2485381  0.6242692
     [ 7.142125, 8.243778] [-2.472083,-0.024993]

V6    7.7870139  0.2891260          0          0
     [ 7.220337, 8.353690] [        0,         0]

V7            0          0          0          0
     [        0,         0] [        0,         0]

V8            0          0  1.8166981  0.5691888
     [        0,         0] [ 0.701109, 2.932288]

V9    0.2220296  0.2247509  2.4150871  0.5772850
     [-0.218474, 0.662533] [ 1.283629, 3.546545]

V10   0.2766660  0.2281057          0          0
     [-0.170413, 0.723745] [        0,         0]

V11           0          0          0          0
     [        0,         0] [        0,         0]

V12  -0.2980340  0.2101544          0          0
     [-0.709929, 0.113861] [        0,         0]

V13           0          0  4.5067282  1.1895548
     [        0,         0] [ 2.175244, 6.838213]

V14           0          0  2.9739798  1.0806625
     [        0,         0] [ 0.855920, 5.092039]

V15           0          0  8.1041119  0.3947306
     [        0,         0] [ 7.330454, 8.877770]

V16           0          0  1.8274981  0.9398649
```

```
     [        0,        0] [-0.014603, 3.669599]

V17          0          0          0          0
     [        0,        0] [        0,        0]

V18          0          0          0          0
     [        0,        0] [        0,        0]


                     FAC_3                FAC_4
V1    3.3150185  0.3912797          0          0
     [ 2.548124, 4.081913] [        0,        0]

V2    2.6437782  0.3597777  0.7592340  0.2513204
     [ 1.938627, 3.348930] [ 0.266655, 1.251813]

V3    2.3283132  0.3517641          0          0
     [ 1.638868, 3.017758] [        0,        0]

V4            0          0  0.8004526  0.2404981
     [        0,        0] [ 0.329085, 1.271820]

V5            0          0          0          0
     [        0,        0] [        0,        0]

V6   -1.6157835  0.3859872          0          0
     [-2.372305,-0.859262] [        0,        0]

V7    1.1626474  0.3260419  7.4778976  0.3350734
     [ 0.523617, 1.801678] [ 6.821166, 8.134629]

V8            0          0  8.2038849  0.3240872
     [        0,        0] [ 7.568686, 8.839084]

V9            0          0  8.3908953  0.3321450
     [        0,        0] [ 7.739903, 9.041888]

V10           0          0  7.9178492  0.3382666
     [        0,        0] [ 7.254859, 8.580840]

V11           0          0  8.2027800  0.3341874
     [        0,        0] [ 7.547785, 8.857775]

V12  -0.3922719  0.2638650  8.1946402  0.3479574
     [-0.909438, 0.124894] [ 7.512656, 8.876624]
```

```
V13   1.3362929  0.3819892  1.8252620  0.4760826
     [ 0.587608, 2.084978] [ 0.892157, 2.758367]

V14   0.5979773  0.4200843  2.2408969  0.5030036
     [-0.225373, 1.421327] [ 1.255028, 3.226766]

V15           0          0  1.5797407  0.5364307
     [        0,         0] [ 0.528356, 2.631126]

V16  -0.9700643  0.4003435  1.6302612  0.5414816
     [-1.754723,-0.185405] [ 0.568977, 2.691546]

V17           0          0  2.0031391  0.5203115
     [        0,         0] [ 0.983347, 3.022931]

V18  -0.6202871  0.4172748  1.0923294  0.5254678
     [-1.438131, 0.197556] [ 0.062431, 2.122227]


                     FAC_5                 FAC_6
V1            0          0  3.5556992  0.3902137
     [        0,         0] [ 2.790894, 4.320504]

V2    0.6916515  0.4076770  1.5668374  0.9287374
     [-0.107381, 1.490684] [-0.253454, 3.387129]

V3    0.5847585  0.3823806  3.8687194  0.4010005
     [-0.164694, 1.334211] [ 3.082773, 4.654666]

V4            0          0  4.2910363  0.3961712
     [        0,         0] [ 3.514555, 5.067518]

V5            0          0  5.5029723  0.7064593
     [        0,         0] [ 4.118338, 6.887607]

V6            0          0  4.6685783  0.3955291
     [        0,         0] [ 3.893356, 5.443801]

V7            0          0  2.2613639  0.5377250
     [        0,         0] [ 1.207442, 3.315285]

V8            0          0          0          0
     [        0,         0] [        0,         0]

V9            0          0          0          0
     [        0,         0] [        0,         0]
```

```
V10  -2.3262888  0.3672806  2.3140842  0.5590919
     [-3.046146,-1.606432] [ 1.218284, 3.409884]

V11  -3.0770671  0.3761633  2.0841849  0.5693505
     [-3.814334,-2.339801] [ 0.968278, 3.200091]

V12  -2.5674990  0.3557420  2.6479295  0.5699106
     [-3.264741,-1.870258] [ 1.530925, 3.764934]

V13           0          0  2.0032229  1.1527076
     [        0,        0] [-0.256042, 4.262488]

V14   1.5996717  0.5018659  3.7016316  1.0390729
     [ 0.616033, 2.583311] [ 1.665086, 5.738177]

V15           0          0          0          0
     [        0,        0] [        0,        0]

V16           0          0  6.1955204  0.9294210
     [        0,        0] [ 4.373889, 8.017152]

V17           0          0  7.4938882  0.3582164
     [        0,        0] [ 6.791797, 8.195980]

V18           0          0  7.6414969  0.3471791
     [        0,        0] [ 6.961038, 8.321955]


                       U_Var
V1    26.222160  2.4930495
     [ 21.33587, 31.10845]

V2    26.641908  2.1451506
     [ 22.43749, 30.84633]

V3    27.575886  2.0059081
     [ 23.64438, 31.50739]

V4    29.619543  1.8696129
     [ 25.95517, 33.28392]

V5    13.165456  1.2414845
     [ 10.73219, 15.59872]

V6    12.608255  2.1187183
```

```
     [ 8.455643, 16.76087]

V7    28.655486  1.9498993
     [ 24.83375, 32.47722]

V8    22.878885  1.6740535
     [ 19.59780, 26.15997]

V9    19.905897  1.5660904
     [ 16.83642, 22.97538]

V10   22.515727  1.5842072
     [ 19.41074, 25.62072]

V11   15.581128  1.7782928
     [ 12.09574, 19.06652]

V12   16.743815  1.4472403
     [ 13.90728, 19.58035]

V13   36.842973  2.6206218
     [ 31.70665, 41.97930]

V14   44.048633  3.0554245
     [ 38.06011, 50.03715]

V15   26.478451  4.5841282
     [ 17.49373, 35.46318]

V16   33.447490  2.4565987
     [ 28.63264, 38.26233]

V17   36.840295  2.6370810
     [ 31.67171, 42.00888]

V18   36.663896  2.7163183
     [ 31.34001, 41.98778]
```

Factor Correlations Phi
-----------------------

```
                      FAC_1                 FAC_2
FAC_1   1.0000000          0          0          0
       [ 1.000000, 1.000000] [        0,         0]
```

```
FAC_2          0          0  1.0000000          0
       [       0,         0] [ 1.000000, 1.000000]

FAC_3          0          0          0          0
       [       0,         0] [        0,        0]

FAC_4          0          0          0          0
       [       0,         0] [        0,        0]

FAC_5          0          0          0          0
       [       0,         0] [        0,        0]

FAC_6          0          0  0.8595380  0.0375430
       [       0,         0] [ 0.785955, 0.933121]


                      FAC_3                 FAC_4
FAC_1          0          0          0          0
       [       0,         0] [        0,        0]

FAC_2          0          0          0          0
       [       0,         0] [        0,        0]

FAC_3   1.0000000         0          0          0
       [ 1.000000, 1.000000] [        0,        0]

FAC_4          0          0  1.0000000          0
       [       0,         0] [ 1.000000, 1.000000]

FAC_5          0          0          0          0
       [       0,         0] [        0,        0]

FAC_6          0          0          0          0
       [       0,         0] [        0,        0]


                      FAC_5                 FAC_6
FAC_1          0          0          0          0
       [       0,         0] [        0,        0]

FAC_2          0          0  0.8595380  0.0375430
       [       0,         0] [ 0.785955, 0.933121]

FAC_3          0          0          0          0
       [       0,         0] [        0,        0]
```

```
   FAC_4          0          0          0          0
         [        0,         0] [        0,         0]

   FAC_5   1.0000000         0          0          0
         [ 1.000000, 1.000000] [        0,         0]

   FAC_6          0          0  1.0000000          0
         [        0,         0] [ 1.000000, 1.000000]
```

```
(1) Standalone Fit Measures: ------------------------------
Fit criterion . . . . . . . . . . . . . . . . . .    0.1289
Normal Th. Chi-square (df = 90) 83.5408 Prob>chi**2 = 0.6713
Normal Theory Reweighted LS Chi-square  . . . . .   83.9798
Probability of Close Fit  . . . . . . . . . . . .    1.0000
Z-Test of Wilson & Hilferty (1931). . . . . . . .   -0.4437
(2) Incremental Fit Measures: -----------------------------
Null Model Chi-square (df = 153). . . . . . . .   9151.0889
RMSEA Estimate  . . . . . . .  0.0000  90%C.I.[ . ,  0.0177]
ECVI Estimate . . . . . . . .  0.3865  90%C.I.[ . ,  0.4249]
McDonald's (1989) Centrality. . . . . . . . . . .    1.0050
Tucker-Lewis Coefficient TLI. . . . . . . . . . .    1.0012
Bentler & Bonett's (1980) NFI . . . . . . . . . .    0.9909
Bentler's Comparative Fit Index CFI . . . . . . .    1.0000
Parsimonious NFI (James, Mulaik, & Brett,1982). .    0.5829
Bollen (1986) Normed Index Rho1 . . . . . . . . .    0.9845
Bollen (1988) Non-normed Index Delta2 . . . . . .    1.0007
(3) Information Criteria: ---------------------------------
Akaike's Information Criterion. . . . . . . . . .  -96.4592
Bozdogan's (1987) CAIC. . . . . . . . . . . . . . -589.2481
Schwarz's Bayesian Criterion. . . . . . . . . . . -499.2481
(4) Other Fit Measures: -----------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .    0.9858
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .    0.5799
GFI Adjusted for Degrees of Freedom (AGFI). . . .    0.9730
Root Mean Square Residual (RMR) . . . . . . . . .    0.9015
Hoelter's (1983) Critical N . . . . . . . . . . .       879
```

Standardized Factor Loadings

| | FAC_1 | FAC_2 | FAC_3 | FAC_4 | FAC_5 |
|---|---|---|---|---|---|
| V1 | 0.6520184 | 0 | 0.3559749 | 0 | 0 |
| V2 | 0.6713886 | 0.2303663 | 0.2842835 | 0.0816399 | 0.0743728 |

| | | | | | |
|---|---|---|---|---|---|
| V3 | 0.7009628 | 0 | 0.2389194 | 0 | 0.0600049 |
| V4 | 0.7065625 | 0 | 0 | 0.0811913 | 0 |
| V5 | 0.8004060 | -0.1299030 | 0 | 0 | 0 |
| V6 | 0.7880067 | 0 | -0.1635092 | 0 | 0 |
| V7 | 0 | 0 | 0.1218518 | 0.7837246 | 0 |
| V8 | 0 | 0.1878955 | 0 | 0.8485027 | 0 |
| V9 | 0.0226378 | 0.2462389 | 0 | 0.8555239 | 0 |
| V10 | 0.0282296 | 0 | 0 | 0.8078966 | -0.2373626 |
| V11 | 0 | 0 | 0 | 0.8342481 | -0.3129472 |
| V12 | -0.0301457 | 0 | -0.0396777 | 0.8288745 | -0.2596983 |
| V13 | 0 | 0.4982822 | 0.1477460 | 0.2018084 | 0 |
| V14 | 0 | 0.3076308 | 0.0618552 | 0.2318001 | 0.1654713 |
| V15 | 0 | 0.8329973 | 0 | 0.1623768 | 0 |
| V16 | 0 | 0.1843848 | -0.0978743 | 0.1644847 | 0 |
| V17 | 0 | 0 | 0 | 0.2033762 | 0 |
| V18 | 0 | 0 | -0.0630997 | 0.1111189 | 0 |

Standardized Factor Loadings

| | FAC_6 |
|---|---|
| V1 | 0.3818197 |
| V2 | 0.1684808 |
| V3 | 0.3969880 |
| V4 | 0.4352471 |
| V5 | 0.5725517 |
| V6 | 0.4724367 |
| V7 | 0.2370033 |
| V8 | 0 |
| V9 | 0 |
| V10 | 0.2361173 |
| V11 | 0.2119681 |
| V12 | 0.2678337 |
| V13 | 0.2214845 |
| V14 | 0.3828996 |
| V15 | 0 |
| V16 | 0.6250950 |
| V17 | 0.7608451 |
| V18 | 0.7773431 |

Squared Multiple Correlations

| N | Variable | ErrVariance | TotVariance | R-squared |
|---|---|---|---|---|
| 1 | V1 | 26.2221597 | 86.7227919 | 0.69763243 |
| 2 | V2 | 26.6419081 | 86.4861926 | 0.69195189 |

```
 3 V3        27.5758859  94.9685501  0.70963139
 4 V4        29.6195433  97.1969511  0.69526263
 5 V5        13.1654562  92.3773057  0.85748170
 6 V6        12.6082549  97.6522196  0.87088614
 7 V7        28.6554864  91.0399549  0.68524274
 8 V8        22.8788851  93.4830054  0.75526156
 9 V9        19.9058973  96.1949634  0.79306716
10 V10       22.5157266  96.0512122  0.76558623
11 V11       15.5811277  96.6788957  0.83883631
12 V12       16.7438146  97.7422254  0.82869415
13 V13       36.8429733  81.8035200  0.54961628
14 V14       44.0486326  93.4579918  0.52867987
15 V15       26.4784513  94.6506617  0.72025075
16 V16       33.4474896  98.2343848  0.65951342
17 V17       36.8402954  97.0112226  0.62024708
18 V18       36.6638962  96.6343101  0.62059132
```

In `sem33.inp` we specified a model developed by Joereskog (1978, p. 454-5):

```
print "*** CFA Model in SEM33: ***";
l = [" Z1  5 # 0. ,
       Z2  5 # 0. ,
       Z3  0.  0. Z19 0. 0. ,
       Z4  0.  0. Z20 0. 0. ,
       Z5  0.  0. Z21 0. 0. ,
       Z6  0.  0. Z22 0. 0. ,
       0.  Z7  4 # 0. ,
       0.  Z8  4 # 0. ,
       0.  Z9  0. 0. Z23 0. ,
       0.  Z10 0. 0. Z24 0. ,
       0.  Z11 0. 0. Z25 0. ,
       0.  Z12 0. 0. Z26 0. ,
       0.  0. Z13  3 # 0. ,
       0.  0. Z14  3 # 0. ,
       0.  0. Z15  0. 0. Z27 ,
       0.  0. Z16  0. 0. Z28 ,
       0.  0. Z17  0. 0. Z29 ,
       0.  0. Z18  0. 0. Z30 "];
print "L=",l;

p = [" 1., C1 1., C2 C3 1., 0. 0. 0. 1.,
       0. 0. 0. C4 1., 0. 0. 0. C5 C6 1. "];
p = (tri2sym)p; print "P=",p;
u = [" u1:u18 "];
print "U=",u;
```

```
list patt;
patt[1] = l; patt[2] = p; patt[3] = u;
print "Patt=",patt;


print "CFA with VARMAX rotation: p=0.0005, chi=174.50, df=117";
optn = [ "data"      "cor" ,
         "nobs"        649 ,
         "anal"      "cov" ,
         "hey"             ,
         "meth"       "ml" ,
         "frot"   "varmax" ,
         "cl"       "wald" ,
         "nfac"          6 ,
         "tech"   "nrridg" ,
         "pmod"            ,
         "phis"            ,
         "pini"          2 ,
         "prin"          3 ];
< gof,parm,resi,covm,mod1,mod2 > = cfa(lord,optn,patt);


        Loadings and Uni. Vars. with ASE's and Wald CIs
        -----------------------------------------------

                           FAC_1                 FAC_2
       V1    7.6283657  0.3109221          0         0
            [ 7.018970, 8.237762] [        0,        0]

       V2    7.7238755  0.3087426          0         0
            [ 7.118751, 8.329000] [        0,        0]

       V3    8.1770373  0.3255776          0         0
            [ 7.538917, 8.815158] [        0,        0]

       V4    7.5301310  0.3434640          0         0
            [ 6.856954, 8.203308] [        0,        0]

       V5    8.0861675  0.3178589          0         0
            [ 7.463176, 8.709159] [        0,        0]

       V6    7.7649167  0.3411981          0         0
            [ 7.096181, 8.433653] [        0,        0]

       V7            0          0  7.8722749  0.3138617
```

```
    [        0,        0] [ 7.257117, 8.487433]

V8           0          0  8.4189221  0.3092419
    [        0,        0] [ 7.812819, 9.025025]

V9           0          0  8.6224021  0.3160839
    [        0,        0] [ 8.002889, 9.241915]

V10          0          0  8.1935822  0.3249554
    [        0,        0] [ 7.556681, 8.830483]

V11          0          0  8.4023056  0.3228630
    [        0,        0] [ 7.769506, 9.035105]

V12          0          0  8.4909755  0.3236832
    [        0,        0] [ 7.856568, 9.125383]

V13          0          0          0          0
    [        0,        0] [        0,        0]

V14          0          0          0          0
    [        0,        0] [        0,        0]

V15          0          0          0          0
    [        0,        0] [        0,        0]

V16          0          0          0          0
    [        0,        0] [        0,        0]

V17          0          0          0          0
    [        0,        0] [        0,        0]

V18          0          0          0          0
    [        0,        0] [        0,        0]


                    FAC_3                 FAC_4
V1           0          0          0          0
    [        0,        0] [        0,        0]

V2           0          0          0          0
    [        0,        0] [        0,        0]

V3           0          0  0.8296144  0.3850475
    [        0,        0] [ 0.074935, 1.584294]
```

```
V4           0          0  3.2916481  0.3633609
     [        0,         0] [ 2.579474, 4.003822]

V5           0          0  3.7419424  0.3163946
     [        0,         0] [ 3.121820, 4.362064]

V6           0          0  4.8221642  0.3458560
     [        0,         0] [ 4.144299, 5.500030]

V7           0          0          0          0
     [        0,         0] [        0,         0]

V8           0          0          0          0
     [        0,         0] [        0,         0]

V9           0          0          0          0
     [        0,         0] [        0,         0]

V10          0          0          0          0
     [        0,         0] [        0,         0]

V11          0          0          0          0
     [        0,         0] [        0,         0]

V12          0          0          0          0
     [        0,         0] [        0,         0]

V13   6.8307132  0.3228920          0          0
     [ 6.197856, 7.463570] [        0,         0]

V14   7.0978273  0.3481031          0          0
     [ 6.415558, 7.780097] [        0,         0]

V15   7.6032641  0.3460626          0          0
     [ 6.924994, 8.281534] [        0,         0]

V16   7.3072750  0.3606371          0          0
     [ 6.600439, 8.014111] [        0,         0]

V17   7.0930379  0.3610033          0          0
     [ 6.385485, 7.800591] [        0,         0]

V18   6.7901582  0.3674533          0          0
     [ 6.069963, 7.510353] [        0,         0]
```

```
                  FAC_5                   FAC_6
V1             0          0          0          0
     [         0,         0] [         0,         0]

V2             0          0          0          0
     [         0,         0] [         0,         0]

V3             0          0          0          0
     [         0,         0] [         0,         0]

V4             0          0          0          0
     [         0,         0] [         0,         0]

V5             0          0          0          0
     [         0,         0] [         0,         0]

V6             0          0          0          0
     [         0,         0] [         0,         0]

V7             0          0          0          0
     [         0,         0] [         0,         0]

V8             0          0          0          0
     [         0,         0] [         0,         0]

V9     0.5456933  0.4436800          0          0
     [-0.323903, 1.415290] [         0,         0]

V10    2.6128046  0.3885210          0          0
     [ 1.851317, 3.374292] [         0,         0]

V11    3.1089389  0.3852227          0          0
     [ 2.353916, 3.863961] [         0,         0]

V12    3.2239759  0.3860668          0          0
     [ 2.467299, 3.980653] [         0,         0]

V13            0          0          0          0
     [         0,         0] [         0,         0]

V14            0          0          0          0
     [         0,         0] [         0,         0]

V15            0          0  1.0898998  0.4623232
     [         0,         0] [ 0.183763, 1.996037]
```

```
V16           0          0  3.5812933  0.4683548
     [        0,         0] [ 2.663335, 4.499252]

V17           0          0  2.9350545  0.4641535
     [        0,         0] [ 2.025330, 3.844779]

V18           0          0  3.6317776  0.4822470
     [        0,         0] [ 2.686591, 4.576964]


                           U_Var
          V1    28.298036  2.0874177
               [ 24.20677, 32.38930]

          V2    26.645847  2.0348994
               [ 22.65752, 30.63418]

          V3    27.151259  2.0413703
               [ 23.15025, 31.15227]

          V4    29.793435  1.8983181
               [ 26.07280, 33.51407]

          V5    13.100164  1.2644546
               [ 10.62188, 15.57845]

          V6    14.433571  1.8961901
               [ 10.71711, 18.15004]

          V7    29.229787  2.0052323
               [ 25.29960, 33.15997]

          V8    22.630650  1.8128745
               [ 19.07748, 26.18382]

          V9    21.732031  1.7513687
               [ 18.29941, 25.16465]

          V10   22.713296  1.5844860
               [ 19.60776, 25.81883]

          V11   17.139269  1.5802589
               [ 14.04202, 20.23652]

          V12   15.935406  1.6149322
               [ 12.77020, 19.10062]
```

```
V13   35.243857  2.5341835
    [ 30.27695, 40.21077]

V14   43.129748  2.9856493
    [ 37.27798, 48.98151]

V15   35.788956  2.5487612
    [ 30.79348, 40.78444]

V16   32.344787  2.6248803
    [ 27.20012, 37.48946]

V17   38.382053  2.6422283
    [ 33.20338, 43.56073]

V18   37.670778  2.8821251
    [ 32.02192, 43.31964]
```

```
                 Factor Correlations Phi
                 -----------------------

                            FAC_1                     FAC_2
FAC_1   1.0000000           0  0.1618146  0.0419585
       [ 1.000000, 1.000000] [ 0.079578, 0.244052]

FAC_2   0.1618146  0.0419585  1.0000000           0
       [ 0.079578, 0.244052] [ 1.000000, 1.000000]

FAC_3   0.4701773  0.0359095  0.4827407  0.0349155
       [ 0.399796, 0.540559] [ 0.414308, 0.551174]

FAC_4           0          0          0           0
       [        0,         0] [        0,         0]

FAC_5           0          0          0           0
       [        0,         0] [        0,         0]

FAC_6           0          0          0           0
       [        0,         0] [        0,         0]


                            FAC_3                     FAC_4
FAC_1   0.4701773  0.0359095          0           0
```

```
      [ 0.399796, 0.540559] [        0,        0]

FAC_2   0.4827407  0.0349155          0          0
      [ 0.414308, 0.551174] [        0,        0]

FAC_3   1.0000000          0          0          0
      [ 1.000000, 1.000000] [        0,        0]

FAC_4           0          0  1.0000000          0
      [        0,        0] [ 1.000000, 1.000000]

FAC_5           0          0  0.2627967  0.0850491
      [        0,        0] [ 0.096104, 0.429490]

FAC_6           0          0  0.6321221  0.0885425
      [        0,        0] [ 0.458582, 0.805662]


                       FAC_5                 FAC_6
FAC_1           0          0          0          0
      [        0,        0] [        0,        0]

FAC_2           0          0          0          0
      [        0,        0] [        0,        0]

FAC_3           0          0          0          0
      [        0,        0] [        0,        0]

FAC_4   0.2627967  0.0850491  0.6321221  0.0885425
      [ 0.096104, 0.429490] [ 0.458582, 0.805662]

FAC_5   1.0000000          0  0.4208009  0.1070164
      [ 1.000000, 1.000000] [ 0.211053, 0.630549]

FAC_6   0.4208009  0.1070164  1.0000000          0
      [ 0.211053, 0.630549] [ 1.000000, 1.000000]

(1) Standalone Fit Measures: -------------------------------
Fit criterion . . . . . . . . . . . . . . . . . . .    0.2693
Normal Th. Chi-square (df = 117)      174.5032  prob= 0.0005
Normal Theory Reweighted LS Chi-square  . . . . .  174.6905
Probability of Close Fit  . . . . . . . . . . . .    1.0000
Z-Test of Wilson & Hilferty (1931). . . . . . . .    3.3143
(2) Incremental Fit Measures: ------------------------------
Null Model Chi-square (df = 153). . . . . . . .   9151.0889
```

```
RMSEA Estimate  . . . . .  0.0275  90%C.I.[ 0.0185,  0.0358]
ECVI Estimate . . . . . .  0.4410  90%C.I.[ 0.3922,  0.5025]
McDonald's (1989) Centrality. . . . . . . . . . .     0.9567
Tucker-Lewis Coefficient TLI. . . . . . . . . . .     0.9916
Bentler & Bonett's (1980) NFI . . . . . . . . . .     0.9809
Bentler's Comparative Fit Index CFI . . . . . . .     0.9936
Parsimonious NFI (James, Mulaik, & Brett,1982). .     0.7501
Bollen (1986) Normed Index Rho1 . . . . . . . . .     0.9751
Bollen (1988) Non-normed Index Delta2 . . . . . .     0.9936
(3) Information Criteria: --------------------------------
Akaike's Information Criterion. . . . . . . . . .   -59.4968
Bozdogan's (1987) CAIC. . . . . . . . . . . . . .  -700.1224
Schwarz's Bayesian Criterion. . . . . . . . . . .  -583.1224
(4) Other Fit Measures: ----------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .     0.9709
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .     0.7425
GFI Adjusted for Degrees of Freedom (AGFI). . . .     0.9575
Root Mean Square Residual (RMR) . . . . . . . . .     2.4779
Hoelter's (1983) Critical N . . . . . . . . . . .        533
```

Standardized Factor Loadings

| | FAC_1 | FAC_2 | FAC_3 | FAC_4 | FAC_5 |
|---|---|---|---|---|---|
| V1 | 0.8202544 | 0 | 0 | 0 | 0 |
| V2 | 0.8314182 | 0 | 0 | 0 | 0 |
| V3 | 0.8402588 | 0 | 0 | 0.0852498 | 0 |
| V4 | 0.7632668 | 0 | 0 | 0.3336470 | 0 |
| V5 | 0.8408124 | 0 | 0 | 0.3890930 | 0 |
| V6 | 0.7844520 | 0 | 0 | 0.4871599 | 0 |
| V7 | 0 | 0.8243220 | 0 | 0 | 0 |
| V8 | 0 | 0.8706228 | 0 | 0 | 0 |
| V9 | 0 | 0.8783036 | 0 | 0 | 0.0555859 |
| V10 | 0 | 0.8333302 | 0 | 0 | 0.2657359 |
| V11 | 0 | 0.8513559 | 0 | 0 | 0.3150104 |
| V12 | 0 | 0.8558595 | 0 | 0 | 0.3249651 |
| V13 | 0 | 0 | 0.7547749 | 0 | 0 |
| V14 | 0 | 0 | 0.7340049 | 0 | 0 |
| V15 | 0 | 0 | 0.7809566 | 0 | 0 |
| V16 | 0 | 0 | 0.7360212 | 0 | 0 |
| V17 | 0 | 0 | 0.7190490 | 0 | 0 |
| V18 | 0 | 0 | 0.6895540 | 0 | 0 |

Standardized Factor Loadings

| | FAC_6 |
|---|---|
| V1 | 0 |
| V2 | 0 |
| V3 | 0 |
| V4 | 0 |
| V5 | 0 |
| V6 | 0 |
| V7 | 0 |
| V8 | 0 |
| V9 | 0 |
| V10 | 0 |
| V11 | 0 |
| V12 | 0 |
| V13 | 0 |
| V14 | 0 |
| V15 | 0.1119472 |
| V16 | 0.3607238 |
| V17 | 0.2975380 |
| V18 | 0.3688142 |

Squared Multiple Correlations

| N | Variable | ErrVariance | TotVariance | R-squared |
|---|---|---|---|---|
| 1 | V1 | 28.2980362 | 86.4900000 | 0.67281725 |
| 2 | V2 | 26.6458468 | 86.3041000 | 0.69125630 |
| 3 | V3 | 27.1512593 | 94.7034583 | 0.71330235 |
| 4 | V4 | 29.7934352 | 97.3312549 | 0.69389653 |
| 5 | V5 | 13.1001637 | 92.4884014 | 0.85835885 |
| 6 | V6 | 14.4335714 | 97.9807707 | 0.85268975 |
| 7 | V7 | 29.2297872 | 91.2025000 | 0.67950673 |
| 8 | V8 | 22.6306500 | 93.5089000 | 0.75798400 |
| 9 | V9 | 21.7320310 | 96.3756294 | 0.77450699 |
| 10 | V10 | 22.7132957 | 96.6748325 | 0.76505472 |
| 11 | V11 | 17.1392691 | 97.4035089 | 0.82403848 |
| 12 | V12 | 15.9354063 | 98.4260915 | 0.83809774 |
| 13 | V13 | 35.2438570 | 81.9025000 | 0.56968521 |
| 14 | V14 | 43.1297479 | 93.5089000 | 0.53876318 |
| 15 | V15 | 35.7889559 | 94.7864618 | 0.62242545 |
| 16 | V16 | 32.3447871 | 98.5667172 | 0.67184879 |
| 17 | V17 | 38.3820530 | 97.3077852 | 0.60556031 |
| 18 | V18 | 37.6707783 | 96.9668346 | 0.61150863 |

## 2.5 Eight Physical Variables (Harman, 1967): SEM47

```
phys8 = [ 1.0    .846   .805   .859   .473   .398   .301   .382 ,
          .846   1.0    .881   .826   .376   .326   .277   .415 ,
          .805   .881   1.0    .801   .380   .319   .237   .345 ,
          .859   .826   .801   1.0    .436   .329   .327   .365 ,
          .473   .376   .380   .436   1.0    .762   .730   .629 ,
          .398   .326   .319   .329   .762   1.0    .583   .577 ,
          .301   .277   .237   .327   .730   .583   1.0    .539 ,
          .382   .415   .345   .365   .629   .577   .539   1.0 ];
```

The data were analyzed before and we obtain for the EFA solution for $f = 2$:

```
********************************************************************
Starting Model Improvement (ChiSq=76.9612 Pvalue=4.09863e-011 DF=13)
********************************************************************
```

The best CFA result found by our algorithm is only slightly better than that of the EFA:

```
**********************************
Table of 20 Best Pattern Solutions
**********************************

 N  Npar   DF  Chisquared           P

 1    20   16  78.6507507  2.910e-010
 2    20   16  78.6507507  2.910e-010
 3    21   15  76.9807295  2.479e-010
 4    21   15  76.9807295  2.479e-010
 5    21   15  78.0383980  1.592e-010
 6    21   15  78.6105180  1.253e-010
 7    21   15  78.6136805  1.251e-010
 8    22   14  76.9613310  1.032e-010
 9    22   14  76.9613310  1.032e-010
10    22   14  76.9613310  1.032e-010
11    22   14  76.9613310  1.032e-010
12    22   14  76.9803629  1.023e-010
13    22   14  76.9803630  1.023e-010
14    22   14  76.9803630  1.023e-010
15    22   14  76.9803630  1.023e-010
16    22   14  78.0164924  6.589e-011
17    22   14  78.0164925  6.589e-011
18    22   14  78.5709244  5.205e-011
```

| | | | | |
|---|---|---|---|---|
| 19 | 22 | 14 | 78.5709245 | 5.205e-011 |
| 20 | 23 | 13 | 76.9611549 | 4.099e-011 |

| N | RMSEA | ECVI | SRMR | PGFI | SBC |
|---|---|---|---|---|---|
| 1 | 0.11349229 | 0.39431280 | 0.02402992 | 0.53687346 | -12.8742377 |
| 2 | 0.11349229 | 0.39431280 | 0.02402992 | 0.53687346 | -12.8742377 |
| 3 | 0.11658587 | 0.39559897 | 0.02012035 | 0.50394621 | -8.82394714 |
| 4 | 0.11658587 | 0.39559897 | 0.02012037 | 0.50394622 | -8.82394714 |
| 5 | 0.11757640 | 0.39907814 | 0.02258945 | 0.50352476 | -7.76627866 |
| 6 | 0.11810874 | 0.40096011 | 0.02389540 | 0.50334229 | -7.19415866 |
| 7 | 0.11811168 | 0.40097051 | 0.02419009 | 0.50331156 | -7.19099610 |
| 8 | 0.12162872 | 0.40231482 | 0.02025498 | 0.47034959 | -3.12303388 |
| 9 | 0.12162872 | 0.40231482 | 0.02025498 | 0.47034959 | -3.12303388 |
| 10 | 0.12162872 | 0.40231482 | 0.02025508 | 0.47034963 | -3.12303386 |
| 11 | 0.12162872 | 0.40231482 | 0.02025512 | 0.47034962 | -3.12303385 |
| 12 | 0.12164710 | 0.40237742 | 0.02011283 | 0.47035014 | -3.10400197 |
| 13 | 0.12164710 | 0.40237742 | 0.02011292 | 0.47035017 | -3.10400190 |
| 14 | 0.12164710 | 0.40237742 | 0.02011299 | 0.47035013 | -3.10400189 |
| 15 | 0.12164710 | 0.40237742 | 0.02011240 | 0.47034994 | -3.10400185 |
| 16 | 0.12264367 | 0.40578574 | 0.02271931 | 0.46995350 | -2.06787245 |
| 17 | 0.12264367 | 0.40578574 | 0.02271901 | 0.46995330 | -2.06787242 |
| 18 | 0.12317362 | 0.40760953 | 0.02406193 | 0.46977930 | -1.51344043 |
| 19 | 0.12317362 | 0.40760953 | 0.02406194 | 0.46977923 | -1.51344041 |
| 20 | 0.12721833 | 0.40909390 | 0.02024873 | 0.43675329 | 2.59710176 |

The following shows details of the best solution found:

```
Loadings and Uni. Vars. with ASE's and Wald CIs
-----------------------------------------------

                       FAC_1                 FAC_2
X1   0.8501964  0.0446153  0.1270203  0.0319009
    [ 0.762752, 0.937641] [ 0.064496, 0.189545]

X2   0.9453263  0.0431220          0          0
    [ 0.860809, 1.029844] [        0,        0]

X3   0.9130442  0.0442776          0          0
    [ 0.826262, 0.999827] [        0,        0]

X4   0.8534984  0.0460557  0.0895713  0.0336441
    [ 0.763231, 0.943766] [ 0.023630, 0.155513]
```

```
X5          0          0  0.9579197  0.0445318
    [        0,        0] [ 0.870639, 1.045200]

X6          0          0  0.7961673  0.0490769
    [        0,        0] [ 0.699978, 0.892356]

X7          0          0  0.7586894  0.0500183
    [        0,        0] [ 0.660655, 0.856724]

X8   0.1370728  0.0500064  0.6096575  0.0554413
    [ 0.039062, 0.235084] [ 0.500994, 0.718320]


                         U_Var
           X1   0.1704068  0.0175540
               [ 0.136002, 0.204812]

           X2   0.1063582  0.0147247
               [ 0.077498, 0.135218]

           X3   0.1663503  0.0179632
               [ 0.131143, 0.201558]

           X4   0.1993628  0.0197228
               [ 0.160707, 0.238019]

           X5   0.0823898  0.0280919
               [ 0.027331, 0.137449]

           X6   0.3661176  0.0359400
               [ 0.295676, 0.436559]

           X7   0.4243904  0.0393853
               [ 0.347197, 0.501584]

           X8   0.5394005  0.0463537
               [ 0.448549, 0.630252]


            Factor Correlations Phi
            -----------------------

                      FAC_1                FAC_2
FAC_1   1.0000000          0  0.4195913  0.0515996
      [ 1.000000, 1.000000] [ 0.318458, 0.520725]
```

```
    FAC_2   0.4195913  0.0515996  1.0000000         0
          [ 0.318458, 0.520725] [ 1.000000, 1.000000]


(1) Standalone Fit Measures: -------------------------------
Fit criterion . . . . . . . . . . . . . . . . . .    0.2587
Normal Th. Chi-square (df = 16) 78.6508 Prob>chi**2 = 0.0000
Normal Theory Reweighted LS Chi-square  . . . . .   78.2660
Probability of Close Fit  . . . . . . . . . . . .    0.0000
Z-Test of Wilson & Hilferty (1931). . . . . . . .    6.0602
(2) Incremental Fit Measures: ------------------------------
Null Model Chi-square (df = 28) . . . . . . . .   2110.0336
RMSEA Estimate  . . . . .  0.1135  90%C.I.[ 0.0892,  0.1391]
ECVI Estimate . . . . . .  0.3943  90%C.I.[ 0.3146,  0.4996]
McDonald's (1989) Centrality. . . . . . . . . . .    0.9024
Tucker-Lewis Coefficient TLI. . . . . . . . . . .    0.9473
Bentler & Bonett's (1980) NFI . . . . . . . . . .    0.9627
Bentler's Comparative Fit Index CFI . . . . . . .    0.9699
Parsimonious NFI (James, Mulaik, & Brett,1982). .    0.5501
Bollen (1986) Normed Index Rho1 . . . . . . . . .    0.9348
Bollen (1988) Non-normed Index Delta2 . . . . . .    0.9701
(3) Information Criteria: ----------------------------------
Akaike's Information Criterion. . . . . . . . . .   46.6508
Bozdogan's (1987) CAIC. . . . . . . . . . . . . .  -28.8742
Schwarz's Bayesian Criterion. . . . . . . . . . .  -12.8742
(4) Other Fit Measures: ------------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .    0.9395
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .    0.5369
GFI Adjusted for Degrees of Freedom (AGFI). . . .    0.8639
Root Mean Square Residual (RMR) . . . . . . . . .    0.0240
Hoelter's (1983) Critical N . . . . . . . . . . .       103


                Standardized Factor Loadings

                    FAC_1      FAC_2
            X1   0.8501961  0.1270205
            X2   0.9453265          0
            X3   0.9130442          0
            X4   0.8534982  0.0895714
            X5           0  0.9579197
            X6           0  0.7961673
            X7           0  0.7586894
            X8   0.1370728  0.6096574


               Squared Multiple Correlations
```

```
N Variable ErrVariance TotVariance   R-squared
1 X1        0.17040709  1.00000000  0.82959291
2 X2        0.10635785  1.00000000  0.89364215
3 X3        0.16635024  1.00000000  0.83364976
4 X4        0.19936302  1.00000000  0.80063698
5 X5        0.08238983  1.00000000  0.91761017
6 X6        0.36611759  1.00000000  0.63388241
7 X7        0.42439039  1.00000000  0.57560961
8 X8        0.53940050  1.00000000  0.46059950
```

Similar to the model by Maydeu-Olivares for the Lot data, we used in our sem80.inp application zero loadings in $L[1,1], L[2,1], L[3,1], L[4,1], L[5,2], L[6,2], L[7,2], L[8,2]$:

```
Loadings and Uni. Vars. with ASE's and Wald CIs
 ------------------------------------------------

                   FAC_1                  FAC_2
 X1   0.9138431  0.0442390          0          0
    [ 0.827136, 1.000550] [        0,         0]

 X2   0.9367505  0.0434153          0          0
    [ 0.851658, 1.021843] [        0,         0]

 X3   0.9079356  0.0444483          0          0
    [ 0.820818, 0.995053] [        0,         0]

 X4   0.9002127  0.0447201          0          0
    [ 0.812563, 0.987863] [        0,         0]

 X5           0          0  0.9454962  0.0448809
    [         0,         0] [ 0.857531, 1.033461]

 X6           0          0  0.8031657  0.0489491
    [         0,         0] [ 0.707227, 0.899104]

 X7           0          0  0.7631068  0.0499931
    [         0,         0] [ 0.665122, 0.861091]

 X8           0          0  0.6829877  0.0519718
    [         0,         0] [ 0.581125, 0.784851]


                                 U_Var
             X1   0.1648907  0.0177858
```

```
            [ 0.130031, 0.199750]

        X2   0.1224985  0.0153121
            [ 0.092487, 0.152510]

        X3   0.1756529  0.0184803
            [ 0.139432, 0.211874]

        X4   0.1896171  0.0194127
            [ 0.151569, 0.227665]

        X5   0.1060370  0.0277814
            [ 0.051586, 0.160488]

        X6   0.3549249  0.0355108
            [ 0.285325, 0.424525]

        X7   0.4176681  0.0392187
            [ 0.340801, 0.494535]

        X8   0.5335278  0.0469720
            [ 0.441464, 0.625591]


         Factor Correlations Phi
         -----------------------

                      FAC_1                 FAC_2
    FAC_1   1.0000000          0  0.4755122 0.0481644
           [ 1.000000, 1.000000] [ 0.381112, 0.569913]

    FAC_2   0.4755122  0.0481644  1.0000000         0
           [ 0.381112, 0.569913] [ 1.000000, 1.000000]


(1) Standalone Fit Measures: ------------------------------
Fit criterion . . . . . . . . . . . . . . . . .     0.3364
Normal Th. Chi-square (df = 19)       102.2519  prob= 0.0000
Normal Theory Reweighted LS Chi-square . . . . .   105.6593
Probability of Close Fit  . . . . . . . . . . .     0.0000
Z-Test of Wilson & Hilferty (1931). . . . . . . .   7.0655
(2) Incremental Fit Measures: -----------------------------
Null Model Chi-square (df = 28) . . . . . . . .   2110.0336
RMSEA Estimate  . . . . .  0.1201  90%C.I.[ 0.0978,  0.1434]
ECVI Estimate . . . . . .  0.4516  90%C.I.[ 0.3585,  0.5703]
McDonald's (1989) Centrality. . . . . . . . . .     0.8724
```

```
Tucker-Lewis Coefficient TLI. . . . . . . . . . .     0.9411
Bentler & Bonett's (1980) NFI . . . . . . . . . .     0.9515
Bentler's Comparative Fit Index CFI . . . . . . .     0.9600
Parsimonious NFI (James, Mulaik, & Brett,1982). .     0.6457
Bollen (1986) Normed Index Rho1 . . . . . . . . .     0.9286
Bollen (1988) Non-normed Index Delta2 . . . . . .     0.9602
(3) Information Criteria: ----------------------------------
Akaike's Information Criterion. . . . . . . . . .    64.2519
Bozdogan's (1987) CAIC. . . . . . . . . . . . . .   -25.4340
Schwarz's Bayesian Criterion. . . . . . . . . . .    -6.4340
(4) Other Fit Measures: ------------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .     0.9201
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .     0.6243
GFI Adjusted for Degrees of Freedom (AGFI). . . .     0.8485
Root Mean Square Residual (RMR) . . . . . . . . .     0.0404
Hoelter's (1983) Critical N . . . . . . . . . . .         91


               Standardized Factor Loadings

                        FAC_1      FAC_2
                 X1   0.9138431          0
                 X2   0.9367505          0
                 X3   0.9079356          0
                 X4   0.9002127          0
                 X5           0  0.9454962
                 X6           0  0.8031657
                 X7           0  0.7631068
                 X8           0  0.6829877

               Squared Multiple Correlations


        N Variable ErrVariance TotVariance   R-squared
        1 X1        0.16489073  1.00000000  0.83510927
        2 X2        0.12249846  1.00000000  0.87750154
        3 X3        0.17565293  1.00000000  0.82434707
        4 X4        0.18961707  1.00000000  0.81038293
        5 X5        0.10603700  1.00000000  0.89396300
        6 X6        0.35492486  1.00000000  0.64507514
        7 X7        0.41766807  1.00000000  0.58233193
        8 X8        0.53352777  1.00000000  0.46647223
```

Finally, for $f = 3$ we obtain slightly better results. This output shows the EFA model fit:

```
*****************************************************************
Starting Model Improvement (ChiSq=23.0148 Pvalue=0.00169455 DF=7)
*****************************************************************
```

The following shows the result of the optimization:

```
*********************************
Table of 20 Best Pattern Solutions
*********************************

 N  Npar    DF  Chisquared           P

 1    23    13  25.2517652  0.02138621
 2    24    12  24.1218575  0.01957841
 3    22    14  27.2238927  0.01800267
 4    24    12  24.6786237  0.01642138
 5    25    11  23.5660773  0.01469193
 6    23    13  26.4985822  0.01456105
 7    24    12  25.1172959  0.01427672
 8    25    11  23.7363159  0.01389186
 9    25    11  23.7363159  0.01389186
10    25    11  23.8729910  0.01327942
11    23    13  26.9282636  0.01272552
12    25    11  24.2106974  0.01187358
13    25    11  24.6428060  0.01027925
14    22    14  29.0722357  0.01021769
15    26    10  23.2429488  0.00988448
16    26    10  23.2429488  0.00988448
17    25    11  24.9076075  0.00940482
18    26    10  23.4142151  0.00931670
19    26    10  23.4142151  0.00931670
20    24    12  26.4870590  0.00915264


N       RMSEA        ECVI        SRMR        PGFI         SBC

1  0.05567893  0.23899722  0.01835913  0.45508251 -49.1122879
2  0.05764441  0.24206008  0.01510296  0.42042403 -44.5218839
3  0.05574153  0.23870482  0.02009596  0.48921722 -52.8604722
4  0.05895337  0.24389155  0.01715112  0.42017008 -43.9651176
5  0.06130088  0.24701152  0.01335737  0.38556998 -39.3573522
6  0.05844342  0.24309859  0.01936837  0.45463393 -47.8654709
7  0.05996457  0.24533455  0.01641867  0.42015391 -43.5264454
8  0.06171472  0.24757151  0.01387635  0.38548115 -39.1871136
9  0.06171472  0.24757151  0.01387629  0.38548116 -39.1871136
```

```
10  0.06204497  0.24802110  0.01493086  0.38541288 -39.0504386
11  0.05936631  0.24451202  0.01976952  0.45441674 -47.4357895
12  0.06285354  0.24913198  0.01607712  0.38533984 -38.7127322
13  0.06387321  0.25055339  0.01467080  0.38521030 -38.2806236
14  0.05950974  0.24478490  0.02445962  0.48880577 -51.0121291
15  0.06600177  0.25272825  0.01247652  0.35059034 -33.9601690
16  0.06600177  0.25272825  0.01247652  0.35059034 -33.9601690
17  0.06449010  0.25142444  0.01730490  0.38513222 -38.0158220
18  0.06642718  0.25329163  0.01366534  0.35047236 -33.7889026
19  0.06642718  0.25329163  0.01366557  0.35047238 -33.7889026
20  0.06301772  0.24984035  0.01956135  0.41966544 -42.1566823
```

The following shows details of the best solution found:

```
Loadings and Uni. Vars. with ASE's and Wald CIs
-----------------------------------------------

                     FAC_1                 FAC_2
X1   0.9129584  0.0464207          0           0
    [ 0.821976, 1.003941] [        0,         0]

X2   0.8971286  0.0441942  0.4313792  0.0736614
    [ 0.810510, 0.983748] [ 0.287006, 0.575753]

X3   0.8652646  0.0459524  0.2271388  0.0476379
    [ 0.775200, 0.955330] [ 0.133770, 0.320507]

X4   0.9160986  0.0443564          0           0
    [ 0.829162, 1.003035] [        0,         0]

X5           0          0 -0.0936520  0.0400532
    [        0,         0] [-0.172155,-0.015149]

X6           0          0          0           0
    [        0,         0] [        0,         0]

X7  -0.0888122  0.0486732          0           0
    [-0.184210, 6.6e-003] [        0,         0]

X8           0          0  0.1844940  0.0609446
    [        0,         0] [ 0.065045, 0.303943]


                     FAC_3                 U_Var
X1   0.0465898  0.0328774  0.1220461  0.0171892
```

```
    [-0.017849, 0.111028] [ 0.088356, 0.155736]

X2          0          0  1.00e-008  0.0590448
    [        0,        0] [-0.115726, 0.115726]

X3          0          0  0.1951180  0.0244503
    [        0,        0] [ 0.147196, 0.243040]

X4          0          0  0.1607634  0.0191277
    [        0,        0] [ 0.123274, 0.198253]

X5  0.9482640  0.0444340  0.0895822  0.0275840
    [ 0.861175, 1.035353] [ 0.035519, 0.143646]

X6  0.7985225  0.0488685  0.3623618  0.0352110
    [ 0.702742, 0.894303] [ 0.293349, 0.431374]

X7  0.8079509  0.0577669  0.4106716  0.0393788
    [ 0.694730, 0.921172] [ 0.333491, 0.487853]

X8  0.6940129  0.0513876  0.4878293  0.0474781
    [ 0.593295, 0.794731] [ 0.394774, 0.580885]
```

```
        Factor Correlations Phi
        -----------------------
```

```
                      FAC_1                 FAC_2
FAC_1   1.0000000          0          0          0
       [ 1.000000, 1.000000] [        0,        0]

FAC_2           0          0  1.0000000          0
       [        0,        0] [ 1.000000, 1.000000]

FAC_3   0.4971289  0.0491747          0          0
       [ 0.400748, 0.593509] [        0,        0]
```

```
                      FAC_3
FAC_1   0.4971289  0.0491747
       [ 0.400748, 0.593509]

FAC_2           0          0
       [        0,        0]

FAC_3   1.0000000          0
```

```
                    [ 1.000000, 1.000000]


                                   FAC_3
               X1   0.0976812  0.0334375
                   [ 0.032145, 0.163217]

               X2           0          0
                   [        0,         0]

               X3           0          0
                   [        0,         0]

               X4   0.0616953  0.0356190
                   [-8.1e-003, 0.131507]

               X5   0.8557350  0.0436450
                   [ 0.770192, 0.941278]

               X6   0.7274247  0.0478260
                   [ 0.633687, 0.821162]

               X7   0.7128989  0.0492949
                   [ 0.616283, 0.809515]

               X8   0.5771781  0.0491124
                   [ 0.480920, 0.673437]


(1) Standalone Fit Measures: ------------------------------
Fit criterion . . . . . . . . . . . . . . . . . .     0.0831
Normal Th. Chi-square (df = 13) 25.2518 Prob>chi**2 = 0.0214
Normal Theory Reweighted LS Chi-square  . . . . .    24.5913
Probability of Close Fit  . . . . . . . . . . . .     0.3482
Z-Test of Wilson & Hilferty (1931). . . . . . . .     2.0254
(2) Incremental Fit Measures: -----------------------------
Null Model Chi-square (df = 28) . . . . . . . .    2110.0336
RMSEA Estimate  . . . . .  0.0557  90%C.I.[ 0.0209,  0.0879]
ECVI Estimate . . . . . .  0.2390  90%C.I.[ 0.2043,  0.3000]
McDonald's (1989) Centrality. . . . . . . . . . .     0.9801
Tucker-Lewis Coefficient TLI. . . . . . . . . . .     0.9873
Bentler & Bonett's (1980) NFI . . . . . . . . . .     0.9880
Bentler's Comparative Fit Index CFI . . . . . . .     0.9941
Parsimonious NFI (James, Mulaik, & Brett,1982). .     0.4587
Bollen (1986) Normed Index Rho1 . . . . . . . . .     0.9742
Bollen (1988) Non-normed Index Delta2 . . . . . .     0.9942
```

```
(3) Information Criteria: ---------------------------------
Akaike's Information Criterion. . . . . . . . . .    -0.7482
Bozdogan's (1987) CAIC. . . . . . . . . . . . . .   -62.1123
Schwarz's Bayesian Criterion. . . . . . . . . . .   -49.1123
(4) Other Fit Measures: -----------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .     0.9802
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .     0.4551
GFI Adjusted for Degrees of Freedom (AGFI). . . .     0.9451
Root Mean Square Residual (RMR) . . . . . . . . .     0.0184
Hoelter's (1983) Critical N . . . . . . . . . . .        271


                Standardized Factor Loadings

                   FAC_1      FAC_2      FAC_3
          X1   0.9129584          0  0.0465898
          X2   0.9012260  0.4333494          0
          X3   0.8672647  0.2276638          0
          X4   0.9160986          0          0
          X5           0 -0.0937666  0.9494242
          X6           0          0  0.7985225
          X7  -0.0888122          0  0.8079509
          X8           0  0.1841701  0.6927942


               Squared Multiple Correlations

      N Variable ErrVariance TotVariance   R-squared
      1 X1        0.12204609  1.00000000  0.87795391
      2 X2        1.000e-008  0.99092777  0.99999999
      3 X3        0.19511797  0.99539281  0.80397892
      4 X4        0.16076341  1.00000000  0.83923659
      5 X5        0.08958223  0.99755756  0.91019844
      6 X6        0.36236178  1.00000000  0.63763822
      7 X7        0.41067164  1.00000000  0.58932836
      8 X8        0.48782932  1.00352123  0.51388241
        Total Determination of All Equations = 1.000
     Total Determination of Manifest Variables = 1.000
```

## 2.6 Data by McArdle (2011)

The following data is derived from raw data with $N = 17351$ observations and $n = 7$ variables:

```
sethi = [  1. ,
```

```
          .773  1. ,
          .371 .359  1. ,
          .189 .170 .227 1. ,
          .283 .280 .255 .201  1. ,
          .362 .345 .381 .221 .308  1. ,
          .385 .352 .393 .185 .202 .403  1. ,
          55.7 43.9 70.5 95.2 94.2 91.3 55.4 ,
          18.5 22.3 34.1 21.1 14.3 16.7 21.2 ];
sethi = (tri2sym)sethi;
name = [" ID DR S7 BC NA DA VO "];
sethi = cname(sethi,name);
```

### 2.6.1 ML Analysis of the Covariance Matrix: nf=2

The following shows the input data when the covariance matrix is analyzed:

```
Name             Mean     Std Dev
---------------------------------
ID        55.70000000  18.5000000
DR        43.90000000  22.3000000
S7        70.50000000  34.1000000
BC        95.20000000  21.1000000
NA        94.20000000  14.3000000
DA        91.30000000  16.7000000
VO        55.40000000  21.2000000

    Variance Divisor = 17350


                 Covariance Matrix

             ID          DR          S7          BC          NA
ID    342.25000
DR    318.90115   497.29000
S7    234.04535   272.99437   1162.8100
BC    73.776150   79.990100   163.32877   445.21000
NA    74.867650   89.289200   124.34565   60.647730   204.49000
DA    111.83990   128.48145   216.96807   77.873770   73.553480
VO    150.99700   166.41152   284.10756   82.754200   61.238320

                 Covariance Matrix

                      DA          VO
              DA   278.89000
              VO   142.67812   449.44000
```

```
Determinant =  3.48541071e+017 (Ln =  4.0393e+001)
```

Due to the many observations $N = 17351$ (which is a factor in the formula for the $\chi^2$ value) we would have to start at an EFA model with $p = 2.1e-62$. Our test computations showed that simple structures are relatively robust against reducing large numbers of observations for obtaining some "decent" $\chi^2$ and therefore $p$ value. Therefore, here, only for the optimization of the CFA zero pattern we divide the number of observations by 10. The starting $\chi^2 = 31.06$ and $p = 1.e-4$ values then will be more suitable for the optimization.

```
**************************************************************
Starting Model Improvement (ChiSq=310.763 p=2.10424e-062 DF=8)
         Maximizing the p Value for 1735 Observations
**************************************************************
```

The 20 best solutions found:

```
**********************************
Table of 20 Best Pattern Solutions
**********************************
```

| N | Npar | DF | Chisquared | P |
|---|---|---|---|---|
| 1 | 17 | 11 | 32.2834799 | 6.868e-004 |
| 2 | 17 | 11 | 32.2834799 | 6.868e-004 |
| 3 | 17 | 11 | 32.2834799 | 6.868e-004 |
| 4 | 18 | 10 | 31.4142977 | 5.011e-004 |
| 5 | 18 | 10 | 31.4143019 | 5.011e-004 |
| 6 | 18 | 10 | 31.4143603 | 5.010e-004 |
| 7 | 18 | 10 | 31.4143605 | 5.010e-004 |
| 8 | 18 | 10 | 31.5589186 | 4.741e-004 |
| 9 | 16 | 12 | 35.4800636 | 3.926e-004 |
| 10 | 16 | 12 | 35.4800636 | 3.926e-004 |
| 11 | 16 | 12 | 35.4800639 | 3.926e-004 |
| 12 | 17 | 11 | 33.7839373 | 3.923e-004 |
| 13 | 18 | 10 | 32.2450398 | 3.645e-004 |
| 14 | 18 | 10 | 32.2450399 | 3.645e-004 |
| 15 | 18 | 10 | 32.2450604 | 3.645e-004 |
| 16 | 18 | 10 | 32.9358925 | 2.792e-004 |
| 17 | 19 | 9 | 31.1487163 | 2.791e-004 |
| 18 | 19 | 9 | 31.1487259 | 2.791e-004 |
| 19 | 19 | 9 | 31.1488122 | 2.791e-004 |
| 20 | 19 | 9 | 31.1488268 | 2.791e-004 |

| N | RMSEA | ECVI | SRMR | PGFI | SBC |
|---|---|---|---|---|---|
| 1 | 0.03340415 | 0.03831665 | 0.01830940 | 0.52103212 | -49.7629098 |
| 2 | 0.03340415 | 0.03831665 | 0.01830940 | 0.52103212 | -49.7629098 |
| 3 | 0.03340415 | 0.03831665 | 0.01830940 | 0.52103212 | -49.7629098 |
| 4 | 0.03514207 | 0.03897414 | 0.01813111 | 0.47373431 | -43.1733292 |
| 5 | 0.03514208 | 0.03897414 | 0.01813080 | 0.47373431 | -43.1733250 |
| 6 | 0.03514212 | 0.03897418 | 0.01812838 | 0.47373448 | -43.1732666 |
| 7 | 0.03514212 | 0.03897418 | 0.01812839 | 0.47373448 | -43.1732665 |
| 8 | 0.03526054 | 0.03905754 | 0.01803283 | 0.47372611 | -43.0287083 |
| 9 | 0.03359189 | 0.03900137 | 0.01998525 | 0.56812185 | -54.0250887 |
| 10 | 0.03359189 | 0.03900137 | 0.01998525 | 0.56812185 | -54.0250887 |
| 11 | 0.03359189 | 0.03900137 | 0.01998541 | 0.56812185 | -54.0250884 |
| 12 | 0.03456157 | 0.03918196 | 0.01931998 | 0.52089873 | -48.2624523 |
| 13 | 0.03581723 | 0.03945323 | 0.01829648 | 0.47366644 | -42.3425871 |
| 14 | 0.03581723 | 0.03945323 | 0.01829648 | 0.47366644 | -42.3425870 |
| 15 | 0.03581725 | 0.03945324 | 0.01829611 | 0.47366646 | -42.3425665 |
| 16 | 0.03636916 | 0.03985164 | 0.01891900 | 0.47362444 | -41.6517345 |
| 17 | 0.03767285 | 0.03997973 | 0.01800462 | 0.42638279 | -35.9801479 |
| 18 | 0.03767286 | 0.03997973 | 0.01800395 | 0.42638273 | -35.9801383 |
| 19 | 0.03767293 | 0.03997978 | 0.01800199 | 0.42638288 | -35.9800521 |
| 20 | 0.03767294 | 0.03997979 | 0.01800233 | 0.42638274 | -35.9800374 |

The following is the best solution found which has two still questionable small loadings for BC and DA of the first factor:

```
Loadings and Uni. Vars. with ASE's and Wald CIs
-----------------------------------------------

                   FAC_1                FAC_2
ID   16.701895  0.1315636          0          0
    [ 16.44404, 16.95976] [        0,         0]

DR   19.094064  0.1596966          0          0
    [ 18.78106, 19.40706] [        0,         0]

S7           0          0  20.625411  0.2761890
    [        0,         0] [ 20.08409, 21.16673]

BC  -1.6217927  0.2930368  8.5249130  0.3107876
    [-2.196134,-1.047451] [ 7.915781, 9.134045]

NA           0          0  6.2470714  0.1191234
    [        0,         0] [ 6.013594, 6.480549]
```

```
DA  -1.8444896  0.2650423  12.240044  0.2784739
    [-2.363963,-1.325016] [ 11.69424, 12.78584]

VO           0          0  12.707867  0.1717861
    [        0,         0] [ 12.37117, 13.04456]


                              U_Var
           ID   63.296695  2.5951222
                [ 58.21035, 68.38304]

           DR   132.70672  3.5700381
                [ 125.7096, 139.7039]

           S7   737.40242  9.9649026
                [ 717.8716, 756.9333]

           BC   389.23422  4.5227308
                [ 380.3698, 398.0986]

           NA   165.46410  1.9253255
                [ 161.6905, 169.2377]

           DA   157.23185  2.8118622
                [ 151.7207, 162.7430]

           VO   287.95011  3.8592242
                [ 280.3862, 295.5140]


            Factor Correlations Phi
            -----------------------

                        FAC_1                  FAC_2
  FAC_1   1.0000000          0  0.6990117  0.0075979
        [ 1.000000, 1.000000] [ 0.684120, 0.713903]

  FAC_2   0.6990117  0.0075979  1.0000000          0
        [ 0.684120, 0.713903] [ 1.000000, 1.000000]


(1) Standalone Fit Measures: -------------------------------
Fit criterion . . . . . . . . . . . . . . . . . .     0.0186
Normal Th. Chi-square (df = 11)       323.0210  prob= 0.0000
Normal Theory Reweighted LS Chi-square  . . . . .   323.6993
Probability of Close Fit  . . . . . . . . . . . .     1.0000
```

```
Z-Test of Wilson & Hilferty (1931). . . . . . . .   14.8126
(2) Incremental Fit Measures: -----------------------------
Null Model Chi-square (df = 21) . . . . . . . .  32421.2870
RMSEA Estimate  . . . . .  0.0404  90%C.I.[ 0.0367,  0.0443]
ECVI Estimate . . . . . .  0.0206  90%C.I.[ 0.0174,  0.0242]
McDonald's (1989) Centrality. . . . . . . . . . .    0.9910
Tucker-Lewis Coefficient TLI. . . . . . . . . . .    0.9816
Bentler & Bonett's (1980) NFI . . . . . . . . . .    0.9900
Bentler's Comparative Fit Index CFI . . . . . . .    0.9904
Parsimonious NFI (James, Mulaik, & Brett,1982). .    0.5186
Bollen (1986) Normed Index Rho1 . . . . . . . . .    0.9810
Bollen (1988) Non-normed Index Delta2 . . . . . .    0.9904
(3) Information Criteria: ---------------------------------
Akaike's Information Criterion. . . . . . . . . .  301.0210
Bozdogan's (1987) CAIC. . . . . . . . . . . . . .  204.6455
Schwarz's Bayesian Criterion. . . . . . . . . . .  215.6455
(4) Other Fit Measures: -----------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .    0.9947
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .    0.5210
GFI Adjusted for Degrees of Freedom (AGFI). . . .    0.9865
Root Mean Square Residual (RMR) . . . . . . . . .    7.4215
Hoelter's (1983) Critical N . . . . . . . . . . .      1058
```

Standardized Factor Loadings

| | FAC_1 | FAC_2 |
|---|---|---|
| ID | 0.9028051 | 0 |
| DR | 0.8562361 | 0 |
| S7 | 0 | 0.6048508 |
| BC | -0.0768622 | 0.4040243 |
| NA | 0 | 0.4368581 |
| DA | -0.1104485 | 0.7329367 |
| VO | 0 | 0.5994277 |

Squared Multiple Correlations

| N | Variable | ErrVariance | TotVariance | R-squared |
|---|---|---|---|---|
| 1 | ID | 63.2966950 | 342.250000 | 0.81505714 |
| 2 | DR | 132.706722 | 497.290000 | 0.73314018 |
| 3 | S7 | 737.402421 | 1162.81000 | 0.36584445 |
| 4 | BC | 389.234217 | 445.210000 | 0.12572894 |
| 5 | NA | 165.464099 | 204.490000 | 0.19084504 |
| 6 | DA | 157.231855 | 278.890000 | 0.43622269 |
| 7 | VO | 287.950109 | 449.440001 | 0.35931357 |

We now run an CFA with the two small loadings set to zero. The model would be:

```
l = [" L11   0., L21  0.,  0. L32, 0. L42,
       0.   L52, 0.  L62,  0. L72 "];
p = [" 1.  P21 , P21 1. "];
u = [" U1:U7 "];
list patt;
patt[1] = l; patt[2] = p; patt[3] = u;
```

First (for comparison of the $p$ value) we use $N = 1735$:

```
Loadings and Uni. Vars. with ASE's and Wald CIs
-----------------------------------------------

                    FAC_1                  FAC_2
ID   16.709322  0.4170521          0           0
    [ 15.89191, 17.52673] [        0,         0]

DR   19.085224  0.5061653          0           0
    [ 18.09316, 20.07729] [        0,         0]

S7           0          0  21.126660  0.8701078
    [        0,         0] [ 19.42128, 22.83204]

BC           0          0  7.2358149  0.5710882
    [        0,         0] [ 6.116503, 8.355127]

NA           0          0  6.2487945  0.3802425
    [        0,         0] [ 5.503533, 6.994056]

DA           0          0  10.606960  0.4247605
    [        0,         0] [ 9.774445, 11.43948]

VO           0          0  12.946389  0.5419721
    [        0,         0] [ 11.88414, 14.00864]


                                   U_Var
           ID   63.048560  8.2652702
               [ 46.84893, 79.24819]

           DR   133.04423  11.352653
               [ 110.7934, 155.2950]
```

```
S7   716.47425  31.245459
    [ 655.2343, 777.7142]

BC   392.85298  13.989364
    [ 365.4343, 420.2716]

NA   165.44257  6.1230591
    [ 153.4416, 177.4435]

DA   166.38240  7.4393188
    [ 151.8016, 180.9632]

VO   281.83100  12.131495
    [ 258.0537, 305.6083]


 Factor Correlations Phi
 -----------------------

                    FAC_1                 FAC_2
FAC_1   1.0000000          0   0.6651695  0.0211041
      [ 1.000000, 1.000000] [ 0.623806, 0.706533]

FAC_2   0.6651695  0.0211041  1.0000000          0
      [ 0.623806, 0.706533] [ 1.000000, 1.000000]
```

Note, that this solution was not included in the best twenty listed above since its $p$ is smaller:

```
(1) Standalone Fit Measures: -------------------------------
Fit criterion . . . . . . . . . . . . . . . . . . .    0.0230
Normal Th. Chi-square (df = 13) 39.9100 Prob>chi**2 = 0.0001
Normal Theory Reweighted LS Chi-square  . . . . .    39.2820
Probability of Close Fit  . . . . . . . . . . . .     0.9804
Z-Test of Wilson & Hilferty (1931). . . . . . . .     3.5984
(2) Incremental Fit Measures: ------------------------------
Null Model Chi-square (df = 21) . . . . . . . .    3240.2600
RMSEA Estimate  . . . . .  0.0346  90%C.I.[ 0.0227,  0.0470]
ECVI Estimate . . . . . .  0.0404  90%C.I.[ 0.0315,  0.0537]
McDonald's (1989) Centrality. . . . . . . . . . .     0.9923
Tucker-Lewis Coefficient TLI. . . . . . . . . . .     0.9865
Bentler & Bonett's (1980) NFI . . . . . . . . . .     0.9877
Bentler's Comparative Fit Index CFI . . . . . . .     0.9916
Parsimonious NFI (James, Mulaik, & Brett,1982). .     0.6114
```

```
Bollen (1986) Normed Index Rho1 . . . . . . . . . .     0.9801
Bollen (1988) Non-normed Index Delta2 . . . . . .     0.9917
(3) Information Criteria: ---------------------------------
Akaike's Information Criterion. . . . . . . . . . .    13.9100
Bozdogan's (1987) CAIC. . . . . . . . . . . . . . .   -70.0539
Schwarz's Bayesian Criterion. . . . . . . . . . . .   -57.0539
(4) Other Fit Measures: -----------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . . .     0.9936
Parsimonious GFI (Mulaik, 1989) . . . . . . . . . .     0.6151
GFI Adjusted for Degrees of Freedom (AGFI). . . .     0.9861
Root Mean Square Residual (RMR) . . . . . . . . . .     7.6173
Hoelter's (1983) Critical N . . . . . . . . . . . .        973


                Standardized Factor Loadings

                     FAC_1      FAC_2
            ID   0.9032066          0
            DR   0.8558396          0
            S7           0  0.6195501
            BC           0  0.3429296
            NA           0  0.4369786
            DA           0  0.6351473
            VO           0  0.6106787


               Squared Multiple Correlations

        N Variable ErrVariance TotVariance   R-squared
        1 ID        63.0485601  342.250000  0.81578215
        2 DR        133.044230  497.290000  0.73246148
        3 S7        716.474250  1162.81000  0.38384237
        4 BC        392.852982  445.210000  0.11760072
        5 NA        165.442567  204.490000  0.19095033
        6 DA        166.382400  278.890000  0.40341210
        7 VO        281.830999  449.440000  0.37292854
```

For $N = 1735$ the $p$ value is zero and there is not much difference of this sparse model with the optimal model found above:

```
Loadings and Uni. Vars. with ASE's and Wald CIs
-----------------------------------------------

                     FAC_1                 FAC_2
ID   16.709322  0.1318454         0           0
    [ 16.45091, 16.96773] [       0,          0]
```

```
DR   19.085224  0.1600174          0          0
    [ 18.77160, 19.39885] [        0,        0]

S7          0          0  21.126660  0.2750729
    [        0,        0] [ 20.58753, 21.66579]

BC          0          0  7.2358149  0.1805419
    [        0,        0] [ 6.881959, 7.589671]

NA          0          0  6.2487945  0.1202086
    [        0,        0] [ 6.013190, 6.484399]

DA          0          0  10.606960  0.1342824
    [        0,        0] [ 10.34377, 10.87015]

VO          0          0  12.946389  0.1713372
    [        0,        0] [ 12.61057, 13.28220]


                         U_Var
          ID   63.048560  2.6129546
              [ 57.92726, 68.16986]

          DR   133.04423  3.5889894
              [ 126.0099, 140.0785]

          S7   716.47425  9.8778338
              [ 697.1141, 735.8344]

          BC   392.85298  4.4225502
              [ 384.1849, 401.5210]

          NA   165.44257  1.9357232
              [ 161.6486, 169.2365]

          DA   166.38240  2.3518411
              [ 161.7729, 170.9919]

          VO   281.83100  3.8352098
              [ 274.3141, 289.3479]


           Factor Correlations Phi
           -----------------------
```

```
                    FAC_1                 FAC_2
FAC_1   1.0000000         0  0.6651695  0.0066718
       [ 1.000000, 1.000000] [ 0.652093, 0.678246]

FAC_2   0.6651695  0.0066718  1.0000000         0
       [ 0.652093, 0.678246] [ 1.000000, 1.000000]


(1) Standalone Fit Measures: -------------------------------
Fit criterion . . . . . . . . . . . . . . . . . .    0.0230
Normal Th. Chi-square (df = 13)       399.3305  prob= 0.0000
Normal Theory Reweighted LS Chi-square  . . . . .  393.0466
Probability of Close Fit  . . . . . . . . . . . .    1.0000
Z-Test of Wilson & Hilferty (1931). . . . . . . .   16.4360
(2) Incremental Fit Measures: -----------------------------
Null Model Chi-square (df = 21) . . . . . . . .  32421.2870
RMSEA Estimate  . . . . .  0.0414  90%C.I.[ 0.0379,  0.0449]
ECVI Estimate . . . . . .  0.0247  90%C.I.[ 0.0212,  0.0287]
McDonald's (1989) Centrality. . . . . . . . . . .    0.9889
Tucker-Lewis Coefficient TLI. . . . . . . . . . .    0.9807
Bentler & Bonett's (1980) NFI . . . . . . . . . .    0.9877
Bentler's Comparative Fit Index CFI . . . . . . .    0.9881
Parsimonious NFI (James, Mulaik, & Brett,1982). .    0.6114
Bollen (1986) Normed Index Rho1 . . . . . . . . .    0.9801
Bollen (1988) Non-normed Index Delta2 . . . . . .    0.9881
(3) Information Criteria: ---------------------------------
Akaike's Information Criterion. . . . . . . . . .  373.3305
Bozdogan's (1987) CAIC. . . . . . . . . . . . . .  259.4322
Schwarz's Bayesian Criterion. . . . . . . . . . .  272.4322
(4) Other Fit Measures: -----------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .    0.9936
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .    0.6151
GFI Adjusted for Degrees of Freedom (AGFI). . . .    0.9861
Root Mean Square Residual (RMR) . . . . . . . . .    7.6173
Hoelter's (1983) Critical N . . . . . . . . . . .       973


                Standardized Factor Loadings

                      FAC_1      FAC_2
                ID   0.9032066          0
                DR   0.8558396          0
                S7           0  0.6195501
                BC           0  0.3429296
                NA           0  0.4369786
                DA           0  0.6351473
```

```
VO          0  0.6106787
```

```
              Squared Multiple Correlations

N Variable ErrVariance TotVariance   R-squared
1 ID        63.0485601  342.250000  0.81578215
2 DR        133.044230  497.290000  0.73246148
3 S7        716.474250  1162.81000  0.38384237
4 BC        392.852982  445.210000  0.11760072
5 NA        165.442567  204.490000  0.19095033
6 DA        166.382400  278.890000  0.40341210
7 VO        281.830999  449.440000  0.37292854
```

### 2.6.2 ML Analysis of the Correlation Matrix: nf=2

We start with the EFA result, which again has a very small $p$ value, and therefore the number of observations is reduced by dividing with 10, but only for the purpose of a meaningful optimization:

```
*************************************************************
Starting Model Improvement (ChiSq=310.762 p=2.10495e-062 DF=8)
         Maximizing the p Value for 1735 Observations
*************************************************************
```

That changes the to some better values of $\chi^2 = 31.06$ and $p = 1.e-4$, the same as for analysing the covariance matrix. The following results are very similar to those of the analysis of the covariance matrix:

```
**********************************
Table of 20 Best Pattern Solutions
**********************************

N  Npar    DF  Chisquared          P

1    18    10  31.3322096  5.170e-004
2    17    11  33.8494784  3.828e-004
3    19     9  31.1487068  2.791e-004
4    19     9  31.1487069  2.791e-004
5    19     9  31.1487070  2.791e-004
6    19     9  31.2947461  2.635e-004
7    19     9  31.2947461  2.635e-004
8    18    10  33.8306223  1.973e-004
9    20     8  31.0583333  1.372e-004
10   20     8  31.0583335  1.372e-004
```

| | | | | |
|---|---|---|---|---|
| 11 | 20 | 8 | 31.0583338 | 1.372e-004 |
| 12 | 20 | 8 | 31.0583338 | 1.372e-004 |
| 13 | 20 | 8 | 31.0583341 | 1.372e-004 |
| 14 | 20 | 8 | 31.0583343 | 1.372e-004 |
| 15 | 20 | 8 | 31.0583344 | 1.372e-004 |
| 16 | 20 | 8 | 31.0583346 | 1.372e-004 |
| 17 | 20 | 8 | 31.0583346 | 1.372e-004 |
| 18 | 18 | 10 | 96.6995912 | 2.489e-016 |
| 19 | 17 | 11 | 183.284372 | 2.150e-033 |
| 20 | 20 | 8 | 310.762474 | 2.105e-062 |

| N | RMSEA | ECVI | SRMR | PGFI | SBC |
|---|---|---|---|---|---|
| 1 | 0.03507465 | 0.03892680 | 0.01811613 | 0.47374445 | -43.2554174 |
| 2 | 0.03461125 | 0.03921976 | 0.01908267 | 0.52090991 | -48.1969113 |
| 3 | 0.03767284 | 0.03997972 | 0.01800534 | 0.42638272 | -35.9801574 |
| 4 | 0.03767284 | 0.03997972 | 0.01800547 | 0.42638271 | -35.9801573 |
| 5 | 0.03767284 | 0.03997972 | 0.01800548 | 0.42638271 | -35.9801572 |
| 6 | 0.03779684 | 0.04006394 | 0.01812341 | 0.42637095 | -35.8341182 |
| 7 | 0.03779684 | 0.04006394 | 0.01812344 | 0.42637095 | -35.8341181 |
| 8 | 0.03707175 | 0.04036764 | 0.01907936 | 0.47355761 | -40.7570046 |
| 9 | 0.04077035 | 0.04108635 | 0.01801949 | 0.37901031 | -28.6117682 |
| 10 | 0.04077035 | 0.04108635 | 0.01801944 | 0.37901031 | -28.6117680 |
| 11 | 0.04077035 | 0.04108635 | 0.01801934 | 0.37901031 | -28.6117678 |
| 12 | 0.04077035 | 0.04108635 | 0.01801956 | 0.37901031 | -28.6117677 |
| 13 | 0.04077035 | 0.04108635 | 0.01801969 | 0.37901030 | -28.6117675 |
| 14 | 0.04077035 | 0.04108635 | 0.01801971 | 0.37901030 | -28.6117672 |
| 15 | 0.04077035 | 0.04108635 | 0.01801930 | 0.37901031 | -28.6117672 |
| 16 | 0.04077035 | 0.04108635 | 0.01801970 | 0.37901030 | -28.6117670 |
| 17 | 0.04077035 | 0.04108635 | 0.01801933 | 0.37901031 | -28.6117669 |
| 18 | 0.07071051 | 0.07662425 | 0.06008571 | 0.46882931 | 22.1119642 |
| 19 | 0.09503904 | 0.12539906 | 0.08023390 | 0.50895354 | 101.237983 |
| 20 | 0 | 0 | 0 | 0 | 0 |

The solution has three small loadings:

```
Loadings and Uni. Vars. with ASE's and Wald CIs
-----------------------------------------------

                   FAC_1                 FAC_2
ID   0.9028836  0.0071108          0          0
    [ 0.888947, 0.916820] [        0,         0]

DR   0.8561902  0.0071607          0          0
```

```
    [ 0.842155, 0.870225] [        0,        0]

S7   0.0717923  0.0131783  0.5653448  0.0136323
    [ 0.045963, 0.097621] [ 0.538626, 0.592064]

BC          0          0  0.3503089  0.0087428
    [        0,        0] [ 0.333173, 0.367444]

NA   0.0955753  0.0120580  0.3658937  0.0128627
    [ 0.071942, 0.119209] [ 0.340683, 0.391104]

DA          0          0  0.6594657  0.0088863
    [        0,        0] [ 0.642049, 0.676883]

VO   0.0926820  0.0128616  0.5399763  0.0133247
    [ 0.067474, 0.117890] [ 0.513860, 0.566092]


                           U_Var
           ID   0.1848012  0.0075812
               [ 0.169942, 0.199660]

           DR   0.2669384  0.0071769
               [ 0.252872, 0.281005]

           S7   0.6260197  0.0091663
               [ 0.608054, 0.643985]

           BC   0.8772837  0.0099841
               [ 0.857715, 0.896852]

           NA   0.8145862  0.0095345
               [ 0.795899, 0.833273]

           DA   0.5651050  0.0097836
               [ 0.545930, 0.584280]

           VO   0.6391557  0.0089937
               [ 0.621528, 0.656783]


            Factor Correlations Phi
            -----------------------

                     FAC_1                FAC_2
```

```
    FAC_1   1.0000000          0  0.6062407  0.0100635
          [ 1.000000, 1.000000] [ 0.586517, 0.625965]

    FAC_2   0.6062407  0.0100635  1.0000000          0
          [ 0.586517, 0.625965] [ 1.000000, 1.000000]
```

```
(1) Standalone Fit Measures: ------------------------------
Fit criterion . . . . . . . . . . . . . . . . . .    0.0181
Normal Th. Chi-square (df = 10)       313.5028  prob= 0.0000
Normal Theory Reweighted LS Chi-square  . . . . .  313.5333
Probability of Close Fit  . . . . . . . . . . . .    0.9996
Z-Test of Wilson & Hilferty (1931). . . . . . . .   14.5930
(2) Incremental Fit Measures: -----------------------------
Null Model Chi-square (df = 21) . . . . . . . .  32421.2870
RMSEA Estimate  . . . . .  0.0418  90%C.I.[ 0.0379,  0.0459]
ECVI Estimate . . . . . .  0.0201  90%C.I.[ 0.0170,  0.0237]
McDonald's (1989) Centrality. . . . . . . . . . .    0.9913
Tucker-Lewis Coefficient TLI. . . . . . . . . . .    0.9803
Bentler & Bonett's (1980) NFI . . . . . . . . . .    0.9903
Bentler's Comparative Fit Index CFI . . . . . . .    0.9906
Parsimonious NFI (James, Mulaik, & Brett,1982). .    0.4716
Bollen (1986) Normed Index Rho1 . . . . . . . . .    0.9797
Bollen (1988) Non-normed Index Delta2 . . . . . .    0.9906
(3) Information Criteria: ---------------------------------
Akaike's Information Criterion. . . . . . . . . .  293.5028
Bozdogan's (1987) CAIC. . . . . . . . . . . . . .  205.8887
Schwarz's Bayesian Criterion. . . . . . . . . . .  215.8887
(4) Other Fit Measures: -----------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .    0.9949
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .    0.4737
GFI Adjusted for Degrees of Freedom (AGFI). . . .    0.9856
Root Mean Square Residual (RMR) . . . . . . . . .    0.0181
Hoelter's (1983) Critical N . . . . . . . . . . .      1015
```

```
                 Standardized Factor Loadings

                          FAC_1      FAC_2
                  ID   0.9028836          0
                  DR   0.8561902          0
                  S7   0.0717923  0.5653448
                  BC           0  0.3503089
                  NA   0.0955753  0.3658937
                  DA           0  0.6594657
                  VO   0.0926820  0.5399763
```

```
                Squared Multiple Correlations

   N Variable ErrVariance TotVariance   R-squared
   1 ID        0.18480123  1.00000000  0.81519877
   2 DR        0.26693836  1.00000000  0.73306164
   3 S7        0.62601966  1.00000000  0.37398034
   4 BC        0.87728368  1.00000000  0.12271633
   5 NA        0.81458619  1.00000000  0.18541381
   6 DA        0.56510499  1.00000000  0.43489501
   7 VO        0.63915573  1.00000000  0.36084427
```

Now similar to the analysis of the covariance matrix, we run an CFA for $N = 1735$ with a specified pattern where the small zero loadings are set to zero to compare the $p$ value of that solution with that obtained by optimization:

```
Loadings and Uni. Vars. with ASE's and Wald CIs
-----------------------------------------------

                   FAC_1                 FAC_2
ID   0.9032066  0.0225434          0         0
    [ 0.859022, 0.947391] [        0,        0]

DR   0.8558396  0.0226980          0         0
    [ 0.811352, 0.900327] [        0,        0]

S7           0          0  0.6195501  0.0255164
    [        0,         0] [ 0.569539, 0.669561]

BC           0          0  0.3429296  0.0270658
    [        0,         0] [ 0.289882, 0.395978]

NA           0          0  0.4369786  0.0265904
    [        0,         0] [ 0.384862, 0.489095]

DA           0          0  0.6351473  0.0254348
    [        0,         0] [ 0.585296, 0.684999]

VO           0          0  0.6106787  0.0255647
    [        0,         0] [ 0.560573, 0.660785]


                                U_Var
           ID   0.1842179  0.0241498
               [ 0.136885, 0.231551]
```

```
              DR   0.2675385  0.0228290
                  [ 0.222794, 0.312283]

              S7   0.6161576  0.0268706
                  [ 0.563492, 0.668823]

              BC   0.8823993  0.0314219
                  [ 0.820813, 0.943985]

              NA   0.8090497  0.0299431
                  [ 0.750362, 0.867737]

              DA   0.5965879  0.0266747
                  [ 0.544306, 0.648869]

              VO   0.6270715  0.0269925
                  [ 0.574167, 0.679976]


               Factor Correlations Phi
               -----------------------

                         FAC_1                 FAC_2
   FAC_1   1.0000000          0  0.6651695  0.0211041
          [ 1.000000, 1.000000] [ 0.623806, 0.706533]

   FAC_2   0.6651695  0.0211041  1.0000000          0
          [ 0.623806, 0.706533] [ 1.000000, 1.000000]



(1) Standalone Fit Measures: ------------------------------
Fit criterion . . . . . . . . . . . . . . . . . . .    0.0230
Normal Th. Chi-square (df = 13) 39.9100 Prob>chi**2 = 0.0001
Normal Theory Reweighted LS Chi-square  . . . . .   39.2820
Probability of Close Fit  . . . . . . . . . . . .    0.9804
Z-Test of Wilson & Hilferty (1931). . . . . . . .    3.5984
(2) Incremental Fit Measures: -----------------------------
Null Model Chi-square (df = 21) . . . . . . . .   3240.2600
RMSEA Estimate  . . . . .  0.0346  90%C.I.[ 0.0227,  0.0470]
ECVI Estimate . . . . . .  0.0404  90%C.I.[ 0.0315,  0.0537]
McDonald's (1989) Centrality. . . . . . . . . . .    0.9923
Tucker-Lewis Coefficient TLI. . . . . . . . . . .    0.9865
Bentler & Bonett's (1980) NFI . . . . . . . . . .    0.9877
```

```
Bentler's Comparative Fit Index CFI . . . . . . .     0.9916
Parsimonious NFI (James, Mulaik, & Brett,1982). .     0.6114
Bollen (1986) Normed Index Rho1 . . . . . . . . .     0.9801
Bollen (1988) Non-normed Index Delta2 . . . . . .     0.9917
(3) Information Criteria: --------------------------------
Akaike's Information Criterion. . . . . . . . . .    13.9100
Bozdogan's (1987) CAIC. . . . . . . . . . . . . .   -70.0539
Schwarz's Bayesian Criterion. . . . . . . . . . .   -57.0539
(4) Other Fit Measures: ----------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .     0.9936
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .     0.6151
GFI Adjusted for Degrees of Freedom (AGFI). . . .     0.9861
Root Mean Square Residual (RMR) . . . . . . . . .     0.0214
Hoelter's (1983) Critical N . . . . . . . . . . .        973


                Standardized Factor Loadings

                         FAC_1      FAC_2
                 ID   0.9032066          0
                 DR   0.8558396          0
                 S7           0  0.6195501
                 BC           0  0.3429296
                 NA           0  0.4369786
                 DA           0  0.6351473
                 VO           0  0.6106787


               Squared Multiple Correlations

        N Variable ErrVariance TotVariance   R-squared
        1 ID        0.18421785  1.00000000  0.81578215
        2 DR        0.26753852  1.00000000  0.73246148
        3 S7        0.61615763  1.00000000  0.38384237
        4 BC        0.88239928  1.00000000  0.11760072
        5 NA        0.80904967  1.00000000  0.19095033
        6 DA        0.59658790  1.00000000  0.40341210
        7 VO        0.62707146  1.00000000  0.37292854
```

If we specify all $N = 17351$ observations the $p$ value becomes practically the same for both models, the optimized and the very sparse:

```
         Loadings and Uni. Vars. with ASE's and Wald CIs
          -----------------------------------------------

                         FAC_1                  FAC_2
```

```
ID   0.9032066  0.0071268          0          0
    [ 0.889238, 0.917175] [        0,         0]

DR   0.8558396  0.0071757          0          0
    [ 0.841776, 0.869904] [        0,         0]

S7           0          0  0.6195501  0.0080667
    [        0,         0] [ 0.603740, 0.635360]

BC           0          0  0.3429296  0.0085565
    [        0,         0] [ 0.326159, 0.359700]

NA           0          0  0.4369786  0.0084062
    [        0,         0] [ 0.420503, 0.453454]

DA           0          0  0.6351473  0.0080409
    [        0,         0] [ 0.619388, 0.650907]

VO           0          0  0.6106787  0.0080819
    [        0,         0] [ 0.594838, 0.626519]


                       U_Var
           ID   0.1842179  0.0076346
               [ 0.169254, 0.199181]

           DR   0.2675385  0.0072171
               [ 0.253393, 0.281684]

           S7   0.6161576  0.0084948
               [ 0.599508, 0.632807]

           BC   0.8823993  0.0099336
               [ 0.862930, 0.901869]

           NA   0.8090497  0.0094661
               [ 0.790496, 0.827603]

           DA   0.5965879  0.0084329
               [ 0.580060, 0.613116]

           VO   0.6270715  0.0085333
               [ 0.610346, 0.643796]
```

Factor Correlations Phi

```
                      ----------------------

                    FAC_1                FAC_2
    FAC_1   1.0000000          0  0.6651695  0.0066718
           [ 1.000000, 1.000000] [ 0.652093, 0.678246]

    FAC_2   0.6651695  0.0066718  1.0000000          0
           [ 0.652093, 0.678246] [ 1.000000, 1.000000]


(1) Standalone Fit Measures: ------------------------------
Fit criterion . . . . . . . . . . . . . . . . . .    0.0230
Normal Th. Chi-square (df = 13)       399.3305  prob= 0.0000
Normal Theory Reweighted LS Chi-square  . . . . .  393.0466
Probability of Close Fit  . . . . . . . . . . . .    1.0000
Z-Test of Wilson & Hilferty (1931). . . . . . . .   16.4360
(2) Incremental Fit Measures: -----------------------------
Null Model Chi-square (df = 21) . . . . . . . .  32421.2870
RMSEA Estimate  . . . . .  0.0414  90%C.I.[ 0.0379,  0.0449]
ECVI Estimate . . . . . .  0.0247  90%C.I.[ 0.0212,  0.0287]
McDonald's (1989) Centrality. . . . . . . . . . .    0.9889
Tucker-Lewis Coefficient TLI. . . . . . . . . . .    0.9807
Bentler & Bonett's (1980) NFI . . . . . . . . . .    0.9877
Bentler's Comparative Fit Index CFI . . . . . . .    0.9881
Parsimonious NFI (James, Mulaik, & Brett,1982). .    0.6114
Bollen (1986) Normed Index Rho1 . . . . . . . . .    0.9801
Bollen (1988) Non-normed Index Delta2 . . . . . .    0.9881
(3) Information Criteria: ---------------------------------
Akaike's Information Criterion. . . . . . . . . .  373.3305
Bozdogan's (1987) CAIC. . . . . . . . . . . . . .  259.4322
Schwarz's Bayesian Criterion. . . . . . . . . . .  272.4322
(4) Other Fit Measures: -----------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .    0.9936
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .    0.6151
GFI Adjusted for Degrees of Freedom (AGFI). . . .    0.9861
Root Mean Square Residual (RMR) . . . . . . . . .    0.0214
Hoelter's (1983) Critical N . . . . . . . . . . .       973


               Standardized Factor Loadings

                    FAC_1      FAC_2
               ID   0.9032066          0
               DR   0.8558396          0
               S7           0  0.6195501
               BC           0  0.3429296
```

```
NA          0  0.4369786
DA          0  0.6351473
VO          0  0.6106787
```

```
         Squared Multiple Correlations

N Variable ErrVariance TotVariance   R-squared
1 ID        0.18421785  1.00000000  0.81578215
2 DR        0.26753852  1.00000000  0.73246148
3 S7        0.61615763  1.00000000  0.38384237
4 BC        0.88239928  1.00000000  0.11760072
5 NA        0.80904967  1.00000000  0.19095033
6 DA        0.59658790  1.00000000  0.40341210
7 VO        0.62707146  1.00000000  0.37292854
```

### 2.6.3 EFA of ML with Oblique GEOMIN Rotation: nf=2

First we show the results of the analysis of the covariance matrix. Jennrichs iterative algorithm converges after 19 iterations:

```
   Gradient Projection Algorithm (Bernaards & Jennrich)

          Oblique Rotation: Geomin (Yates, 1984)

          Iter     OptCrit      Termin        Alfa

             1  425.046675  2.63051184  1.00000000
             2  657.110929  3.71264402  0.00195313
             3  240.936733  3.27096778  3.052e-005
             4  68.0141593  2.94238086  6.104e-005
             5  58.8152967  2.80519448  1.526e-005
             6  55.3097263  2.28521252  1.526e-005
             7  55.1668107  1.02729717  7.629e-006
             8  55.1665080  0.63125714  3.815e-006
             9  55.1664587  0.25091438  3.815e-006
            10  55.1664500 -0.09719278  3.815e-006
            11  55.1664482 -0.39158929  3.815e-006
            12  55.1664475 -0.90761390  7.629e-006
            13  55.1664474 -1.38209697  7.629e-006
            14  55.1664474 -1.83779342  7.629e-006
            15  55.1664474 -2.28654657  7.629e-006
            16  55.1664474 -2.73293150  7.629e-006
            17  55.1664474 -3.17853158  7.629e-006
            18  55.1664474 -3.62387402  7.629e-006
```

```
19  55.1664474 -4.06913215  7.629e-006
```

```
 Rotated Factor Loadings with Standard Errors
 --------------------------------------------

                  FAC_1                 FAC_2
ID   15.530995  0.9537941  1.0518155  0.8702244
    [ 13.66159, 17.40040] [-0.653793, 2.757424]

DR   19.697458  0.4837415 -0.0613617  0.0182379
    [ 18.74934, 20.64557] [-0.097107,-0.025616]

S7   0.0130254  0.0032414  20.870963  0.2914997
    [ 6.7e-003, 0.019378] [ 20.29963, 21.44229]

BC  -1.2889542  0.3005380  8.2662530  0.3068495
    [-1.877998,-0.699910] [ 7.664839, 8.867667]

NA   0.7323044  0.1982060  5.6447326  0.2014838
    [ 0.343828, 1.120781] [ 5.249832, 6.039634]

DA  -1.2280460  0.2957971  11.722542  0.2843065
    [-1.807798,-0.648294] [ 11.16531, 12.27977]

VO   0.3902479  0.3277066  12.501844  0.3160164
    [-0.252045, 1.032541] [ 11.88246, 13.12122]
```

```
                      U_Var
ID   78.338930  11.769961
    [ 55.27023, 101.4076]

DR   110.89552  18.835943
    [ 73.97775, 147.8133]

S7   726.85354  10.646377
    [ 705.9870, 747.7201]

BC   389.30167  4.5229628
    [ 380.4368, 398.1665]

NA   166.62672  1.9481911
    [ 162.8083, 170.4451]
```

```
                DA   158.99286  2.7792911
                    [ 153.5456, 164.4402]

                VO   286.54292  4.0563810
                    [ 278.5926, 294.4933]


         Factor Correlations with Standard Errors
         ----------------------------------------

                         FAC_1                 FAC_2
   FAC_1   1.0000000         0  0.6609174  0.0196315
          [ 1.000000, 1.000000] [ 0.622440, 0.699394]

   FAC_2   0.6609174  0.0196315  1.0000000         0
          [ 0.622440, 0.699394] [ 1.000000, 1.000000]


(1) Standalone Fit Measures: -------------------------------
Fit criterion . . . . . . . . . . . . . . . . . .     0.0179
Normal Th. Chi-square (df = 8) 310.7632 Prob>chi**2 = 0.0000
Bartlett Corrected Chi-square (df = 8)    310.6826 p= 0.0000
Normal Theory Reweighted LS Chi-square  . . . . .   311.1505
Probability of Close Fit  . . . . . . . . . . . .     0.8835
Z-Test of Wilson & Hilferty (1931). . . . . . . .    14.4870
(2) Incremental Fit Measures: ------------------------------
Null Model Chi-square (df = 21) . . . . . . . .   32421.2870
RMSEA Estimate  . . . . .  0.0467  90%C.I.[ 0.0423,  0.0512]
ECVI Estimate . . . . . .  0.0202  90%C.I.[ 0.0171,  0.0238]
McDonald's (1989) Centrality. . . . . . . . . . .     0.9913
Tucker-Lewis Coefficient TLI. . . . . . . . . . .     0.9755
Bentler & Bonett's (1980) NFI . . . . . . . . . .     0.9904
Bentler's Comparative Fit Index CFI . . . . . . .     0.9907
Parsimonious NFI (James, Mulaik, & Brett,1982). .     0.3773
Bollen (1986) Normed Index Rho1 . . . . . . . . .     0.9748
Bollen (1988) Non-normed Index Delta2 . . . . . .     0.9907
(3) Information Criteria: ----------------------------------
Akaike's Information Criterion. . . . . . . . . .   294.7632
Bozdogan's (1987) CAIC. . . . . . . . . . . . . .   224.6719
Schwarz's Bayesian Criterion. . . . . . . . . . .   232.6719
(4) Other Fit Measures: ------------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .     0.9949
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .     0.3790
GFI Adjusted for Degrees of Freedom (AGFI). . . .     0.9822
Root Mean Square Residual (RMR) . . . . . . . . .     6.8481
Hoelter's (1983) Critical N . . . . . . . . . . .        867
```

Standardized Factor Loadings

| | FAC_1 | FAC_2 |
|---|---|---|
| ID | 0.8395130 | 0.0568549 |
| DR | 0.8832928 | -0.0027516 |
| S7 | 3.82e-004 | 0.6120517 |
| BC | -0.0610879 | 0.3917655 |
| NA | 0.0512101 | 0.3947365 |
| DA | -0.0735357 | 0.7019486 |
| VO | 0.0184079 | 0.5897094 |

Squared Multiple Correlations

| N | Variable | ErrVariance | TotVariance | R-squared |
|---|---|---|---|---|
| 1 | ID | 78.3389298 | 342.250212 | 0.77110626 |
| 2 | DR | 110.895518 | 497.291463 | 0.77700096 |
| 3 | S7 | 726.853535 | 1162.81014 | 0.37491641 |
| 4 | BC | 389.301666 | 445.210095 | 0.12557763 |
| 5 | NA | 166.626719 | 204.490014 | 0.18515963 |
| 6 | DA | 158.992862 | 278.890055 | 0.42990846 |
| 7 | VO | 286.542923 | 449.440306 | 0.36244498 |

```
    Total Determination of All Equations = 0.947
Total Determination of Manifest Variables = 0.946
```

Now we show the results of the analysis of the correlation matrix. The GEOMIN algorithm converges after 17 iterations:

Gradient Projection Algorithm (Bernaards & Jennrich)

Oblique Rotation: Geomin (Yates, 1984)

| Iter | OptCrit | Termin | Alfa |
|---|---|---|---|
| 1 | 0.96691234 | -0.06275166 | 1.00000000 |
| 2 | 0.90040283 | -0.14032784 | 0.12500000 |
| 3 | 0.79895666 | -0.03464587 | 0.25000000 |
| 4 | 0.73227476 | -0.06755507 | 0.12500000 |
| 5 | 0.66581678 | -0.07297478 | 0.12500000 |
| 6 | 0.56335375 | 0.16482321 | 0.25000000 |
| 7 | 0.51285785 | -0.09648090 | 0.03125000 |
| 8 | 0.48638071 | -0.36551510 | 0.06250000 |
| 9 | 0.47762486 | -0.65724936 | 0.06250000 |

```
   10  0.47545658 -1.03236771  0.06250000
   11  0.47508596 -1.46133437  0.06250000
   12  0.47503540 -1.91434928  0.06250000
   13  0.47502916 -2.37652824  0.06250000
   14  0.47502842 -2.84196567  0.06250000
   15  0.47502833 -3.30852866  0.06250000
   16  0.47502832 -3.77547702  0.06250000
   17  0.47502832 -4.24255665  0.06250000

 Rotated Factor Loadings with Standard Errors
  --------------------------------------------

                    FAC_1                FAC_2
ID   0.8527124  0.0385122  0.0398485  0.0291566
    [ 0.777230, 0.928195] [-0.017297, 0.096994]

DR   0.8917996  0.0319867 -0.0171253  0.0172974
    [ 0.829107, 0.954492] [-0.051027, 0.016777]

S7   0.0384313  0.0099825  0.5871692  0.0114282
    [ 0.018866, 0.057997] [ 0.564770, 0.609568]

BC  -0.0374927  0.0112693  0.3769952  0.0124535
    [-0.059580,-0.015405] [ 0.352587, 0.401404]

NA   0.0762644  0.0117185  0.3778781  0.0125682
    [ 0.053296, 0.099232] [ 0.353245, 0.402511]

DA  -0.0307357  0.0070806  0.6747248  0.0100797
    [-0.044613,-0.016858] [ 0.654969, 0.694481]

VO   0.0551056  0.0106668  0.5655979  0.0117597
    [ 0.034199, 0.076012] [ 0.542549, 0.588646]


                              U_Var
           ID   0.2281843  0.0053683
               [ 0.217663, 0.238706]

           DR   0.2237762  0.0053683
               [ 0.213255, 0.234298]

           S7   0.6251264  0.0053683
               [ 0.614605, 0.635648]
```

```
BC   0.8744014  0.0053683
    [ 0.863880, 0.884923]

NA   0.8148298  0.0053683
    [ 0.804308, 0.825351]

DA   0.5701122  0.0053683
    [ 0.559591, 0.580634]

VO   0.6375202  0.0053683
    [ 0.626999, 0.648042]
```

Factor Correlations with Standard Errors
----------------------------------------

```
                        FAC_1                   FAC_2
FAC_1   1.0000000           0   0.6343478  0.0089093
       [ 1.000000, 1.000000] [ 0.616886, 0.651810]

FAC_2   0.6343478  0.0089093   1.0000000           0
       [ 0.616886, 0.651810] [ 1.000000, 1.000000]
```

Standardized Factor Loadings

| | FAC_1 | FAC_2 |
|---|---|---|
| ID | 0.8527124 | 0.0398485 |
| DR | 0.8917996 | -0.0171253 |
| S7 | 0.0384313 | 0.5871692 |
| BC | -0.0374927 | 0.3769952 |
| NA | 0.0762644 | 0.3778781 |
| DA | -0.0307357 | 0.6747248 |
| VO | 0.0551056 | 0.5655979 |

Squared Multiple Correlations

| N | Variable | ErrVariance | TotVariance | R-squared |
|---|---|---|---|---|
| 1 | ID | 0.22818428 | 1.00000000 | 0.77181572 |
| 2 | DR | 0.22377615 | 1.00000001 | 0.77622385 |
| 3 | S7 | 0.62512636 | 1.00000000 | 0.37487364 |
| 4 | BC | 0.87440136 | 1.00000000 | 0.12559864 |
| 5 | NA | 0.81482981 | 1.00000000 | 0.18517019 |
| 6 | DA | 0.57011220 | 1.00000000 | 0.42988780 |
| 7 | VO | 0.63752022 | 1.00000000 | 0.36247978 |

```
      Total Determination of All Equations = 0.947
   Total Determination of Manifest Variables = 0.946
```

As I was told, there are some differences between these results and those reported by McArdle (2011).

#### 2.6.4 ML Analysis of the Covariance Matrix: nf=3

Now, we optimize w.r.t. three factors. For ML estimates with the EFA model we obtain.

```
***************************************************************
 Starting Model Improvement (ChiSq=26.9934 p=5.90626e-006 DF=3)
          Maximizing the p Value for 173 Observations
***************************************************************
```

Due to the large number of observations (17251) the $p$ value is close to zero:

```
(1) Standalone Fit Measures: ------------------------------
Fit criterion . . . . . . . . . . . . . . . . . .    0.0016
Normal Th. Chi-square (df = 3). 26.9934 Prob>chi**2 = 0.0000
Bartlett Corrected Chi-square (df = 3) 26.9853  prob= 0.0000
Normal Theory Reweighted LS Chi-square  . . . . .   26.9637
Probability of Close Fit  . . . . . . . . . . . .    1.0000
Z-Test of Wilson & Hilferty (1931). . . . . . . .    4.2400
(2) Incremental Fit Measures: -----------------------------
Null Model Chi-square (df = 21) . . . . . . . .   32421.2870
RMSEA Estimate  . . . . .  0.0215  90%C.I.[ 0.0145,  0.0292]
ECVI Estimate . . . . . .  0.0044  90%C.I.[ 0.0037,  0.0056]
McDonald's (1989) Centrality. . . . . . . . . . .    0.9993
Tucker-Lewis Coefficient TLI. . . . . . . . . . .    0.9948
Bentler & Bonett's (1980) NFI . . . . . . . . . .    0.9992
Bentler's Comparative Fit Index CFI . . . . . . .    0.9993
Parsimonious NFI (James, Mulaik, & Brett,1982). .    0.1427
Bollen (1986) Normed Index Rho1 . . . . . . . . .    0.9942
Bollen (1988) Non-normed Index Delta2 . . . . . .    0.9993
(3) Information Criteria: ---------------------------------
Akaike's Information Criterion. . . . . . . . . .   20.9934
Bozdogan's (1987) CAIC. . . . . . . . . . . . . .   -5.2909
Schwarz's Bayesian Criterion. . . . . . . . . . .   -2.2909
(4) Other Fit Measures: -----------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .    0.9996
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .    0.1428
GFI Adjusted for Degrees of Freedom (AGFI). . . .    0.9959
```

```
Root Mean Square Residual (RMR) . . . . . . . . .     2.8867
Hoelter's (1983) Critical N . . . . . . . . . . .       5024


               Rotated Factor Loadings with Standard Errors
               --------------------------------------------

                             FAC_1                  FAC_2
          ID    13.832286  0.6006860  6.2238630  0.2906096
               [ 12.65496, 15.00961] [ 5.654279, 6.793447]

          DR    20.773764  0.9603427  5.0615724  0.3990759
               [ 18.89153, 22.65600] [ 4.279398, 5.843747]

          S7    8.6338812  0.2898870  18.367544  0.4274393
               [ 8.065713, 9.202049] [ 17.52978, 19.20531]

          BC    2.1987415  0.1733463  6.6514697  0.2423777
               [ 1.858989, 2.538494] [ 6.176418, 7.126521]

          NA    3.0198151  0.1302705  5.0364373  0.2173754
               [ 2.764490, 3.275141] [ 4.610389, 5.462485]

          DA    3.7812980  0.1460617  9.7032629  0.1809474
               [ 3.495022, 4.067574] [ 9.348613, 10.05791]

          VO    4.6803104  0.1775939  13.913024  0.4117647
               [ 4.332233, 5.028388] [ 13.10598, 14.72007]


                             FAC_3                  U_Var
          ID    0.2196889  0.2547574  112.13365  13.241422
               [-0.279626, 0.719004] [ 86.18094, 138.0864]

          DR    0.2346563  0.1131624  40.072776  35.887195
               [ 0.012862, 0.456451] [-30.26483, 110.4104]

          S7    2.2157410  0.3931470  745.99176  10.725870
               [ 1.445187, 2.986295] [ 724.9694, 767.0141]

          BC    4.0870160  0.4108408  379.43008  4.9970858
               [ 3.281783, 4.892249] [ 369.6360, 389.2242]

          NA    5.0501879  0.4741234  144.50192  4.3968393
               [ 4.120923, 5.979453] [ 135.8843, 153.1196]
```

```
DA   2.3019830  0.4085772  165.14046  2.6474320
    [ 1.501186, 3.102780] [ 159.9516, 170.3293]

VO  -4.6034008  0.8544240  212.77823  15.388701
    [-6.278041,-2.928761] [ 182.6169, 242.9395]
```

The best solutions for simple structure are the following:

```
**********************************
Table of 20 Best Pattern Solutions
**********************************
```

| N | Npar | DF | Chisquared | P |
|---|---|---|---|---|
| 1 | 18 | 10 | 1.05681274 | 0.99977835 |
| 2 | 19 | 9 | 0.82973802 | 0.99973986 |
| 3 | 20 | 8 | 0.60803423 | 0.99972055 |
| 4 | 20 | 8 | 0.64736686 | 0.99964647 |
| 5 | 20 | 8 | 0.64737613 | 0.99964645 |
| 6 | 19 | 9 | 0.89844351 | 0.99963807 |
| 7 | 20 | 8 | 0.66900373 | 0.99960022 |
| 8 | 19 | 9 | 0.95283616 | 0.99953872 |
| 9 | 20 | 8 | 0.71358863 | 0.99949155 |
| 10 | 21 | 7 | 0.50743232 | 0.99941881 |
| 11 | 20 | 8 | 0.75414496 | 0.99937580 |
| 12 | 21 | 7 | 0.51960765 | 0.99937147 |
| 13 | 20 | 8 | 0.76623130 | 0.99933798 |
| 14 | 19 | 9 | 1.04468144 | 0.99932744 |
| 15 | 20 | 8 | 0.78024974 | 0.99929212 |
| 16 | 22 | 6 | 0.34907179 | 0.99922214 |
| 17 | 21 | 7 | 0.56105438 | 0.99919080 |
| 18 | 22 | 6 | 0.35450764 | 0.99918687 |
| 19 | 21 | 7 | 0.57594440 | 0.99911817 |
| 20 | 21 | 7 | 0.59480211 | 0.99902003 |

| N | RMSEA | ECVI | SRMR | PGFI | SBC |
|---|---|---|---|---|---|
| 1 | 0 | 0.22565646 | 0.01073546 | 0.47537045 | -50.4761032 |
| 2 | 0 | 0.23653138 | 0.00871209 | 0.42798995 | -45.5498863 |
| 3 | 0 | 0.24743752 | 0.00675025 | 0.38057055 | -40.6182985 |
| 4 | 0 | 0.24766620 | 0.00736474 | 0.38054458 | -40.5789659 |
| 5 | 0 | 0.24766625 | 0.00736282 | 0.38054460 | -40.5789566 |
| 6 | 0 | 0.23693083 | 0.00961663 | 0.42793966 | -45.4811808 |
| 7 | 0 | 0.24779200 | 0.00801976 | 0.38053456 | -40.5573290 |

```
 8          0  0.23724706  0.00984211  0.42789788 -45.4267882
 9          0  0.24805121  0.00777729  0.38050760 -40.5127441
10          0  0.25904775  0.00621947  0.33305397 -35.5656088
11          0  0.24828700  0.00836732  0.38047915 -40.4721878
12          0  0.25911854  0.00651920  0.33304843 -35.5534335
13          0  0.24835727  0.00931404  0.38047243 -40.4601015
14          0  0.23778105  0.01058736  0.42783979 -45.3349429
15          0  0.24843877  0.00846600  0.38046371 -40.4460830
16          0  0.27032217  0.00575254  0.28554832 -30.5706778
17          0  0.25935951  0.00713220  0.33302723 -35.5119868
18          0  0.27035377  0.00576968  0.28554580 -30.5652419
19          0  0.25944607  0.00716497  0.33301646 -35.4970968
20          0  0.25955571  0.00805226  0.33300380 -35.4782391
```

```
Loadings and Uni. Vars. with ASE's and Wald CIs
-----------------------------------------------

                   FAC_1                 FAC_2
ID   16.719466  0.1316275          0         0
    [ 16.46148, 16.97745] [        0,        0]

DR   19.073644  0.1598069          0         0
    [ 18.76043, 19.38686] [        0,        0]

S7           0          0  20.759541  0.2732631
    [        0,         0] [ 20.22395, 21.29513]

BC           0          0  7.2883277  0.1923605
    [        0,         0] [ 6.911308, 7.665347]

NA           0          0  6.6592971  0.1266958
    [        0,         0] [ 6.410978, 6.907616]

DA           0          0  10.494758  0.1333456
    [        0,         0] [ 10.23341, 10.75611]

VO           0          0  13.614394  0.1764118
    [        0,         0] [ 13.26863, 13.96015]


                   FAC_3                 U_Var
ID           0          0  62.709453  2.6032825
    [        0,         0] [ 57.60711, 67.81179]

DR           0          0  133.48609  3.5727845
```

```
    [        0,        0] [ 126.4836, 140.4886]

S7          0          0  731.85147  9.8153664
    [        0,        0] [ 712.6137, 751.0892]

BC   2.6039975  0.3235364  385.30948  4.7548129
    [ 1.969878, 3.238117] [ 375.9902, 394.6287]

NA   4.6515370  0.6099560  138.50697  5.7785682
    [ 3.456045, 5.847029] [ 127.1812, 149.8328]

DA          0          0  168.75005  2.3308522
    [        0,        0] [ 164.1817, 173.3184]

VO  -6.3256458  0.9098426  224.07448  12.727777
    [-8.108905,-4.542387] [ 199.1285, 249.0205]


              Factor Correlations Phi
              -----------------------

                       FAC_1                FAC_2
FAC_1   1.0000000          0  0.6571400  0.0066259
       [ 1.000000, 1.000000] [ 0.644153, 0.670127]

FAC_2   0.6571400  0.0066259  1.0000000          0
       [ 0.644153, 0.670127] [ 1.000000, 1.000000]

FAC_3          0          0          0          0
       [        0,        0] [        0,        0]

                                 FAC_3
           FAC_1           0         0
                  [        0,        0]

           FAC_2           0         0
                  [        0,        0]

           FAC_3   1.0000000         0
                  [ 1.000000, 1.000000]


(1) Standalone Fit Measures: ------------------------------
Fit criterion . . . . . . . . . . . . . . . . . .    0.0061
Normal Th. Chi-square (df = 10)       106.6029  prob= 0.0000
Normal Theory Reweighted LS Chi-square  . . . . .  104.7526
```

```
Probability of Close Fit . . . . . . . . . . . .     1.0000
Z-Test of Wilson & Hilferty (1931). . . . . . . .    8.2046
(2) Incremental Fit Measures: -----------------------------
Null Model Chi-square (df = 21) . . . . . . . .  32421.2870
RMSEA Estimate  . . . . .  0.0236  90%C.I.[ 0.0197,  0.0277]
ECVI Estimate . . . . . .  0.0082  90%C.I.[ 0.0065,  0.0104]
McDonald's (1989) Centrality. . . . . . . . . . .    0.9972
Tucker-Lewis Coefficient TLI. . . . . . . . . . .    0.9937
Bentler & Bonett's (1980) NFI . . . . . . . . . .    0.9967
Bentler's Comparative Fit Index CFI . . . . . . .    0.9970
Parsimonious NFI (James, Mulaik, & Brett,1982). .    0.4746
Bollen (1986) Normed Index Rho1 . . . . . . . . .    0.9931
Bollen (1988) Non-normed Index Delta2 . . . . . .    0.9970
(3) Information Criteria: ---------------------------------
Akaike's Information Criterion. . . . . . . . . .   86.6029
Bozdogan's (1987) CAIC. . . . . . . . . . . . . .   -1.0111
Schwarz's Bayesian Criterion. . . . . . . . . . .    8.9889
(4) Other Fit Measures: -----------------------------------
Goodness of Fit Index (GFI) . . . . . . . . . . .    0.9983
Parsimonious GFI (Mulaik, 1989) . . . . . . . . .    0.4754
GFI Adjusted for Degrees of Freedom (AGFI). . . .    0.9952
Root Mean Square Residual (RMR) . . . . . . . . .    5.3527
Hoelter's (1983) Critical N . . . . . . . . . . .      2981
```

Standardized Factor Loadings

| | FAC_1 | FAC_2 | FAC_3 |
|---|---|---|---|
| ID | 0.9037553 | 0 | 0 |
| DR | 0.8553200 | 0 | 0 |
| S7 | 0 | 0.6087781 | 0 |
| BC | 0 | 0.3454182 | 0.1234089 |
| NA | 0 | 0.4656885 | 0.3252822 |
| DA | 0 | 0.6284330 | 0 |
| VO | 0 | 0.6421882 | -0.2983861 |

Squared Multiple Correlations

| N | Variable | ErrVariance | TotVariance | R-squared |
|---|---|---|---|---|
| 1 | ID | 62.7091975 | 342.250000 | 0.81677371 |
| 2 | DR | 133.486425 | 497.290000 | 0.73157227 |
| 3 | S7 | 731.860053 | 1162.81000 | 0.37061080 |
| 4 | BC | 385.309895 | 445.210002 | 0.13454349 |
| 5 | NA | 138.506337 | 204.490003 | 0.32267429 |
| 6 | DA | 168.748525 | 278.890000 | 0.39492802 |

```
7 VO        224.072829  449.440043  0.50144000
```

There are other local maxima with different zero patterns in **P** when optimizing for $N = 173$.
However, this nice simple structure which maximizes the $p$ value for $N = 173$ would not maximize the $p$ value for $N = 17531$. When using $N = 17531$ this zero pattern gives $p = 2.6e - 18$ which is even worse than the $p = 5.9e - 006$ for EFA.
Maybe one of the loadings L43, L53, or L73 could be set to zero? If we set only L73 to zero we obtain $p = 2.9e - 63$ and if we set both L53 and L73 to zero we obtain $p = 5.2e - 78$. Our statement, that the $p$ optimal CFA structure is very robust when the number of observations is reduced is contradicted by this example.
On the other side, when optimizing the $p$ value for a $\chi^2$ value with $N = 17531$ we would be able to obtain a slightly better $p$ value that that of the EFA solution, but only by generating one additional zero loading. That means the $p$ optimal pattern for the large $N = 17531$ is almost as dense as EFA. What that means is, that for such large $N$ an optimization for maximum $p$ may not get very sparse factor loadings patterns. And the reason is probably, that for the large $\chi^2$ the small increase of the degrees of freedom by introducing a few zero loadings does not contribute to much increase of the $p$ value.

## 3 The Bibliography

## References


[1] Bentler, P.M. (1989): *EQS, Structural Equations, Program Manual*, Program Version 3.0, Los Angeles: BMDP Statistical Software, Inc.

[2] Harman, H.H. (1976), *Modern Factor Analysis*, Chicago: University of Chicago Press.

[3] Jöreskog, K.G. (1978), "Structural Analysis of Covariance and Correlation Matrices", *Psychometrika*, **43**, 443-477.

[4] Jöreskog, K.G. & Sörbom, D. (1988), *LISREL 7: A Guide to the Program and Applications*, SPSS Inc., Chicago, Illinois.

[5] Maydeu-Olivares, A., Garcia-Forero, C., Gallardo-Pujol, D., & Renom, J. (2009), "Testing categorized bivariate normality with two-stage polychoric correlation estimates", *Methodology*, **5(4)**, p. 131-136.

[6] McArdle, J.J. (2011), " Some Ethical Issues in Factor Analysis", in: A.T. Panter & S.K. Sterba (eds.): *Handbook of Ethics in Quantitative Methodology*, Routledge.

[7] McDonald, R.P. (1985), *Factor Analysis and Related Methods*, Hillsdale NJ and London: Lawrence Erlbaum Associates.

[8] *SAS/STAT User's Guide*, (1990), Version 6, Second Printing, SAS Institute Inc., Cary, NC.

[9] Thurstone, L.L. (1931), "Multiple factor analysis"; *Psych. Rev.*, **38**, 406-427.